Institutions and Equilibrium in the United States Supreme Court*

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Abstract

Over the last decade the scholarship on judicial politics has increasingly emphasized the strategic aspects of decision-making in the United States Supreme Court. This scholarship, however, has struggled with two significant limitations—the restriction to unidimensional policy spaces and the assumption of binary comparisons of alternatives. These two assumptions have the advantage of implying stable, predictable outcomes, but have the disadvantages of lacking a theoretical foundation and potentially assuming away important aspects of strategic maneuvering on the Court. In this article, we identify institutional features of the Court that allow us to relax these two assumptions without sacrificing stable, predictable policy outcomes. In particular, we incorporate the “part-by-part” opinion voting used by the justices, a feature that has not appeared in previous models of the Court and leads to a stable, predictable policy outcome at the issue-by-issue median of the justices.
1 Introduction

Over the last decade the social science literature on judicial politics has increasingly emphasized the strategic aspects of decision-making on the United States Supreme Court. The literature on strategic Supreme Court decision-making is extensive, with prominent examples covering the study of opinion assignments (Maltzman and Wahlbeck 1996a), coalition formation (Wahlbeck, Spriggs, and Maltzman 1998), certiorari (Caldeira, Wright, and Zorn 1999), hierarchical relationships between the higher and lower courts (Lax 2003; Cameron, Segal, and Songer 2000; Songer, Segal, and Cameron 1994) as well as a handful of books incorporating most or all of these stages (Hammond, Bonneau, and Sheehan 2005; Epstein and Knight 1998; Maltzman, Spriggs, and Wahlbeck 2000). The move toward strategic accounts is so significant that two prominent scholars even talk of a “strategic revolution” in the field of judicial politics (Epstein and Knight 2000).

The literature of the strategic revolution, however, still has not confronted two theoretically troublesome assumptions that underlie virtually all models of the Court’s policymaking process. The first is the assumption that Supreme Court decision-making takes place in a one-dimensional policy space. That is, the justices are restricted to choose a policy along a single issue dimension, an assumption that (together with single-peaked utility functions) ensures the social transitivity of the justices’ preferences. The second is the assumption that justices face only binary choices over alternative policies. That is, the justices may face a choice of whether to reverse or affirm a lower court, or whether to endorse a majority opinion or a single alternative opinion, but never face a simultaneous choice among three or more competing opinion proposals. The (often implicit) assumption is that the Supreme Court employs some procedure for pairing alternative opinions in an agenda, such as an amendment procedure in Congress.\footnote{In fact, each of these assumptions is analogous to an assumption in the literature on Congress, where arguments about structure-induced stability are now nearly thirty years old (see, e.g., Shepsle 1979; Shepsle...}

The twin assumptions of unidimensionality and binary comparisons of alternatives have considerable an-
alytical advantages. Together, these assumptions produce median-voter-type results that allow for tractable models and stable, predictable outcomes. In particular, these assumptions preclude cycling over alternatives or “chaos” results of the type identified in Arrow (1963) and McKelvey (1976). In contrast, relaxing these assumptions leads to social intransitivity (in the case of unidimensionality) and loss of the median voter result (in the case of binary comparison of alternatives). As a result, virtually all rational-choice models of Supreme Court decision-making restrict their assumptions to unidimensional policy spaces (e.g., Hammond, Bonneau, and Sheehan 2005; Moraski and Shipan 1999; Segal 1997) or to treat the Court as a unitary actor in a single- or multi-dimensional space (Gely and Spiller 1990; Spiller and Gely 1992; Rogers 2001; Spiller and Spitzer 1992; Bailey, Kamoie, and Maltzman 2005).

The question that has not been resolved is whether these assumptions, while not specifically tailored to the Supreme Court’s institutional features, nonetheless provide a reasonable working approximation of the institutional environment of Supreme Court justices’ decision-making.

The potential problems associated with these assumptions in the context of the Supreme Court are significant. The first problem is that the literature on the Court has not worked out any solid theoretical basis for these assumptions. In the case of both unidimensional policy spaces and binary comparison of alternatives, the principal reason for the assumption is analytical convenience rather than a theory about the Court as an institution. The second problem is that in both cases, part of the reason for the assumption is that strategic maneuvering causes the median voter model to break down. But this strategic maneuvering is the very feature that the strategic account of Supreme Court decision-making should aspire to explain. The addition of alternatives and dimensions is one of the most important ways of strategically manipulating the

The rare exceptions incorporating more than one dimension include Schwartz (1992), whose second dimension is not a “policy” dimension but rather a level of “precedent.” Similarly, in Cohen and Spitzer (1994), the second dimension is not policy but “deference.”
outcome of a political situation (e.g., Riker 1986). Thus, without a coherent theoretical account supporting these assumptions, we not only have little confidence in a model’s results, but also assume away much of what the strategic account should strive to explain.

In this paper, we develop a model of the Supreme Court that predicts stable, unique policy outcomes without the confining assumptions of unidimensional policy spaces or binary comparisons of policy alternatives. Our model incorporates institutional features that are specific to the Supreme Court and, to our knowledge, have not previously appeared in other models of judicial decision-making. In particular, we focus on the practice of “part-by-part voting” that has evolved in the Supreme Court, which allows a justice to vote for individual parts of an opinion without voting for the opinion as a whole. This institutionalized opinion-voting procedure, when coupled with separable preferences over judicial policy, implies a stable, unique outcome at the issue-by-issue median of the justices’ ideal points. Our result has important implications for many areas of active empirical and theoretical research on the Court. On the one hand, our model formally establishes what most strategic models of the Supreme Court assume—that the Supreme Court’s decision-making often can be reasonably represented in a single-dimensional policy space. On the other hand, however, our model suggests that Supreme Court policy-making behavior is significantly closer to sincere behavior than some of the strategic choice models might suggest. Thus, in this sense our model is a foundational challenge to much of the modern strategic literature of Supreme Court policymaking.

We develop our argument in this paper as follows. In Part 2, we introduce our assumptions and notation and describe some institutional features of the Supreme Court’s policymaking process, particularly the “part-by-part voting” used in the Court. In Part 3, we relax the unidimensionality assumption in the literature and demonstrate how sincere part-by-part voting leads to a stable equilibrium in multidimensional space. In Part 4, we also relax the assumption of binary comparisons of alternatives, showing how a fully strategic model of Supreme Court policymaking leads to a unique equilibrium at the issue-by-issue median of the justices. In Part 5, we describe some of the implications and extensions of our theory, including the interface of our
theory with empirical results. Part 6 concludes with our overall lesson: that an accurate strategic account of Supreme Court policymaking is actually fully consistent with sincere behavior on the part of the justices.

2 Institutional Background, Notation, and Assumptions

The starting point for our analysis is the same as for the dominant attitudinal and strategic models of Supreme Court decision-making—that the justices are primarily interested in setting legal policy, not merely in resolving legal disputes among individual parties. The courts scholarship has recognized that the Court is motivated by policy at least as far back as Pritchett (1948), a perspective that gained momentum with the rise of the attitudinal approach (Schubert 1965; Segal and Spaeth 1993) and the strategic approach (Epstein and Knight 1998; Maltzman, Spriggs, and Wahlbeck 2000). While these competing approaches differ in their assumptions about how justices achieve their policy ends, they all agree on one basic idea that distinguishes them from traditional “legal” approaches: that policy is the motivating goal of the justices.

Our model, like the attitudinal and strategic approaches, assumes that policy is the motivating goal of the justices and that the Court makes this policy primarily through its opinions (e.g., Segal and Spaeth 2002, 357), rather than through its judgments in individual cases. The key point sometimes overlooked in political science literature is that the “opinions matter” (Maltzman, Spriggs, and Wahlbeck 2000; Hammond, Bonneau, and Sheehan 2005; Rohde 1972), since they carry most of the ideological content of the justices’ decisions. As a result, the justices seek to craft opinions that reflect their policy preferences (Maltzman, 2000).

3 The key observation here is that in the attitudinal and strategic models, justices are interested in advancing their policy preferences, not in deciding cases to satisfy purely legal considerations or in pleasing some constituency. While it is certainly true that judging is on occasion about “the common search for right answers” or “power struggles among personalities” (Levmore 2002), consistently with the attitudinal and strategic approaches, we model policy preferences as the dominant consideration in deciding cases.
Spriggs, and Wahlbeck 2000, 17, 93), preferably embodying those preferred policy positions in the majority opinion of the Court. Thus, again our approach parallels the mainstream position of both the attitudinal and the strategic models of Supreme Court decision-making; the justices have preferences over policy and that they attempt to embody that policy in law through their opinions.

The key difference between our model and previous models of the Court is that we relax two confining assumptions that underlie other models. The first is the assumption that Supreme Court policymaking takes place in a single-dimensional issue space. In most theoretical work, this unidimensional policy space is interpreted as a left-right ideological dimension, and often justified on the basis of empirical studies suggesting that a single dimension can account for a significant proportion of the Court’s voting outcomes (e.g., Grofman and Brazill 2002). There are several significant problems, however, with this justification of a unidimensional policy space. First, while a single dimension may explain much of the variance in voting, there is always a significant residuum not explained, and it only takes one additional dimension to completely destabilize a unidimensional model. Second, and perhaps most importantly, relying on the empirical finding of unidimensionality amounts to little more than observing that the Plott (1967) conditions are satisfied for some specific group of justices. This approach means that the empirical justification for a unidimensional model may have some credibility as a model of specific set of justices, but not of the Supreme Court as an institution.

