Pre-electoral Debate: The Case of a Large Election∗

(Preliminary and Incomplete)

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Abstract

The model presented in this paper captures some of the effects of pre-electoral debate on the incentives for information acquisition for voters that belong to different ideological strands. We introduce the option to publicly share information into a fairly standard model of information aggregation through an election with costly information acquisition. We find that this option dramatically changes the incentive to acquire information. Without the option to share one’s signal no extremist has any incentive to acquire information. With this option the extremists’ incentive to acquire information is even stronger than the independents’ incentive to acquire information. In equilibrium the extremists acquire more information than the independents. We use this to explain the empirically observed correlation between extremism and information.

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1 Introduction

“Highly informed voters consist of a significantly more polarized subset of the electorate than uninformed voters”. This is one of the main conclusions of “The Relationship between Information, Ideology and Voting Behavior” by Palfrey and Poole (1987).¹ In this paper we suggest that extremists might have higher incentives for information acquisition if elections are preceded by debates. In our model extremist voters acquire significantly more information in equilibrium than independent voters. The main modeling innovation that allows us to obtain this result is that voters can engage in a debate before the vote is taken.

National elections are social events. Pre-electoral debates are a key characteristic of national elections that has been overlooked in the literature of game theoretic models of elections. Such pre-electoral debates are relevant: According to a poll conducted by The American National Election Studies (ANES) poll 34% of respondents in 2004 reported yes to the question, “During the campaign, did you talk to any people and try to show them why they should vote for or against one of the parties or candidates?” (The National Election Studies 2004) This desire to convince can lead to the desire to be more informed. This leads us to believe that a model of information acquisition before an election without some form of communication between voters is incomplete.

The following rationales to acquire information about candidates running for an election have been modeled. In Martinelli (2006) or Gerardi and Yariv (2004) (and many others) voters acquire private information to make the correct decision in an election. In David Baron’s model of the media voters become informed since political information is relevant to their private decisions on consumption or investment (Baron 2004). Bernhardt, Krasa and Polborn (2006) model the media as a consumption good: people read or watch for fun. We take up the first rationale and add a fourth: A voter might watch the news to obtain valid arguments to convince their fellow citizens to vote for their preferred party.

To do so we incorporate a pre-electoral debate into a fairly standard model of an election with costly information acquisition. In our model voters

¹There are some similar results in the literature on political psychology see for example Sidanius (1988)
first have to decide whether to acquire a costly signal. Then they have to decide whether to make their private signal public. In our model a “debate” consists of the simultaneous publication of the signals of the voters that choose to publish their signals. There are three different camps of voters in our model: left and right wing extremists and independents. They are distinguished by the amount of pain they suffer from the choice of a wrong candidate. A left wing extremist is assumed to suffer quite a bit if the right wing candidate is chosen in a situation that would have called for the left wing opponent to run the country. Contrary to that, the left wing extremist does not experience any disutility if the left wing candidate wins the election when the right wing candidate would have been the "objectively" better choice. Independents suffer equally much from the erroneous choice of either candidate. So while information about the state of the world matters for all voters, it matters a great deal more for independents. No finite amount of information will induce an extremists to switch allegiance to the opposite candidate.

Without a pre-electoral debate none of the extremists would have incentive to acquire costly information, since this information would have no bearing on their optimal voting strategy. In equilibrium extremists will always vote for their preferred candidate, no matter what kind of information they would see. The option to share information significantly changes the incentive structure for information acquisition. The optimal voting strategy of any extremist remains unchanged. However, they now have a chance to use their knowledge of politics to convince some of the independent voters to vote for their preferred party.

We show this model has an equilibrium that satisfies the following four criteria: First, no voter has an incentive to deviate from their strategy, given everyone else’s strategy. Second, extremists of both camps are equally likely not to acquire information. Third, independents broadcast all signals that they acquire. Lastly, voters vote sincerely.