The second assumption we relax is the restriction of the “agenda” of the Court to binary comparisons of

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4 Another interpretation to which we are more sympathetic is that each case sometimes represents a single dimension (Hammond, Bonneau, and Sheehan 2005). This assumption has the virtue of having some institutional foundation given the Court’s practice of granting certiorari to resolve particular issues. The problem remains, however, that legal disputes are often multidimensional, even in the context of a single case (see, e.g., Easterbrook 1982). Thus, to achieve any satisfactory level of generality, theoretical work must confront the problem of multiple dimensions.
policy alternatives. This assumption takes several forms, but in many cases (especially in empirical work), models make this assumption by modeling the Court’s binary affirm/reverse decision rather than modeling the opinion that the Court adopts. More recent work has advanced the state of the literature by explicitly formalizing and modeling the opinion-writing process (e.g., Hammond, Bonneau, and Sheehan 2005, 93), but even in this work the assumption is maintained that only one alternative can be offered at a time. The assumption of binary comparison of alternatives makes modeling much more convenient, but has little basis in any actual institutional practice or procedure of the Supreme Court. In the Court, the Chief Justice (or the Senior Associate Justice) has the right to assign (or propose) the initial majority opinion in the case. However, once the original draft is circulated, any justice is free to propose an opinion at any time, even when two or more alternatives are already under consideration along the same policy dimensions. Unlike Congress, the Court appears to have no rules or procedures to structure multiple policy alternatives into an agenda of binary decisions. Thus, in contrast to members of Congress, the justices may face a simultaneous choice over many, not just two, competing policy alternatives.

The combination of unidimensionality and binary comparison of alternatives has the felicitous effect of leading to stable outcomes and analytical tractability. These restrictions, however, essentially amount to implicitly imposing a particular institutional structure on the Court that was largely imported from the scholarship on the United States Congress. In fact, these assumptions resemble rather closely the “institutional arrangements” that Shepsle (1979) identified as contributing to “structure-induced equilibrium” in legislative bodies. As suggested above, the problem with applying those assumptions to the Supreme Court is that, unlike in Congress, there is no evidence that the Court uses any type of “committee system,” “jurisdictional arrangement,” or “amendment control rule” to structure the justices’ alternatives. The result is that, as many authors have pointed out, the Court potentially faces the problems of instability and cycling, just like any other pure majority rule institution (e.g., Easterbrook 1982).
2.1 The Institution of Part-by-Part Voting

The solution to the multidimensional dilemma of the Court is found in a previously unstudied but very important institutional feature of the Court’s opinion voting procedure. In each case, each justice must make a decision about what opinions, if any, the justice will write and which other opinions, if any, the justice will “join” or agree to. When a justice joins an opinion, the justice indicates assent to the rationale of that opinion, which is equivalent to “voting” for the policy point expressed in the opinion. An opinion commanding the assent of a majority of the justices becomes law. But the justices need not join an opinion in full; the justices can “concur in part,” joining only specified portions of an opinion while disagreeing with the remainder. This feature allows a justice to vote for the policy positions of a proposed opinion the justice agrees with, but not policy positions of the opinion the justice does not agree with. Only those opinions or parts of opinions that receive the assent of a majority of the justices constitute the “opinion of the Court.” We refer to this institutional feature as “part-by-part voting.”

The justices actually have a rather wide range of options on opinion voting in individual cases, corresponding to any logical combination with respect to the judgment in the case and the opinion in the case. The available choices, in decreasing order of agreement with the majority’s reasoning, are: (1) join the majority opinion, (2) join the majority opinion and write a regular concurrence, (3) concur in part and concur in the judgment, (4) concur in part and dissent in part, (5) concur in part, concur in the judgment in part.

\[\text{\underline{5}}\]We use this terminology to distinguish “part-by-part voting” from “issue-by-issue voting,” which has multiple meanings in political science, none of which is exactly what we mean here. The two concepts are related, but the main difference is that in issue-by-issue voting, some institutional arrangement allows proposals to change the policy only along one dimension at a time. In part-by-part voting, proposals can alter multiple dimensions, but voters can vote on the dimensions separately (but simultaneously). The notion that something like issue-by-issue voting might provide a solution to majority-rule instability on the Court was anticipated in Hammond, Bonneau, and Sheehan (2005, 263–265).
and dissent in part, (6) concur in the judgment, (7) concur in the judgment in part and dissent in part, or (8) dissent. The possibilities are illustrated in Table 1.

The alternatives in this table that are familiar to most readers are those in the upper-left-hand cell (joining the majority opinion in full) and the lower-right-hand cell (dissenting from the majority opinion). The option in the upper-left-hand cell is sometimes accompanied by a “regular” concurrence, in which the justice joins the majority opinion in full but writes separately to emphasize additional points. The lower-right-hand option of dissenting gives the justice the option of joining one or more dissenting opinions written by other justices, or writing his or her own dissenting opinion. A justice who agrees with the result of a decision (e.g., to reverse or affirm) at least in part, but who disagrees with the majority’s reasoning can also write a “special concurrence,” which is denoted “concurring in the judgment” in reported opinions. This possibility of a “special concurrence,” combined with the institution of part-by-part voting, are the main features that are assumed away in other models of the Court. As described in more detail below, the availability of the “special concurrence” means that choices over policy are not necessarily binary, and the availability of part-by-part voting means that opinions are not “take-it-or-leave-it.”

The institutional feature of part-by-part voting allows justices to select the alternatives in the “Agree in Part” column of Table 1. The justices need not join entire opinions; they can join parts and subparts of opinions, which are typically designated by roman numerals and letters. Thus, a justice may, for example, join all of an opinion except “Part II,” “Part III-A,” or “footnote 4.” Indeed, justices may exclude certain specified portions of an opinion, even when those portions are not separated out into parts. For example, Justice Scalia will sometimes join an opinion excepting all portions discussing “legislative history.” The justices can even narrow the Court’s policy by excluding substantive issue areas (e.g., Turner Broadcasting System v. FCC (1977) (joining the opinion “except insofar as Part II-A1 relied on an anticompetitive
part-by-part voting of the justices is often quite intricate, with several justices concurring in part in various portions of their colleagues’ opinions, both concurring and dissenting.

We conceive of this part-by-part voting opinion voting of the Supreme Court in terms of a spatial model. Each opinion constitutes a point in multidimensional policy space over which the justices vote. In contrast to the standard set-up with majority voting with binary alternatives, part-by-part voting allows the justices to accept or reject individual dimensions of each opinion proposal. In effect, by joining an opinion or a part of an opinion, the justice “votes” for the proposed point along each policy dimension in the opinion (or part of opinion) he joined. This means that when an opinion point has \( d \) dimensions, each justice has \( 2^d \) possible votes with respect to that opinion. The justices are able to vote separately on each dimension by joining only those parts of each opinion they agree with.

For example, in *Lorillard Tobacco v. Reilly* (2001), the syllabus of the decision reports the following fractured pattern agreement for Justice O’Connor’s opinion, “O’Connor, J., delivered the opinion of the Court, Parts I, II-C, and II-D of which were unanimous; Parts III-A, III-C, and III-D of which were joined by Rehnquist, C. J., and Scalia, Kennedy, Souter, and Thomas, JJ.; Part III-B-1 of which was joined by Rehnquist, C. J., and Stevens, Souter, Ginsburg, and Breyer, JJ.; and Parts II-A, II-B, III-B-2, and IV of which were joined by Rehnquist, C. J., and Scalia, Kennedy, and Thomas, JJ.”

Even if the portion of an opinion a justice disagrees with is not separated out into a separate textual “part,” the justice can exclude that portion of the opinion by describing it. Thus, even if some sentences or phrases are inherently multidimensional, the justices can still join or reject those sentences or phrases on a single-dimensional level by describing the part the justice does not join. An illustrative example is portrayed in the syllabus for *Adarand Constructors v. Pena* (1995), in which Justice O’Connor’s opinion was the opinion of the Court “except insofar as it might be inconsistent with the views expressed in the concurrence of Scalia.”
The institution of part-by-part voting has attracted almost no scholarly attention in either the legal or the political science literatures. Most strategic models of the Court ignore the possibility of more than two opinions, much less the possibility of part-by-part voting. And even those scholars who discuss the possibility of multiple opinions typically portray the justices as having only four exhaustive choices, such as “join the majority opinion, join or author a regular concurrence, join or author a special concurrence, or join or author a dissent” (Wahlbeck, Spriggs, and Maltzman 1999, 490). We believe that the institution of part-by-part voting implies that the justices have a much richer set of alternatives than the existing literature has recognized. Indeed, we argue that the institution of part-by-part voting, which has been overlooked by the political science literature, is the very feature that produces stable outcomes in a multidimensional policy space.

2.2 Notation and Assumptions

We now introduce some of the assumptions and notation we use in the remainder of this article. We assume a set of justices with single-peaked, separable preferences over the $d$-dimensional Euclidean policy space $\mathbb{R}^d$. The assumption of separable preferences is a common (though typically implicit) feature in multidimensional spatial models. This assumption is particularly apt in the context of legal decision-making at the Supreme Court level, where justices set legal policy at the apex of a hierarchy of lower courts.

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9 One notable exception is Delson (2001). This insightful student law review note traces the history of a voting protocol it calls “voting by part,” which corresponds to the “part-by-part voting” described in this article.

10 Indeed, recent research shows that even beginning law students generally (although not always) express separable preferences over legal issues, even in the context of deciding a single case with no future consequences (Braman 2006). The distinction should be made between the separability of preferences over legal conclusions in individual cases and separability of preferences over legal policy in the Supreme Court.
Court justices are highly cognizant of the influence of their opinions over many future cases. Thus, the fortuitous combination of policy dimensions in particular cases is unlikely to influence preference over those issues.

We assume that each justice has perfect and complete information about the preferences of the other justices, the structure of the game, and so forth. The game begins with nature selecting a policy-making opportunity\textsuperscript{11} that involves policy issues in a $d$-dimensional subset of the policy space. The $d$ issues in the case are determined exogenously (for example, by the issues and cases for which certiorari was granted in a previous stage of the game). The structure of the policy-making game that ensues among the justices differs in Part 3 and Part 4. In Part 3, we examine the case where the Court has some agenda for pairwise comparisons of alternatives in a multidimensional space. The justices vote sincerely over the alternatives, but use part-by-part voting. In Part 4, we relax the assumption of a binary amendment agenda and consider the justices’ strategic behavior in a more unstructured environment. In both cases, part-by-part voting leads to a unique equilibrium outcome at the issue-by-issue median of the justices’ ideal points.

While judges may (and probably do) have non-separable preferences over legal conclusions in individual cases (e.g., a lower court preferring not to find jurisdiction to hear a case when the court disagrees with the plaintiff on substantive issues), the preferences of Supreme Court justices over legal policy is much more likely to be separable across issues. This is because each policy issue decided will likely arise in many future cases in conjunction with a variety of other issues in the future. Thus, while we believe that Supreme Court justices have separable preferences over policy issues, we would not be surprised if the same did not hold at the district or circuit court levels.

\textsuperscript{11}We specifically refer to the Court’s task as a “policy-making opportunity” rather than as a “case” because we need not assume that the various cases and issues being considered by the justices during the same time period are independent of one another. Even if individual cases were one-dimensional, the “policy-making opportunity” of the Court during the same time period could be multi-dimensional.
3 Non-Strategic Part-by-Part Voting With Binary Alternatives

We first illustrate part-by-part voting by relaxing the unidimensionality restriction on the policy space while maintaining sincere pairwise voting over binary policy alternatives. For ease of exposition, we illustrate the case where there are three voters (denoted J1, J2, and J3) in a two-dimensional space with Euclidean preferences (circular indifference contours). For this situation, Figure 1 depicts a standard configuration of preferences where no Condorcet winner exists. That is, for any point in the space, there is some other point that two of the three voters prefer to the original point. This is true even for the issue-by-issue median of the voters, denoted here by \( x_{\text{med}} \). Observe that the point \( y \) defeats \( x_{\text{med}} \) in a pairwise vote, since J2 and J3 prefer \( y \) to \( x_{\text{med}} \). In addition, the point \( z \) then defeats \( y \) in a pairwise vote, since J1 and J2 prefer \( z \) to \( y \). But then we come full circle in the cycle, since \( x_{\text{med}} \), the issue-by-issue median that was defeated in the first vote, would defeat \( z \) in a pairwise comparison. Thus, the social preference order over these three alternatives (and indeed over the whole policy space plane) is intransitive (McKelvey 1976).

We next consider the case of part-by-part voting as practiced in the Supreme Court. Again consider the same three voters (whom we now call justices) with identical ideal points in the same two-dimensional policy space. Suppose the current opinion proposal (which may be thought of as the initial majority opinion or a status quo point as in Hammond, Bonneau, and Sheehan (2005)) were located at \( x_{\text{med}} \), and either J2 or J3 wrote an opinion corresponding to point \( y \). Both J2 and J3 prefer the point \( y \) to \( x_{\text{med}} \), as in the discussion above of regular majority voting. However, J2 prefers the part of \( y \) that moves along the vertical dimension to \( x_{\text{med}} \), but not the part of \( y \) that moves along the horizontal dimension of \( x_{\text{med}} \). In contrast, J3 prefers the part of \( y \) that moves along the horizontal dimension from \( x_{\text{med}} \), but not the part of \( y \) that moves along the vertical dimension from \( x_{\text{med}} \). J1 does not prefer either part of opinion \( y \) to \( x_{\text{med}} \). The result is as follows.

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\[12\] A Condorcet winner is defined as a point \( x \) such that for all other points \( y \) the number of voters who strictly prefer \( x \) to \( y \) exceeds the number of voters who strictly prefer \( y \) to \( x \) (Oreshek 1986, 76).
J1 will not vote for either part of \( y \), but rather will continue to vote for \( x_{\text{med}} \). J2 will vote for the part of \( y \) that moves along the vertical dimension but not the part of \( y \) that moves along the horizontal dimension. J3 will vote for the part of \( y \) that moves along the horizontal dimension, but not the part of \( y \) that moves along the vertical dimension. The result is that \( x_{\text{med}} \) remains the majority outcome with two votes for the horizontal portion of \( x_{\text{med}} \) and two votes for the vertical portion of \( x_{\text{med}} \).

[Figure 1 about here.]

The illustration above is a simplified version of how part-by-part voting works, but it leads to a general result for sincere voting over an agenda of binary alternatives. For any configuration of preferences, the issue-by-issue median will defeat any other point in the policy space under part-by-part voting. Thus, we have the following proposition (all proofs are collected in Appendix A):

**Proposition 3.1.** If justices have convex, separable preferences over the issues in the policy space and always vote sincerely, the median along each issue dimension will defeat any other point in the policy space in part-by-part voting.

Thus, the issue-by-issue median occupies a status in part-by-part voting similar to that of a Condorcet winner in pure majority rule. This fact implies a second proposition:

**Proposition 3.2.** Given the assumptions of Proposition 3.1, if the issue-by-issue median is included in any agenda, the outcome of voting over the agenda will be the issue-by-issue median.

The propositions introduced above illustrate how part-by-part voting extricates the Court from the instability of pure majority rule with sincere voting. In this illustration, even if some agenda setter were empowered to pair opinion alternatives in an agenda but provided that the justices were not limited in their ability to make proposals, the outcome would converge on the issue-by-issue median. In the next section, we show that this same result occurs with strategic voting, even without a restriction to binary comparisons of alternatives.
4 Strategic Part-by-Part Voting

In this part, we relax both the constraint of unidimensionality and of binary comparison of alternatives to model Supreme Court policymaking in a fully strategic multidimensional environment. The Supreme Court has a collegial atmosphere and apparently few formal rules. In particular, there is no restriction on which justices may propose alternatives (special concurrences or dissents) and there is no amendment rule for considering these alternatives. Thus, we believe that the assumption of an agenda process for binary comparisons of multiple alternatives (as assumed in other models of the Court) is not an approximation of the Court’s institutional features, but rather a foreign institutional structure that could distort our understanding of the decision-making process. Furthermore, the justices may (and do) communicate and coordinate in writing opinions, and our model should be consistent with this feature of the Court. Thus, we develop a relatively simple model that has a minimal institutional structure and allows for communication and coordination among the justices. We allow any justice to propose an opinion along any or all dimensions of the policy space, and we allow justices to freely communicate and make non-binding commitments on how to vote.

4.1 Coalition-Proof Nash Equilibrium

Our model, as is the case with virtually any voting model, has multiple Nash equilibria, including many that are Pareto dominated. Obviously, many Nash equilibria (for example, one where everyone unanimously votes for a Pareto dominated outcome) seem implausible, particularly where the voters can communicate and coordinate their strategies, as can the justices in the Supreme Court. Thus, we adopt an equilibrium refinement—coalition proof Nash equilibrium (Bernheim, Peleg, and Whinston 1987)—that captures these aspects of free communication and coordination. The concept of coalition-proof Nash equilibrium is par-

\[13\text{These same problems that occur with Nash equilibria in voting games also tend to occur with equilibrium refinements that are not aimed at capturing communication or coalition formation.}\]
particularly well suited to the Supreme Court’s collegial deliberative environment, as this solution concept is explicitly designed “for strategic environments in which players can freely discuss their strategies, but cannot make binding commitments” (Bernheim and Whinston 1987, 13). In this section, we describe in an intuitive fashion the refinement of coalition-proof Nash equilibrium. The formal definition is provided in Appendix A on page 32 and readers are referred to Bernheim, Peleg, and Whinston (1987) for more details.

The concept of coalition-proof Nash equilibrium is an equilibrium refinement that requires equilibrium strategies to have the property that no coalition of the players can deviate from the equilibrium strategy profile in a self-enforcing manner. This means that coalition-proof strategy profiles, as well as any self-enforcing deviations from non-coalition proof strategies, have the property that the members of the coalitions have incentives not to defect from those strategies. This means that in a coalition-proof Nash equilibrium, no subset of the players in the game can deviate from the equilibrium in such a way that they all improve their position, unless some further subset could deviate from that deviation in a self-enforcing manner, and so forth. The definition is therefore a recursive one that tests each sub-coalition, each sub-sub-coalition, and so on, for incentives to defect from the equilibrium strategy.

The main alternative equilibrium concept that allows for communication and coalition formation is a “strong equilibrium.” Like coalition-proof Nash equilibria, strong equilibria have the advantage over Nash equilibria of allowing coalitions of players to coordinate on a deviation that will make them all better off. However, in a multidimensional setting, strong equilibria only exist under very special configurations.

\[14\text{We do not consider the case where justices were allowed to make binding commitments in this model. However, we note that if justices could make binding commitments (for example, through a reputation mechanism or repeat play), the two most basic results of our model would still hold. The justices would reach a stable policy outcome (through their commitment mechanism) and agenda control would be irrelevant to the policy outcome. The main difference would be that the outcome might not be the issue-by-issue median.}\]
of preferences. Thus, a strong equilibrium would require us to make restrictive assumptions about the distribution of preferences, which would limit the generality of our model.

Moreover, because justices cannot form binding agreements, it also strikes us as unrealistic to think that a justice would be willing to change his vote knowing that those with whom he or she is forming a coalition will defect in a way that makes the justice’s own vote change lead to a worse outcome. Unlike a strong equilibrium, a coalition-proof Nash equilibrium tests coalitional deviations for their own self-enforcing character. As a non-cooperative game theory refinement, a coalition-proof Nash equilibrium is “designed to capture the notion of an efficient self-enforcing agreement for environments with unlimited, but nonbinding, pre-play communication” (Bernheim, Peleg, and Whinston 1987). We believe this equilibrium concept is uniquely suited for the Supreme Court’s bargaining process, and captures much of what other models leave out: communication and coalitions in a non-cooperative game-theoretic environment.