\footnote{We are modeling the national election as an extended Poisson game following Myerson (1998). We cannot use Nash equilibrium due to the fact that we have an unknown number of players, but instead use the Myerson (1998) equilibrium concept as it is conceptually similar to the Nash concept.}

\footnote{The first equilibrium requirement entails that voters vote strategically in equilibrium. It is well known that strategic and sincere voting need not coincide Austin-Smith and Banks (1996) where the first to make this point. We will show that a strategy profile that}
We obtain two major results: First, in every equilibrium extremists acquire more information than independents. This result is quite surprising considering previous models claim exactly the opposite. Secondly, contrary to Feddersen and Pesendorfer’s model on the “Swing Voter’s Curse” a pivotal voter’s preferences are even stronger than the preferences of a sincere voter. (Feddersen and Pesendorfer 1996) Our model exhibits a “Swing Voter’s Boon”.

To obtain some intuition for the first result consider a voter that obtained a signal that the left wing candidate is probably the better candidate given the current state of the world. Should this voter share his signal with the entire electorate? The independent voters take all public signals into account when voting. Therefore a broadcast of this signal increases the likelihood that the left wing candidate will win the election. This diminishes the probability that the right wing candidate is wrongfully chosen. Observe that left wing extremists only care about this probability. So they would always send the signal. For an independent voter matters are a little bit more complicated. A broadcast of the signal also increases the probability that the left wing candidate is chosen wrongly. However, given that the independent obtained this signal that indicates that the left wing candidate is the better choice given the current state of affairs, the first consideration, the consideration of a wrong choice of the right wing candidate outweighs the second, an independent would also send the signal. The independent will also broadcast the signal. Now let us revisit this story: While a left wing extremist see only benefits is sending the signal the independent also sees some disadvantages. The same holds true for comparison for signals in favor of the right wing candidate. In short: signals have a higher value for the extremists, as a result extremists acquire more information in equilibrium.

Concerning the “Swing Voter’s Boon” result observe that a voter must condition, “his action not only on his information but also on what must be true about the world if his action matters.” (Feddersen and Pesendorfer 1996) In our model all independents have the same information in the voting stage. Voting sincerely, they will all vote for the same candidate. If an independent is pivotal this implies that one candidate, say the left wing candidate has all of the votes of the other independents and all the left wing extremists. All right wing extremists vote for the right wing candidate.

satisfies all these criteria exists.
This implies that many of the right wing extremists candidate remained silent, thus creating even stronger incentives for the pivotal independent to vote for the left wing candidate. Interestingly the event of being pivotal in our model carries more positive information about the candidate that the independents are voting for. So information exchange in this model inverts the swing voter’s curse. This is unfortunate insofar as that another main result of Palfrey and Poole (1987) is that more informed voters are more likely to turn out at the polls. The persistence of a “Swing Voter’s Boon” implies that our results cannot simply be combined with Feddersen and Pesendorfer (1996) to obtain a unified theory that explains these two main results of Palfrey and Poole (1987).

There is a large literature about voting in a situation where the best choice is uncertain. The basis for much of this literature is in the writings of Condorcet ([1785] 1976) in which identical jurors must choose an option that coincides with an unknown state of the world based on a private signal. Martinelli (2006) has only one type of voter, however they are allowed to buy a signal with varying strength based on a cost function. Under some conditions on the cost and signal structure Martinelli shows that equilibria exist with the property that information acquisition is positive for any size of an electorate. Oliveros (2006) extends this to have a two dimensional continuum of types and shows that preference is important when considering endogenous information acquisition. Persico (2004) and Mukhopadhaya (2003) both look at a case of endogenous information acquisition in which signal strength and cost are fixed however their results are oriented to small juries rather than an election.

There is a small literature on information exchange before an election. Coughlan (2000) shows that by allowing for “even the most minimal communication can significantly change the conclusions of model analysis.” Austen-Smith and Feddersen (2004) compare unanimity and majority rule under the assumption that the vote is preceded by a cheap talk stage in which all voters can send messages. To the best of our knowledge Mathis (2006) is the only paper in this literature that allows for verifiable messages. The literature on endogenous information acquisition before an election is more established. Martinelli (2006) and Oliveros (2006) provide conditions under which the probability that the correct candidate is being picked is increasing in the

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4In fact this result has been corroborated in various empirical studies. (?)
size of the electorate and converging to 1 when information acquisition is endogenous. Mukhopadhaya (2003) views the acquisition of costly information in a small committee as a public good problem. Finally Persico (2004) studies optimal committee design with endogenous information.