### 4.2 Majority Opinions, Plurality Opinions, and the Status Quo Policy

The fact that our model allows multiple policy proposals along a single issue dimension raises the possibility of plurality opinions. As is common in the literature, we assume that if a particular point along a given dimension receives a majority of the votes (that is, \( \frac{n+1}{2} \) or more), that point becomes the policy of the

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\(^{15}\)In a multidimensional police space, unless the Plott conditions are satisfied (Plott 1967), there always exists a point in the policy space which a majority of voters prefer and can obtain. Thus, there always exists a coalition which can deviate in a self-improving way. As a result, no strategy profile can be a strong equilibrium. In a unidimensional policy space, the unique outcome under all strong equilibria is also the median.

\(^{16}\)Because we do not believe that justices can make binding agreements, we believe that cooperative or coalitional concepts are inappropriate.
Court along that dimension. However, in the event that no policy point along a given dimension receives a majority of the votes (that is, \(
\frac{n-1}{2}
\) or fewer), there is no clear consensus in the literature on what policy prevails, and in fact, virtually no discussion of the matter. Thus, we devote some time in this section to our assumptions about what happens when there is no clear majority on each dimension.

There are several possibilities as to what happens when there is no majority opinion along a dimension. One obvious possibility is that the plurality position—the policy point with the largest number of votes—becomes the Court’s policy. One problem with this interpretation is that sometimes there are ties among plurality opinions (e.g., a 3-3 or 4-4 vote), but some further tie-breaking rule could conceivably be devised for that circumstance. Another possibility is that the relevant legal actors may treat the decision as setting no precedent at all; that is, some exogenous status quo point would prevail. This was at least formally the rule in the federal courts at one time and has been incorporated in some models of the Court (e.g., Hammond, Bonneau, and Sheehan 2005).

In our view, however, neither of these options withstands scrutiny from either a doctrinal perspective or a strategic perspective. From a doctrinal perspective, the Supreme Court rejected the view that non-majority opinions cannot set precedent in the case of Mark v. United States (1977). In that case, the Court wrote that “[w]hen a fragmented court decides a case and no single rationale explaining the result enjoys the assent

17Note that this is true even if some dissenters join that part of the opinion and those dissenters are necessary to give the policy a majority.

18This is probably because most modelers assume away the possibility of plurality opinions by restricting the Court to binary consideration of alternatives (even along a single dimension). This is a problem for models that assume binary comparisons since plurality opinions do occur, and arbitrarily eliminating this possibility for strategic maneuvering is unfortunate.

19See (United States v. Pink 1942) (“the lack of an agreement by a majority of the Court on the principles of law involved prevents it from being an authoritative determination for other cases”).
of five Justices, ‘the holding of the Court may be viewed as that position taken by those Members who concurred in the judgments on the narrowest grounds’” (Marks v. United States 1977, 193). The Supreme Court in Marks specifically rejected the lower court’s application of the status quo approach—that because no opinion in the prior case “commanded the assent of more than three Justices” that it “never became the law” (Marks v. United States 1977, 192). The Court’s decision in Marks also rejected the plurality approach, emphasizing that it is the “narrowest grounds” that prevails, not the grounds with the largest number of votes. Indeed, in several cases since Marks both the lower Courts and the Supreme Court have interpreted an opinion with a single vote as the narrowest grounds of the Court, even when other opinions had more votes. Thus, even from a purely legal approach—that is, if lower courts ignored strategy and merely formally obeyed the Supreme Court—neither the plurality approach nor the status quo approach could ever be the general rule.

These two options (the plurality and the status quo) also fail from a strategic perspective. The reason is that we would expect strategic, forward-looking lower courts grappling with a fractured Supreme Court decision to view that position that no majority of the justices could agree to reverse as the Court’s policy along a dimension. That is, to the extent that the Court can be said to have a single policy along a dimension that policy must be the policy point that a lower court could adopt in a case involving that dimension.

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20 See, for example, O’Dell v. Netherland (1997), which interpreted the Court’s holding in Gardner v. Florida (1977) as Justice White’s opinion, even though no other justices joined that opinion and another opinion had three votes.

21 There is a possibility that the status quo would prevail if the Marks inquiry “fails” as some lower courts and many commentators assert that it can. We need not take a position on whether the Marks inquiry can “fail,” as we are operating in the paradigm of the strategic approach.

22 A good argument could be made that a single point along each dimension is not sufficient for the lower courts to predict whether they will be reversed. This is because if the lower courts’ cases are multidimensional and we regard the policy space as a partitioned fact space, the lower courts may need to know each
alone without fear of reversal. The plurality opinion could happen to be such a position but often is not, as a majority of justices may disagree with a relatively extreme plurality decision, even along a single dimension. The status quo is certainly not such a position, except in the case where the status quo lies precisely at the median of the justices’ ideal points along the dimension. Thus, since rational lower courts in a strategic model would adopt roughly what Kornhauser and Sager term the “predictive approach” (Kornhauser and Sager 1993), the plurality and the status quo are not plausible candidates for the Court’s policy when no policy receives a majority of the votes.

We therefore adopt an approach that follows directly from the strategic model of the court system and is largely consistent with Marks’ “narrowest grounds” doctrinal position. The lower courts regard as the Supreme Court’s policy along a dimension that point that the Supreme Court cannot reverse in a hypothetical unidimensional case involving that dimension alone. The lower courts ascertain what coalitions could form in the Supreme Court to reverse their decisions by looking to the possible coalition-proof Nash equilibria of the Supreme Court. The only point for which the lower courts expect that the Supreme Court could not form a coalition to upset their decision is the median of the actual votes of the justices along the dimension (i.e., not necessarily the median of the justices ideal points). This is because the median of the votes along that dimension would constitute the lower courts’ “best guess” about what policy point the justices could least likely muster a five-vote majority to overturn. This strategic “predictive” approach has the benefit of not only being consistent with the strategic incentives of rational lower courts, but also being similar (if not identical) to the Court’s “narrowest grounds” doctrinal position.23

Justice’s multidimensional position, not merely a median position of the Court along a dimension. In such a case, the notion of a Court “policy” along a single dimension is not meaningful. Thus, in this paper, we consider lower courts that confront unidimensional cases.

23For example, Kornhauser and Sager (1993) observe that “[t]he predictive approach could commend the same result to a lower court as that pointed to by the narrowest grounds test, even if the lower court did
4.3 The Strategic Part-by-Part Voting Game

Our strategic part-by-part voting model proceeds as follows. The justices have a policy-making opportunity in $d$-dimensional Euclidean space. The justices can freely discuss their strategies, write opinions, and make preliminary votes over the opinions, but cannot make binding agreements. Each justice chooses a policy point in $d$-dimensional space that constitutes his “opinion” or “vote” in the case. This captures the fact that any justice may write an opinion (i.e., someone else does not need to have proposed the policy point) and those justices who do not write opinions along a particular dimension may vote for any policy point along that dimension. To reiterate, under part-by-part voting the justices may “pick and choose” which opinions they choose to vote for along various dimensions; they are not required to vote for any one opinion proposed by a particular individual. In fact, since we do not differentiate between a “proposal stage” and a “voting stage,” there is no technical distinction between writing an opinion and voting for an opinion, except that “voting” for an opinion along a particular dimension involves choosing a point that at least one other justice has already chosen. The justices may revise opinions, write new opinions, or switch their votes at any time and no decisions are final if the justices have “finalized” their votes.

Eventually there comes a time when each justice finalizes his opinions and votes, although this time need not be the same for all justices. There is obviously some outer limit for this time as the judgment is eventually read in open Court. We leave the process by which the justices “finalize” their votes completely arbitrary; they may finalize their votes one by one or may all finalize their votes at the same time. The key is that under coalition-proof Nash equilibrium, each player can finalize his vote knowing that the remaining players cannot deviate from the equilibrium in a self-enforcing way.

not perceive that the disagreement of the higher court was entrenched and/or nested. The lower court could simply think that the rationale identified by the narrowest grounds test was in fact the most likely rationale to emerge as the majority view of the higher court.”

24If we wished to introduce a cost for writing opinions, then we would distinguish between the two.
The application of coalition-proof Nash equilibrium under part-by-part voting to this strategic scenario leads to the following Propositions.

**Proposition 4.1.** *If a strategy profile is coalition-proof, then the outcome is the issue-by-issue median.*

**Proposition 4.2.** *There exists a coalition-proof strategy profile whose outcome is the issue-by-issue median.*

These propositions show that as in the case of sincere voting with binary alternatives, the unique outcome in strategic voting is the issue-by-issue median of the justices. This result has significant implications for the prospects for agenda control and strategic manipulation as discussed in the next part. At this point, however, our key point is that the part-by-part voting procedure, combined with the default rule in plurality cases, means that justices are not strategically disadvantaged by simply voting for their own ideal points.\(^{25}\) This conclusion poses a challenge for strategic models of Supreme Court decision-making and suggests an explanation for the persistent failure of empirical studies to show evidence of systematic strategic behavior in the opinion writing (as opposed to the certiorari) stage of Supreme Court policymaking.

### 5 Discussion

The prospect of social intransitivity, cycling, or chaos in collective decision-making is a problem that all collective choice institutions must confront, and the Supreme Court is no exception. In this paper, we have identified specific institutional features that have evolved in the Supreme Court to alleviate the cycling problem and promote stable, predictable outcomes. In the remainder of this paper, we expand on these results in three ways. First, we explain in intuitive terms the mechanism by which part-by-part voting

\(^{25}\) We say “strategically” disadvantaged because there are obviously other disadvantages to writing opinions “seriatim” at each justice’s ideal point. Multiple opinions are costly, more difficult to interpret, and arguably undermine the Court’s authority. Thus, we are not predicting that justices will write opinions at their own ideal points, merely observing that such behavior is not strategically irrational.
creates stability in the Court’s outcomes. Second, we examine some empirical implications of our theory, particularly in the context of opinion assignment. Finally, we expand on how the implications of our model fit into the study of judicial politics more generally.