We are aware of only one paper in the intersection between these two literatures. Gerardi and Yariv (2004) allow for endogenous information acquisition and for a pre-electoral debate. They show that incentives for information acquisition have to be traded off against efficient information aggregation when designing an optimal voting rule for a small committee.

All papers on information exchange before an election focus on the design of a small committee. Our goal is not normative: we hope to explain the observed correlation between extremism and information. Focusing on a large electorate our paper is related to Myerson (1997) and Myerson (1998) in that the game presented in this paper is an extended Poisson game.

2 The Model

We define an extended Poisson game following Myerson (1998) \( \{ \Omega, \alpha, T, n, p, C, U \} \) with the state space \( \Omega = \{ l, r \} \) and a common prior of \( \alpha = \frac{1}{2} \) that the state is \( l \). The parameter \( T \) denotes the set of voter types \( T = T_1 \times T_2 \) where \( t_1 \in T_1 = \{ l, i, r \} \) denotes the preference type of a voter and \( t_2 \in T_2 = \{ l, r \} \) denotes the informational type of a voter. The size of the electorate is a Poisson random variable with mean \( n \). For each state of the world \( \omega \in \{ l, r \} \) the preference and information type of a voter are drawn from independent distributions \( p_1(t_1 | \omega) \) and \( p_2(t_2 | \omega) \) respectively. We assume that the preference type of a voter is independent of the state of the world, and that any voter is equally likely to be of preference type \( l \) or \( r \), we let \( p_1(t_1 = l | \omega) = p_1(t_1 = r | \omega) = \eta < \frac{1}{2} \) for \( \omega = l, r \). We assume that \( n(1 - 2\eta) > 1 \) which amounts to a very weak statement of the requirement that the electorate is “large”. Finally the informational type \( t_2 \) of a voter depends on the state of the world, we assume that \( p_2(t_2 = l | \omega = l) = p_2(t_2 = r | \omega = r) = p > \frac{1}{2} \), \( t_2 \) can be interpreted as an informative signal about the state of the world.

The game proceeds according to the following time-line: First Nature draws a state of the world and an electorate. Secondly voters choose whether to acquire information. Thirdly voters choose whether to publish their information. Fourth voters learn the value of all the information that has been
published. Fifth the Supreme Court throws a fair coin which can come up \( R \) or \( L \). Sixth voters vote for \( L \) or \( R \).

The exchange of information is modelled as a persuasion game following Glaezer and Rubinstein (2001), Milgrom and Roberts (1986) and Shin (1994). A voter that chooses to acquire information learns his informational type \( t_2 \). The voter that knows his signal \( t_2 \) can decide to either share this with the electorate as a whole, or he can stay silent. He cannot lie, reveal partial truths or communicate with subsets of the entire electorate. The electorate learns the total number of \( l \) and \( r \) signals that have been published, denoted by \( s_r, s_l \). No voter learns anything beyond that. This implies in particular that voters do not learn the total number of voters, or the number of voters in any of the camps. They also do not learn how many signals have been acquired.

In the voting stage voter have to pick between \( R \) and \( L \), there is no abstention in the model.\(^5\) The vote is decided by majority rule. In case of a tie the Supreme Court picks on the behalf of independent voters, meaning that the Supreme Court picks the candidate, whom an independent would like better given all the information contained in the public signal vector \( \vec{s} = (s_l, s_r) \). If this does not yield a clear cut result then the Supreme Court picks the candidate that was determined by the fair coin toss in the fifth stage.

The utility of a voter depends on the outcome of the vote \( W = L \) or \( R \) (which in turn depends the votes of all voters and in case of a tie on \( \vec{s} \)) his decision whether to acquire information or not \( (x_1 \in \{0, 1\}) \), the state of the world and his preference type. We have that:

\[
U(W, x_1, t_1, \omega) = \begin{cases} 
-\delta(t_1) - x_1 c & \text{if } \omega = r \text{ and } W = L \\
-(1 - \delta(t_1)) - x_1 c & \text{if } \omega = l \text{ and } W = R \\
-x_1 c & \text{otherwise}
\end{cases}
\]