5.1 Part-by-Part Voting as an Intuitive Solution to Instability

The institution of part-by-part voting is an intuitive solution to the problem of social intransitivity. The reason part-by-part voting solves the social intransitivity problem in the Supreme Court is that it allows justices to “unbundle” a policy proposal into its component parts. In other words, unlike voting on a bill in a legislature, justices are not required to take the “bad” with the “good”. In majority voting over binary choices, the “bad” part of the “winning” alternative is what sets that alternative up to be defeated later, but part-by-part voting allows a justice to reject the “bad” and avoid setting up the cycle. The ability to approve or disapprove of policy dimensions individually through part-by-part voting mean that justices cannot be forced to “compromise” across issues the way they are when they face binary choices over multidimensional alternatives. In this sense, part-by-part voting is similar to “issue-by-issue voting” (as that term is used in the political science literature, not the legal literature on the Supreme Court). Since instability and cycling results in majority voting invariably involve compromises across dimensions, part-by-part voting evades the cycling problem by undermining the basic premise of binary choice over multidimensional options.

5.2 Applications: Agenda Setting and Opinion Assignment

The implications of agenda control for influencing collective choice outcomes are among the central questions in positive political science. In the context of the Supreme Court, the main form of agenda control (of which we are aware) is the right to assign who writes the (initial) majority opinion. In the Supreme Court, the Chief Justice assigns the task of writing the majority opinion when he is in the majority and the senior Associate Justice assigns the task when the Chief Justice is in the minority. The political science literature,
as well as the journalistic accounts of the Court, commonly suppose or conclude that the Chief Justice (and the senior Associate Justice when the chief is in the minority) will assign opinions to ideological allies.\footnote{One notable recent exception is Hammond, Bonneau, and Sheehan (2005), which concludes that under certain conditions rational opinion assigners will not assign opinions to the most ideologically proximate justice.}

While some empirical work has found support for that proposition (Rohde 1972), other work has pointed to the dominance of non-ideological factors in opinion assignment (Rathjen 1974). The distinction matters because, if agenda control is important in determining the Supreme Court’s outcomes, then the Chief Justice (or senior Associate Justice) has the opportunity for strategic manipulation not only in assigning opinions when in the conference majority, but in changing his vote on the merits in order to be in the conference majority (Maltzman, Spriggs, and Wahlbeck 2000, 32).

Our model suggests, however, that the capacity of these agenda setters to manipulate outcomes is limited. The conventional wisdom is that the opinion author (and therefore the opinion assigner) “occupies an agenda setting position” (Maltzman, Spriggs, and Wahlbeck 2000, 15), and that may well be the case. But because there is a unique coalition-proof Nash equilibrium in the strategic game, and this equilibrium is independent of the initial opinion proposal, the right to assign (or write) the initial position has no influence on the outcome.\footnote{This is the same result that Hammond, Bonneau, and Sheehan (2005) reach with respect to their “open-bidding” and “median-holdout” models. The absence of agenda influence also means that there are few reasons for strategic voting in the justices’ conference, or for strategically changing votes on the judgment after the conference. This conclusion is also consistent with the “open-bidding” and “median-holdout” models of Hammond, Bonneau, and Sheehan (2005), but not with their “agenda-control” model. We note that the mere fact that justices change their votes on the merits after conference somewhat frequently (Maltzman and Wahlbeck 1996b) does not necessarily mean they are doing so strategically.} The clear implication of this model is that properly conducted empirical tests should fail

\[26\]
to produce evidence that justices with agenda control wield disproportionate influence on the outcome of Supreme Court policy.

Indeed, the recent empirical studies that have considered the issue of opinion assignment have found only very weak effects of ideology on opinion assignment (Maltzman, Spriggs, and Wahlbeck 2000, 47–56). When the Chief Justice assigns opinions, the authors’ results suggest that the ideological component of opinion assignment is overwhelmed by “contextual factors” such as equity of assignment, legal expertise, workload, and the “end of term” opinion crunch (Maltzman, Spriggs, and Wahlbeck 2000, 47–56). When the senior Associate Justice assigns opinions, ideological considerations are virtually non-existent, and dwarfed by the effect of self-assignment (Maltzman, Spriggs, and Wahlbeck 2000, 48). These results hold up under the Burger Court (Maltzman, Spriggs, and Wahlbeck 2000) and the Rehnquist Court (Maltzman and Wahlbeck 1996a). In short, our model provides a plausible explanation for the seemingly surprising conclusion reached by empirical researchers: that “organizational needs” explain more than ideology in opinion assignment (Maltzman and Wahlbeck 1996a).

5.3 Further Implications and Relation to the Literature

In multidimensional policy settings, the right to control the agenda is a powerful (and sometimes all-powerful) tool (McKelvey 1976). Those with influence over the agenda can affect the outcome of a policymaking process, and therefore the agenda control rights in an institution are often important features in modeling policy outcomes. The problem in Supreme Court is that if the Court has any agenda-setting rules for comparing alternatives, we do not know what they are. Without this information, explaining outcomes in the Supreme Court would be very difficult in a multidimensional environment, since even with socially transitive preferences, if a single player controls the agenda, we may have “anything can happen” chaos.
Moreover, if someone does have agenda power, the other voters might try to deceive the agenda setter about their true preferences. In the Supreme Court context, where rational lower courts rely on predictions of the Supreme Court’s behavior to make decisions, the possibility for such misrepresentation is obviously a problem.

The stability induced by part-by-part voting, therefore, is an important feature not just for the study of the Court’s decision-making process specifically, but for the functioning of the judicial system generally. Our model suggests that the institutional features of the Court, particularly part-by-part voting and the “narrowest grounds” rule, help to minimize the potential for strategic manipulation of the agenda or policy alternatives in the Court’s decision-making. This result suggests that some of the incentives for misrepresentation of preferences are attenuated in the Supreme Court, a fact that is relevant for the hierarchical control of lower courts. Indeed, the outcomes adopted by forward thinking justices in our model are consistent with the outcomes that “sincere” justices would adopt in the attitudinal model. Thus, to a greater extent than has been previously realized, the “attitudinal” and “strategic” schools of Supreme Court decision-making ultimately converge on similar behavioral predictions.

The fact that our model predicts strategically sincere behavior, however, does not mean that other institutional features of the Court are irrelevant. In fact, we would argue that our model allows for a larger role for the briefs, oral arguments, conference discussions, and bargaining among the justices than other strategic models do. The coalition-proof Nash equilibrium solution concept, when combined with the proposal process that allows all justices to participate at any point in the opinion-writing process, means that nothing is final until the justices decide to “sign off” on the final opinions. Thus, in contrast to other models of complete information, we need not commit to the assumption that the justices have perfect information and fixed preferences over legal policy alternatives at the outset of a case. The coalition-proof solution concept

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28 McKelvey (1976) shows that even if social preferences are transitive, the agenda setter could potentially create intransitivity by misrepresenting his own preferences, thereby obtaining his ideal point.
allows for the possibility that the justices use the briefs, oral argument, conference, and discussions with colleagues and clerks to learn about their colleagues’ (and their own) preferences in the case.

In contrast to most models of complete and perfect information, then, our model does not require that the briefs, oral arguments, and conference discussions are empty charades before a Court that deterministically enacts a median policy. Our justices may learn, communicate, and change their preferences during the course of their deliberations and opinion drafting. Because coalition-proof Nash equilibrium concept requires the justices to commit only when they are ready to finalize the opinions in the case, our model does not need to foreclose a meaningful role for legal argumentation, deliberation, and learning. We only assume that the justices are able to develop preferences over the alternative legal policies at some point during the opinion-writing process. Thus, our model suggests an important (indeed, heightened) role for the growing body of work that examines these processes (e.g., Johnson, Wahlbeck, and Spriggs 2006).

6 Conclusion

Our model suggests several important results that should contribute to the direction of future scholarship on the Supreme Court. The first result is that, in general, we would not expect Supreme Court decision-making to exhibit instability or cycling, even in a multidimensional issue space. The consequence is that the Supreme Court should display much more consistency and stability than a pure majority rule model would predict. Second, our model suggests that the Court’s policy outcomes will generally end up at the issue-by-issue median of the justices’ ideal points, a fact that has important implications for theoretical and empirical work on the Supreme Court. Third, our model suggests that agenda control, opinion assignment, conference voting, the Chief Justice’s institutional position, and similar strategic tools should have limited significance in explaining Supreme Court voting outcomes. Thus, in general, our model predicts a stable, unique policy outcome in Supreme Court decision-making attributable to the institutional features the Court has developed over many years.
In addition to these specific results, we see our model as providing two more general contributions to the study of decision-making on the Supreme Court. On the one hand, we regard these results as a first step in providing the necessary theoretical underpinning for the growing body of strategic models of the Court. The institutional arrangements we identify go some of the way toward justifying median-voter-type results that have appeared in most strategic models of the Court. On the other hand, however, a second overall lesson of our model is that the opportunities for strategic behavior in the proposal and voting stage of the Supreme Court’s policymaking are more limited than other models suggest. In particular, we show that the justices are not strategically disadvantaged by simply voting for their own ideal points, an important implication for any strategic model of the Court. Thus, while we believe that opportunities for strategic behavior exist in certain stages Supreme Court decision-making (for example, in certiorari), our model suggests that Supreme Court behavior will ultimately prove significantly more sincere than many strategic accounts predict.

References


A Proofs

Notation A.1. The definition of a Coalition-Proof Nash equilibrium uses the following notation:

- $S$ denotes the set of all possible strategy profiles for a game, $\Gamma$.
- $u^i(s)$ denotes the utility of player $i$ resulting from strategy profile $s$.
- $\Gamma|s^*_{\_J}$ denotes the game played by a coalition or subset of players, $J$, with the strategies of all players not in $J$ fixed at their strategies under strategy profile $s^*$.

Definition A.2. (Bernheim, Peleg, and Whinston 1987) (i) In a single player game $\Gamma$, $s^* \in S$ is a Coalition-Proof Nash equilibrium if and only if $s^*$ maximizes $u(s)$.

(ii) Let $n > 1$ and assume that Coalition-Proof Nash equilibrium has been defined for games with fewer than $n$ players. Then,

1. For any game $\Gamma$ with $n$ players, $s^* \in S$ is self-enforcing if, for all $J \in J$, $s^*_J$ is a Coalition-Proof Nash equilibrium in the game $\Gamma|s^*_{\_J}$.

2. For any game $\Gamma$ with $n$ players, $s^* \in S$ is a Coalition-Proof Nash equilibrium if it is self-enforcing and if there does not exist another self-enforcing strategy vector $s \in S$ such that $u^i(s) > u^i(s^*)$ for all $i = 1, \ldots, n$.

A.1 Notation

Notation A.3. The following notation is used throughout:

- $n$ justices, where $n$ is odd
- $N = \{1, \ldots, n\}$ represents the set of justices
• \( \Omega = \Omega_1 \times \cdots \times \Omega_d \) is the policy space where, \( \forall i, \Omega_i \) has an ordering function, denoted \(<\), that is defined over all ordered pairs of unique elements of \( \Omega_i \) and is transitive, with \( >, \leq, \text{ and } \geq \) also defined in terms of this ordering function, \(<\).

• \( D = \{1, \ldots, d\} \) represents the set of dimensions of the policy space

• Let each justice have ideal point \( \ell_i \in \Omega \ (i \in \{1, \ldots, n\}) \)

• \( L = (\ell_1, \ldots, \ell_n) \)

• \( \ell_i^{(j)} \) is the ideal point of justice \( i \) along dimension \( j \)

• \( \sigma \in \Omega^n \) is a strategy profile

• \( \sigma_i \) is the strategy of player \( i \) under strategy profile \( \sigma \)

• \( \sigma^{(j)} \) is the strategy of all players along dimension \( j \)

• \( o(\sigma) \) is the outcome under strategy profile \( \sigma \)

• \( o_j(\sigma) \) is the outcome along dimension \( j \)

• \( t(\sigma, A \rightarrow x) \) is the strategy profile after coalition \( A \) deviates to \( x \) from strategy profile \( \sigma \)

  – When \( x \) specifies a single point, it means that all members deviate to voting for \( x \).

  – When \( x \) is not specified, it can represent any possible new voting arrangement for members of \( A \). This can be but is not necessarily a single point. Thus, \( x \) can specify different voting strategies for different members of \( A \).

• \( m_j(x) \) is the median of vector \( x \) along dimension \( j \)

• \( m(x) = (m_1(x), \ldots, m_d(x)) \)

• \(|A|\) denotes the cardinality of set \( A \)
• $u_i(x)$ denotes the utility of justice $i$ from outcome $x$

• $u_i^{(j)}(x_j)$ denotes the utility of justice $i$ from outcome $x$ along dimension $j$, where this utility is defined to be any function such that

$$u_i(x) = \sum_{i=1}^{d} u_i^{(j)}(x_j)$$

– Note that, by separability of preferences, such a utility function must exist.

• $aR_i b$ denotes that $a$ is weakly preferred by justice $i$ to $b$ and is defined as $aR_i b \iff u_i(a) \geq u_i(b)$.

• $aP_i b$ denotes that $a$ is strictly preferred by justice $i$ to $b$ and is defined as $aP_i b \iff u_i(a) > u_i(b)$.

A.2 Assumptions

The following assumptions are made throughout:

• Justices are assumed to have separable utility functions. That is, for any justice $i$, there exists a set of

functions $u_i^{(1)}(\cdot), \ldots, u_i^{(d)}(\cdot)$ such that, for all outcomes $x$,

$$u_i(x) = \sum_{i=1}^{d} u_i^{(j)}(x_j)$$

• Justices are assumed to have single peaked preferences along each dimension, peaked at their ideal points, $\ell_i^{(j)}$. That is, for all $x \neq \ell_i^{(j)} \in \Omega_j$, $u_i^{(j)}(\ell_i^{(j)}) > u_i^{(j)}(x)$ and, if $a < b \leq \ell_i^{(j)}$ or $\ell_i^{(j)} \leq b < a$,

then $u_i^{(j)}(a) < u_i^{(j)}(b)$.

• The number of justices, $n$, is odd.

– Note that, when $n$ is even, the median is not unique. This is an unnecessary complication, particularly since most courts involve an odd number of justices.
A.3 Strategic Voting Results

**Lemma A.4.** Assume \(d = 1\). If a strategy profile, \(\sigma\), is coalition-proof, then \(o(\sigma) = m(\sigma)\).

**Proof.**

Case 1. \(\exists x : |\{i : \sigma_i = x\}| > \frac{n}{2}\) (majority opinion)

Since \(x\) has majority support, it is the opinion of the court. Thus, by assumption, \(o(\sigma) = x\). Since \(|\{i : \sigma_i = x\}| > \frac{n}{2}\), \(|\{i : \sigma_i > x\}| < \frac{n}{2}\) and \(|\{i : \sigma_i < x\}| < \frac{n}{2}\). That is, since the majority opinion, \(x\), has more than half of the votes, fewer than half of the votes can be for outcomes on either side of it. So, \(m(\sigma) = x\), indicating that \(x\) is the median of the votes. Since \(x\) is the majority opinion, it is also the outcome under strategy profile \(\sigma\). Thus, \(o(\sigma) = m(\sigma)\).

Case 2. \(\nexists x : |\{i : \sigma_i = x\}| > \frac{n}{2}\) (plurality opinion)

By assumption, when there is no majority opinion, the outcome is \(o(\sigma) = m(\sigma)\).

\(\therefore\), in all cases, \(o(\sigma) = m(\sigma)\).

**Lemma A.5.** Assume \(d = 1\). If a strategy profile, \(\sigma\), is coalition-proof, then \(o(\sigma) = m(\ell)\).

**Proof.** Assume that \(\sigma\) is a strategy profile such that \(o(\sigma) \neq m(\ell)\). Thus, by **Lemma A.4**, \(m(\sigma) \neq m(\ell)\).

Without loss of generality, assume \(o(\sigma) > m(\ell)\). Since the outcome is a point greater than the median of the ideal points, **Lemma A.4** implies that more than half of the votes must be for points greater than the median of the ideal points. So, \(|\{i : \sigma_i > m(\ell)\}| > \frac{n}{2}\). But, by definition of \(m(\ell)\) being the median of the ideal points, \(|\{i : \ell_i > m(\ell)\}| < \frac{n}{2}\). So, there must be a justice who voted for a point greater than his ideal point—\(\exists i : \sigma_i > m(\ell)\geq \ell_i\). Let \(A \subseteq \{i : \sigma_i > m(\ell)\}\) be the smallest set such that \(A\) could change the outcome by voting for the greatest ideal point among members of \(A\)—that is, such that \(o(t(\sigma, A \rightarrow \max_{i \in A} \ell_i)) < o(\sigma)\) and \(\forall i \in A, j \in \{j : \sigma_j > m(\ell)\}, \ell_i < \ell_j\). Note that such a set must exist as, should all justices with ideal points \(\ell_i \leq m(\ell)\) vote for \(m(\ell)\), \(m(\ell)\) would have a majority and would therefore be the outcome.
Claim. \( A \rightarrow \max_{i \in A} \ell_i \) is a self-enforcing.

Let \( \hat{\sigma} = t(\sigma, A \rightarrow \max_{i \in A} \ell_i) \) be the deviation of coalition \( A \) to the greatest ideal point of members of the coalition. Note that \( o(\hat{\sigma}) \geq \max_{i \in A} \ell_i \)—that is, the new outcome cannot be greater than the greatest ideal point of members of the coalition.

If \( o(\hat{\sigma}) > \max_{i \in A} \ell_i \), then, for any deviation by a subset of \( A \), \( \hat{\sigma}, o(\hat{\sigma}) \geq o(\hat{\sigma}) \) as over half of the justices not in \( A \) must be voting for points greater than \( \max_{i \in A} \ell_i \). So, since all members have single peaked preferences, they cannot prefer \( o(\hat{\sigma}) \) to \( o(\hat{\sigma}) \). Thus, \( \hat{\sigma} \) is self-enforcing.

If \( o(\hat{\sigma}) = \max_{i \in A} \ell_i \), then the justice with the largest ideal point in \( A \), \( \arg \max_{i \in A} \ell_i \), cannot be part of any deviating coalition since \( o(\hat{\sigma}) \) achieves its ideal point and, consequently, no other outcome can improve his utility. But, since, for any \( B \subset A \) and deviation \( x \), \( o(t(\sigma, B \rightarrow x)) \geq o(\sigma), o(t(\hat{\sigma}, B \rightarrow x)) \geq o(\hat{\sigma}) \).

That is, since, by definition \( A \), the outcome under the deviation by subcoalition \( B \) must be greater than the outcome under \( \sigma \), the outcome if \( B \) deviates to \( x \) from \( \hat{\sigma} \) also cannot be greater than the outcome under \( \hat{\sigma} \) as more than half of the justices would still be voting for points equal to or greater than the outcome under \( \hat{\sigma} \). Since, \( \forall i \in B \subset A, \ell_i \leq o(\hat{\sigma}) \leq o(t(\hat{\sigma}, B \rightarrow x)) \), by single peakedness of preferences, it must be the case that \( u_i(o(\hat{\sigma})) \leq u_i(o(t(\hat{\sigma}, B \rightarrow x))) \) or, equivalently, \( o(\hat{\sigma}) R_i o(t(\hat{\sigma}, B \rightarrow x)) \). That is, no member of a subcoalition, \( B \), could improve itself by any deviation from \( \hat{\sigma} \). So, \( \hat{\sigma} \) is self-enforcing.