The preference type of a voter, \( t_1 \), determines the disutility a voter incurs when candidate \( X \) is picked in state \( Y \). For leftist voters we have that \( \delta(l) = 0 \). A leftist does not receive any disutility from a wrong choice of

\(^5\)We will show in section 7 that our assumption that voters cannot abstain is without loss of generality in the context of the present model, in the sense that we would obtain the same equilibrium result if we where to allow voter’s to abstain.
candidate $L$ however he receives the maximal disutility when candidate $R$ wins the election in state $l$. Conversely we set $\delta(r) = 1$, so a rightist’s utility is minimized when $L$ wins in state $r$. Finally we assume that independents suffer an equal amount of disutility from either mistake, $\delta(i) = \frac{1}{2}$.


***********preference picture************

3 Strategies and Equilibrium

The strategy set of a voter $C = C_1 \times C_2 \times C_3$ is composed of an information acquisition-strategy $C_1 = \{0, 1\}$ a broadcasting strategy $C_2 = (L \times R)$ and a voting-strategy $C_2 = \{f : N_0 \times N_0 \times T \rightarrow \{L, R\}\}$ where $f$ is allowed to depend on $T_2$ only if the voter chose to acquire information in the first step. The broadcasting strategy consists of two elements: $L = \{\emptyset, l\}$ is the choice to broadcast a signal with value $l$, and $R = \{\emptyset, r\}$ is the decision to broadcast a signal with value $r$, in each case $\emptyset$ stands for the suppression of the signal. $C_2$ is the set of all voting rules given vector of broadcast signals $s$ and $(t_1, x_1 t_2)$.

We denote a mixed strategy of a a voter of type $t_1$ by $\tau_{t_1}$. We define $\tau = (\tau_l, \tau_i, \tau_r)$ as a mixed strategy profile for the game. So $\tau_l(1, \emptyset, r, f)$, for example, denotes the probability that a leftist acquires information, only broadcasts right wing signals and follows the voting rule $f$. We define $\tau_{t_1}(\cdot, l, r, f)$ as the probability that a voter of type $t_1$ passes on both signals and plays voting strategy $f$. Formally $\tau_{t_1}(\cdot, l, r, f) = \tau_{t_1}(0, l, r, f) + \tau_{t_1}(1, l, r, f)$, the expressions $\tau_{t_1}(\cdot, \cdot, r, f)$, $\tau_{t_1}(\cdot, l, \cdot, \cdot)$ and so forth are defined analogously. We define $EU_{t_1}(\tau, x)$ as the expected utility of a voter of preference type $t_1$ when this voter uses the pure strategy $x$ while all other voters follow the profile $\tau$. The probability that candidate $L$ wins in state $r$ when all other voters follow strategy $\tau$ and the voter under consideration follows strategy $x$ is denoted by $Pr(L, r|\tau, x)$. Conversely the probability that $R$ wins in state $l$ when the voter uses the pure strategy $x$ and all other voters follow the strategy profile $\tau$ is denoted by $Pr(R, l|\tau, x)$. Given these definitions we can express the expected utility of a voter of type $t_1$ $EU_{t_1}(\tau, x)$ as follows:

$$EU_{t_1}(\tau, x) = -\delta(t_1)Pr(L, r|\tau, x) - (1 - \delta(t_1))Pr(R, l|\tau, x) - cx_1. \quad (1)$$
where $x_1$ denotes the first component of the player’s strategy, $x_1$ equals 1 if and only if the voter acquired information.

**Definition 1** A strategy profile $\tau$ is an equilibrium if:

1. $EU_{t_1}(\tau, x) \geq EU_{t_1}(\tau, y)$ for all $x \in C : \tau_{t_1}(x) > 0$ and all $y \in C$ for all $t_1 \in T_1$.

2. The strategy profile is symmetric in the sense that extremists of both camps are equally likely not to acquire any information, we have that $\tau_l(0, \cdot, \cdot, \cdot) = \tau_r(0, \cdot, \cdot, \cdot)$.

3. The independents will pass on any signal that they acquire, we have that $\tau_i(1, l, \emptyset, f) = \tau_i(1, \emptyset, r, f) = \tau_i(1, \emptyset, \emptyset, f) = 0$ for all $f \in C_2$.