Thus, \( \sigma \) is not coalition-proof.

\[ \therefore \text{if a strategy profile, } \sigma, \text{ is coalition-proof, } o(\sigma) = m(\ell). \]

Lemma A.6. Assume \( d = 1 \). If \( \sigma \) be a strategy profile such that \( \sigma_i \neq \ell_i \), then \( o(\sigma, \{i\} \rightarrow \ell_i) R_i o(\sigma) \). That is, no justice can be made worse off by voting for his most preferred outcome.

Proof. Let \( \hat{\sigma} \equiv t(\sigma, \{i\} \rightarrow \ell_i) \) be the deviation where \( i \) votes for \( \ell_i \). Without loss of generality, assume \( \ell_i < m(\sigma) \).
Case 1. $\sigma_i < m(\sigma)$

By definition of $m(\sigma)$ being the median of the votes, at least half of the votes must be for $m(\sigma)$ or points greater than or equal to $m(\sigma)$ and at least half of the votes must be for points less than or equal to $m(\sigma)$. Formally, $|\{j : \sigma_j \geq m(\sigma)\}| \geq \frac{n}{2}$ and $|\{j : \sigma_j \geq m(\sigma)\}| \geq \frac{n}{2}$. Since $\ell_i < m(\sigma)$ and $\sigma_i < m(\sigma)$, under $\hat{\sigma}$, $|\{j : \hat{\sigma}_j \geq m(\sigma)\}| \geq \frac{n}{2}$ and $|\{j : \hat{\sigma}_j \geq m(\sigma)\}| \geq \frac{n}{2}$. That is, at least half of the votes under $\hat{\sigma}$ must be for points greater than or equal to $m(\sigma)$ and at least half of the votes under $\hat{\sigma}$ must be for points less than or equal to $m(\sigma)$. Since this is also the definition of the median under $\hat{\sigma}$, $m(\hat{\sigma}) = m(\sigma)$. Thus, by Lemma A.4, the outcomes, $o(\hat{\sigma})$ and $o(\sigma)$, must be the same. So, $o(\hat{\sigma}) R_i o(\sigma)$.

Case 2. $\sigma_i > m(\sigma)$

Let $m^-(\sigma)$ denote the largest opinion under $\sigma$ for which at least $\frac{n}{2} + 1$ opinions are still at least as large—i.e., $m^-(\sigma) \equiv \max_j \{j : \sigma_j \geq x\} \geq \frac{n}{2} + 1$. It must be the case that $|\{j : \sigma_j \leq m^-(\sigma)\}| \geq \frac{n}{2} - 1$ since, otherwise, there would be a larger value of $x$ satisfying $|\{j : \sigma_j \geq x\}| \geq \frac{n}{2} + 1$. The number of opinions equal to or greater than $m(\sigma)$ will be at least $\frac{n}{2}$. The number of opinions less than or equal to $\max(m^- (\hat{\sigma}), \ell_i)$ will be the number less than or equal to $m^-(\hat{\sigma})$ plus $\ell_i$, giving a total of $|\{j : \sigma_j \geq x\}|+1$.

Since $|\{j : \sigma_j \leq m^-(\sigma)\}| \geq \frac{n}{2} - 1$, this number must be at least $\frac{n}{2}$—i.e., $|\{j : \sigma_j \leq \max(m^- (\hat{\sigma}), \ell_i)\}| \geq \frac{n}{2}$. So, under $\hat{\sigma}$, the outcome must be $o(\hat{\sigma}) = \max(m^-(\hat{\sigma}), \ell_i)$. Since $m^-(\hat{\sigma}) \leq m(\sigma)$ and $\ell_i \leq m(\sigma)$, $\max(m^-(\hat{\sigma}), \ell_i) \leq m(\sigma)$. So, $\ell_i \leq \max(m^-(\hat{\sigma}), \ell_i) \leq m(\sigma)$ or, equivalently, $\ell_i \leq o(\hat{\sigma}) \leq o(\sigma)$.

Thus, by single peakedness of preferences, $u_i (o(\hat{\sigma})) \geq u_i (o(\sigma))$ or, equivalently, $o(\hat{\sigma}) R_i o(\sigma)$.

$\therefore o(\hat{\sigma}) R_i o(\sigma)$. □

Lemma A.7. Assume $d = 1$. Let $\sigma$ be a strategy profile, $A \subseteq N$, and $x$ be a deviation such that $\forall i \in A, o(t(\sigma, A \rightarrow x)) P_i o(\sigma)$ but $\forall B \subset A, \forall y : \forall i \in B, o(t(\sigma, A \rightarrow y)) P_i o(\sigma)$. Then, there exists a self-enforcing deviation from $\sigma$, $t(\sigma, A \rightarrow z)$. That is, if $A$ is a coalition which can deviate to a unanimously preferred point, but no subcoalition of $B$ can make such a deviation, then there also exists a self-enforcing
deviation by $A$ from $\sigma$.

Proof. Without loss of generality, assume $o(t(\sigma, A \to x)) < o(\sigma)$. Consider the deviation $\hat{\sigma} \equiv t(\sigma, A \to \max_{i \in A} \ell_i)$ in which all members of $A$ vote for the ideal point of the member of $A$ with the largest ideal point. Assume there exists a subcoalition, $B \subseteq A$, that can make a self-enforcing deviation, $t(\hat{\sigma}, B \to u)$. If $i = \arg \max_{i \in A} \ell_i$, then, by Lemma A.6, $o(\hat{\sigma}) R_i o(\sigma, B \to u)$. So, $\arg \max_{i \in A} \ell_i \notin B$. Thus, $B \neq A$ and, consequently, $B \subset A$. Note that $o(\hat{\sigma}) \geq \max_{i \in A} \ell_i \geq \ell_i$. Since $\forall i \in B, o(\hat{\sigma}) \geq \max_{i \in A} \ell_i \geq \ell_i$ and $\forall i \in B, o(t(\hat{\sigma}, B \to u)) P_i o(\hat{\sigma}), o(t(\hat{\sigma}, B \to u)) < o(\hat{\sigma})$. That is, since, for all members of $B$, the outcome under $\hat{\sigma}$ is greater than that justice’s ideal point and the outcome under the deviation from $\hat{\sigma}$ must be prefered to the outcome under $\sigma$, the outcome under the deviation from $\hat{\sigma}$ must be less than the outcome under $\hat{\sigma}$ by single peakedness of preferences. So, $\max_{i \in A} \ell_i > o(t(\hat{\sigma}, B \to u))$. So, by Lemma A.4, $\max_{i \in A} \ell_i > m(t(\hat{\sigma}, B \to u))$. So, $\forall i \in A \setminus B, \hat{\sigma}_i > m(t(\hat{\sigma}, B \to u))$—all justices in $A$ but not $B$ must be voting for points greater than the outcome under the deviation from $\hat{\sigma}$. Since these justices must also have been voting for an even greater point under $\sigma$, they must have been voting under $\sigma$ for points greater than the outcome under this deviation from $\hat{\sigma}$. That is, since $\forall i \in A, \sigma_i > \hat{\sigma}_i$, we have that $\forall i \in A \setminus B, \sigma_i > m(t(\hat{\sigma}, B \to u))$. Since the votes of the justices in $A$ but not in $B$ are greater than the outcome under $t(\hat{\sigma}, B \to u)$ and are the only difference between $t(\hat{\sigma}, B \to u)$ and $t(\sigma, B \to u)$, the outcomes under $t(\hat{\sigma}, B \to u)$ and $t(\sigma, B \to u)$ must be the same. Therefore, $m(t(\sigma, B \to u)) = m(t(\hat{\sigma}, B \to u))$. So, $B \subset A$ and $\forall i \in B, o(t(\sigma, B \to u)) P_i o(\sigma)$, which is a contradiction.

\[ \therefore A \to \max_{i \in A} \ell_i \text{ is a self-enforcing deviation from } \sigma. \]

Lemma A.8. If, for a strategy profile $\sigma, \sigma_j$ is self-enforcing on the game restricted to dimension $j$ for all $j \in D$, then $\sigma$ is self-enforcing.

Proof. Assume there exists a subcoalition, $A$, and deviation, $x$, such that $\forall i \in A, o(t(\sigma, A \to x)) S_i o(\sigma)$. Take any such coalition $A$ with the smallest cardinality and corresponding deviation, $x$. That is, $\frac{3}{2} B \subset$
A, y : \forall i \in B, o(t(\sigma, B \rightarrow y)) P_i o(\sigma). Note that, since all such sets have finite cardinality, such a set must exist (although it may not be unique). Let J be the set of all dimensions along which A can deviate such a manner — \( J = \{ j : \exists x : \forall i \in A, o(t(\sigma, A \rightarrow x)) P_i o(\sigma) \}. \) By Lemma A.7 for all dimensions along which A that allows such a deviation, there also exists a self-enforcing deviation along that dimension. Let \( z \) be the deviation by A to a self-enforcing deviation along each dimension \( j \in J \). So, there must be a self-enforcing deviation from \( t(\sigma, A \rightarrow z) \) by \( B \subseteq A \). Thus, B can move to an outcome, \( u \), which is preferred by all members of \( B \)—\( \forall i \in B, o(t(\bar{\sigma}, B \rightarrow u)) P_i o(\bar{\sigma}) \). Thus, there must be some dimension, \( k \), along which all members of \( B \) are made better off—\( \forall i \in B, o_k(t(\bar{\sigma}, B \rightarrow u)) P_i o_k(\bar{\sigma}) \). Thus, by Lemma A.7 there exists a self-enforcing deviation along dimension \( k \) by \( B \). If \( k \in J \), \( \bar{\sigma} \) is not coalition-proof along dimension \( k \), which is a contradiction. If \( k \notin J \), then the first deviation did not change along dimension \( k \), so \( \bar{\sigma}^{(k)} = \sigma^{(k)} \) and, thus, \( B \) can made the same deviation from \( \sigma_k \) along \( k \), \( t(\sigma_k, B \rightarrow u_k) \), which is preferred by all members of \( B \) along \( k \). Since, by assumption, no proper subset can make such a deviation, \( B = A \). So, A can move to a unanimously preferred outcome along \( k \), so \( k \in J \), which contradicts \( k \notin J \).