4. Voters vote sincerely.

The first condition says that any strategy that a voter plays with a positive probability in $\tau$ has to be a best response to the strategy profile $\tau$. This is Roger Myerson’s definition of an equilibrium in an extended Poisson game (Myerson 1998). This definition alone does not rule out certain odd behaviors. A profile $\tau$ that prescribes that all voters vote $L$ if more than 2 signals have been sent can be an equilibrium profile. In this case voters can infer that there are more than 2 voters in the electorate. So no voter has a chance to change the outcome of the vote.

To deal with such problems we require conditions 2, 3 and 4. An important question arises: is the set of all 4 conditions compatible? It is, for example, well known that sincere and strategic voting need not coincide. It is easy to show that the second requirement is compatible with the first. It turns out that the most involved proofs in this paper are devoted to showing that the third and fourth requirements are compatible with the first see section 7 and 8.

**4 Voting Strategies**
4.1 Sincere Voting

Sincere voting requires a voter to vote for the candidate whom he would choose if he alone had to determine the winner based on all information available to him. We define $Pr[\vec{s}, x_1 t_2 | \omega]$ as the probability that the signal vector and the voter’s own information is $\vec{s}, x_1 t_2$ is state $\omega$, where $x_1 t_2$ is equal to 0 if $x_1 = 0$ meaning that the voter has no private information beyond the public vector of signals and $x_1 t_2 \in \{ l, r \}$ for the case that the voter has private information that is the case $x_1 = 1$. We calculate a voter’s expected utility of candidates $L, R$ when the information available to the voter is the signal vector $\vec{s}$ and the voter’s own private information $\delta t_2$ respectively as

$$-\delta(t_1) \frac{Pr[\vec{s}, x_1 t_2 | r]}{Pr[\vec{s}, x_1 t_2 | l] + Pr[\vec{s}, x_1 t_2 | r]}$$

and

$$-(1 - \delta(t_1)) \frac{Pr[\vec{s}, x_1 t_2 | l]}{Pr[\vec{s}, x_1 t_2 | l] + Pr[\vec{s}, x_1 t_2 | r]}.$$  

If candidate $L$ is chosen only one type of mistake matters: namely the case that $L$ is chosen in state $r$. To obtain a voter’s expected utility the probability of this mistake has to be multiplied with the disutility that this voter receives if $L$ is chosen in state $r$, this disutility is $-\delta(t_1)$. Analogously we obtain the expected utility of candidate $R$ given $\vec{s}, x_1 t_2, t_1$.

Voting sincerely requires a voter to vote for candidate $L$ if expression $2$ is larger than expression $3$. If the opposite inequality holds true a voter that votes sincerely has to vote for candidate $R$. If the two expressions are equal the voter is indifferent between the two candidates. We assume that indifferent voters base their vote on the coin thrown by the Supreme Court, if the coin comes up on $X$ they vote for $X$.

4.2 Extremists

Observe that $Pr[\vec{s}, x_1 t_2 | \omega] \neq 0$ for all possible signal profiles $(\vec{s}, x_1 t_2)$ and both states $\omega = l, r$. Our assumption that extremists vote sincerely implies the following Lemma:

**Lemma 1** In equilibrium all leftists vote for candidate $L$ and rightists vote for candidate $R$. 

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**Proof** Expression 2 equals to 0 if $\delta(t_1) = 0$. On the other hand for no vector $(\vec{s}, x_1 t_2)$ does the probability $Pr(\vec{s}, x_1 t_2|l)$ equal 0. So expression 3 is always negative for a leftist. Consequently expression 2 is larger than expression 3 for any $(\vec{s}, x_1 t_2)$. A leftist votes sincerely if he votes for L. The same argument holds mutatis mutandum for a rightist. □

### 4.3 Independents

We show next that in equilibrium the independent voters vote according to a simple cutoff rule.

**Lemma 2** There exists a cutoff $g(n, p, \tau)$ such that an independent voter votes for L if $s_l > s_r + g(n, p, \tau)$ and votes for R if $s_l < s_r + g(n, p, \tau)$, otherwise the independent is indifferent.