Thus, there does not exist a subcoalition, \( A \), and deviation, \( x \), such that \( \forall i \in A, o(t(\sigma, A \rightarrow x)) P_i o(\sigma) \).

\[ \therefore, \sigma \text{ is self-enforcing.} \]

Lemma A.9. If a strategy profile \( \sigma \) is self-enforcing, then \( \sigma_j \) is self-enforcing on the game restricted to dimension \( j \) for all \( j \in D \).

Proof. Let \( \sigma \) be a self-enforcing strategy profile. Assume, for some \( j, \sigma_j \) is not self-enforcing on the game restricted to dimension \( j \). Let \( k \) be a dimension with a deviating coalition that is minimum in cardinality over all deviating coalitions on all dimensions (where a deviating coalition means any coalition which can make a self-enforcing deviation which is preferred by all members of the coalition along a dimension) and let \( A \) be such a coalition. Let \( J_A \) be the set of all dimensions on which coalition \( A \) can make a self-enforcing deviation. Note that, since \( A \) is the smallest possible size a deviating coalition, no subcoalition \( B \subseteq A \) can make a self-enforcing deviation. Consider the deviation, \( \bar{\sigma} \), in which \( A \) makes each of these self-enforcing
deviations along all dimensions \( j \in J_A \) and no deviation along other dimensions. By \textit{Lemma A.8} this deviation is self-enforcing. This contradicts the assumption that \( \sigma \) is self-enforcing. So, there cannot exists any \( j \) such \( \sigma_j \) is not self-enforcing on the game restricted to dimension \( j \).

\[ \therefore, \sigma \text{ is coalition-proof.} \]

**Theorem A.10.** If a strategy profile, \( \sigma \), is coalition-proof, then \( o(\sigma) = m(\ell) \)—i.e., the outcome is the issue-by-issue median.

**Proof.** Let \( \sigma \) be a coalition-proof strategy profile. Assume, for some dimension \( j \in D \), \( o_j(\sigma) \neq m_j(\ell) \). Thus, by \textit{Lemma A.5} \( o_j(\sigma) \neq m_j(\ell) \). So, by \textit{Lemma A.9} \( \sigma \) is not coalition-proof, which is a contradiction. So, for all \( j \in D \), \( o_j(\sigma) \neq m_j(\ell) \).

\[ \therefore, o(\sigma) = m(\ell). \]

**Theorem A.11.** There exists a coalition-proof strategy profile, \( \sigma \).

**Proof.** Consider the strategy profile, \( \sigma \), in which all justices vote for the median along each dimension—i.e., \( \forall i \in N, j \in D, \sigma_i^{(j)} = m_j(\ell) \). Along any dimension, \( j \), any deviating coalition, \( A \), must have at least \( \frac{n}{2} \) justices to change the outcome (as, otherwise, the median on dimension \( j \), \( m_j(\ell) \), would retain a majority). However, any such coalition \( A \) must include either the median justice on dimension \( j \) or justices whose ideal points are on both sides of the median. No outcome can be preferred to \( m_j(\ell) \) by the median justice since \( m_j(\ell) \) is, by definition, his most preferred outcome. No outcome, \( x \neq m_j(\ell) \), can be preferred by justices with ideal points on both sides of the median since, if \( x < m_j(\ell) \), justices with ideal points greater than the median, \( m_j(\ell) \), must prefer \( m_j(\ell) \) to \( x \) and, likewise, if \( x > m_j(\ell) \), justices with ideal points less than the median, \( m_j(\ell) \), must prefer \( m_j(\ell) \) to \( x \). So, no coalition can deviate along a given dimension in a way that produces an outcome preferred by all members of the coalition. Thus, \( \sigma \) is coalition-proof along all dimensions.

\[ \therefore, \text{by Lemma A.8, it is coalition-proof.} \]
A.4 Sincere Voting Results

Notation A.12. \( o_j (x, y) \) denotes the outcome in sincere part-by-part voting between \( x \) and \( y \). All other notation is as before.

Theorem A.13. Consider sincere part-by-part voting between \( m(\ell_i) \) and \( x \neq m(\ell_i) \). Then, along all dimensions, the result will be \( o_j (m(\ell), x) = m(\ell) \).

Proof. Assume the outcome is not \( m(\ell) \). Thus, along some dimension \( j \), \( o_j (m(\ell), x) \neq m_j (\ell) \). Take any such \( j \). So, \( |\{i : x_j R_i m_j (\ell)\}| \geq \frac{n}{2} \). That is, at least half of the justices must weakly prefer \( x \) to the median of the ideal points along dimension \( j \). Note that \( x \neq m_j (\ell) \) or else \( o_j (m(\ell), x) = m_j (\ell) = x \). That is, \( x \) and the median of the ideal points cannot be the same along dimension \( j \) or else the outcome would be the median along dimension \( j \), regardless of whether the votes were for \( x \) or \( m_j (\ell) \). Without loss of generality, assume \( x_j > m_j (\ell) \). So, \( x_j R_i m_j (\ell) \) implies \( \ell_j (i) > m_j (\ell) \). That is, if we assume that \( x_j \) is greater than the median of the ideal points, then a justice can only weakly prefer \( x_j \) to the median of the ideal points along \( j \) when his ideal point along \( j \) is greater than the median of the ideal points along \( j \). Thus, since \( x_j \) receives at least half of the votes, at least half of the justices must have ideal points along dimension \( j \), \( \ell_j (i) \), equal to or greater than \( x_j \)—\( |\{i : \ell_i > x_j\}| \geq \frac{n}{2} \). Since \( m_j (\ell) \) is the median along dimension \( j \), \( |\{i : \ell_i \leq m_j (\ell)\}| \geq \frac{n}{2} \). That is, by definition, at least half of the justices must have ideal points equal to or less than the median of the ideal points. Since \( x_j > m_j (\ell) \), \( |\{i : \ell_i \leq x\}| \geq \frac{n}{2} \)—at least half of the ideal points along \( j \) must be less than or equal to \( x_j \), as well. Thus, as \( |\{i : \ell_i \geq x\}| \geq \frac{n}{2} \) and \( |\{i : \ell_i \leq x\}| \geq \frac{n}{2} \), \( x_j \) is the median along dimension \( j \). So, \( o_j (m(\ell), x) = x_j = m_j (\ell) \), which is a contradiction.

\[ \therefore \] the outcome along all dimensions is \( m(\ell) \). \(\Box\)

Theorem A.14. For any agenda that includes the issue-by-issue median, the result of sincere voting on the agenda in which the winner of each vote is voted on against the next item on the agenda is the issue-by-issue
median. Formally, let \( y_i \) be the outcome after \( i \) votes, defined recursively as

\[
\begin{align*}
y_1 &= x_1 \\
y_i &= o(y_{i-1}, x_i) & \text{if } i > 1.
\end{align*}
\]

Then if \( \exists i \in \{1, \ldots, n\} : x_i = m(\ell) \),

\[
y_n = m(\ell).
\]

Proof. \( y_{i-1} \) be the outcome which is first voted on against \( x_i \),

\[
y_{i-1} = \begin{cases} 
  x_2 & \text{if } i = 1 \\
  x_1 & \text{if } x = 2 \\
  o(\ldots o(o(x_1, x_2), x_3), \ldots, x_{i-1}) & \text{otherwise}
\end{cases}
\]

Assume the issue-by-issue median of the ideal points lies somewhere on the agenda—\( \exists j \in \{1, \ldots, n\} : x_j = m(\ell) \). Take any such \( j \), a place on the agenda where the alternative under consideration is the issue-by-issue median. By Theorem A.13, \( y_j = o(y_{j-1}, x_j) = x_j = m(\ell) \). Likewise, for any \( i \) such that \( j < i \leq n \), given that \( o(y_{i-2}, x_{i-1}) = m(\ell) \), \( o(y_{i-1}, x_i) = o(m(\ell), x_i) = m(\ell) \). That is, given that the outcome up until vote \( i \) is the issue-by-issue median of the ideal points, the result of vote \( i \), pairing the issue-by-issue median against an alternative, must be the issue-by-issue median. Thus, by induction, \( o(y_n) = m(\ell) \).

\[\therefore y_n = m(\ell)—\text{the final outcome is the issue-by-issue median of the ideal points.} \]
Figure 1: Preference Configuration with No Condorcet winner
Table 1: Configurations of Opinion Types

<table>
<thead>
<tr>
<th>Judgment in the Case</th>
<th>Agree in Full</th>
<th>Agree in Part</th>
<th>Disagree in Full</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agree in Full</td>
<td>Join the Majority Opinion</td>
<td>Concur in Part and Concur in the Judgment</td>
<td>Concur in the Judgment (Special Concurrence)</td>
</tr>
<tr>
<td>Agree in Part</td>
<td>Not Applicable</td>
<td>Concur in Part, Concur in the Judgment in Part, and Dissent in Part</td>
<td>Concur in the Judgment in Part and Dissent in Part</td>
</tr>
<tr>
<td>Disagree in Full</td>
<td>Not Applicable</td>
<td>Concur in Part and Dissent in Part</td>
<td>Dissent</td>
</tr>
</tbody>
</table>