**Proof** We need to show that

$$Pr[\vec{s}, x_1 t_2|l] > Pr[\vec{s}, x_1 t_2|r]$$

if and only if $s_l > s_r + g(n, p, \tau)$ for some expression $g(n, p, \tau)$.

If the voter has not invested in information acquisition, that is if $x_1 t_2 = 0$, then we have that:

$$Pr[\vec{s}, x_1 t_2|l] = \frac{e^{-npS_l}[npS_l]^{s_l}}{s_l!} \cdot \frac{e^{-n(1-p)S_r}[n(1-p)S_r]^{s_r}}{s_r!}$$

$$Pr[\vec{s}, x_1 t_2|r] = \frac{e^{-n(1-p)S_l}[n(1-p)S_l]^{s_l}}{s_l!} \cdot \frac{e^{-npS_r}[npS_r]^{s_r}}{s_r!}$$

for

$$S_l = \eta[\tau_l(1, l, \emptyset, \cdot) + \tau_l(1, l, r, \cdot) + \tau_r(1, l, \emptyset, \cdot) + \tau_r(1, l, r, \cdot)] + (1 - 2\eta)[\tau_l(1, l, \emptyset, \cdot) + \tau_l(1, l, r, \cdot)]$$

$$S_r = \eta[\tau_l(1, \emptyset, r, \cdot) + \tau_l(1, l, \cdot, \cdot) + \tau_r(1, \emptyset, r, \cdot) + \tau_r(1, l, \cdot, \cdot)] + (1 - 2\eta)[\tau_l(1, \emptyset, r, \cdot) + \tau_l(1, l, \cdot, \cdot)]$$

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6 One might ask: isn’t it obvious that $\tau_r(1, l, \cdot, \cdot) = \tau_l(1, \cdot, r, \cdot) = 0$? Yes, this is obvious but we believe that there should be no loose ends in a proof. At this point we believe that this setup helps us to show most straightforwardly that this “obvious” statement indeed holds true.
Observe that expression 2 is larger than expression 3 if and only if
\[
\frac{Pr[\vec{s}|l]}{Pr[\vec{s}|r]} = \left(\frac{p}{1 - p}\right)^{|s_l - s_r|} e^{[n(1-2p)(s_l - s_r)]} < 1
\]
we can rearrange to find the cutoff condition
\[
s_l < \frac{n(2p - 1)}{\ln\left(\frac{p}{1-p}\right)} (S_l - S_r) + s_r
\]
which is always well defined in the given environment. If equation 6 holds with equality then the voter is indifferent.

We next need to show that an independent voter that has observed a signal follows the same rule when voting sincerely. To see this assume that the voter has observed an \(l\)-signal. So the public signal count without his signal then becomes \(s_l - 1\) and \(s_r\). In this case the voter estimates the probabilities \(Pr[\vec{s}, tx_2|l]\) and \(Pr[\vec{s}, tx_2|r]\) as
\[
Pr[\vec{s}, tx_2|l] = p \frac{e^{-npS_l} [npS_l]^{(s_l-1)}}{(s_l-1)!} \cdot \frac{e^{-n(1-p)S_r} [n(1-p)S_r]^{s_r}}{s_r!}
\]
\[
Pr[\vec{s}, tx_2|r] = (1 - p) \frac{e^{-n(1-p)S_l} [n(1-p)S_l]^{(s_l-1)}}{(s_l-1)!} \cdot \frac{e^{-npS_r} [npS_r]^{s_r}}{s_r!}
\]
which yields the exact same condition on the voter’s behavior. So we find that the statement of the Lemma holds true for \(g(n, p, \tau) = \frac{n(2p - 1)}{\ln\left(\frac{p}{1-p}\right)} (S_l - S_r)\) \(\square\)

The assumption of the Poisson distribution is essential for the proof to hold. Under the assumption of the Poisson distribution a voter’s own signal carries as much information as anyone else’s. For other assumptions the case in which the independent has not acquired any information differs significantly from the alternative case in which he has acquired information.

In the next section on optimal communication we will formally show that no extremists will ever send evidence that favors the opposing candidates or \(\tau_l(1, \cdot, r, \cdot) = \tau_r(1, l, \cdot, \cdot) = 0\). This implies that \(S_l = S_r\) and the independents decision rule becomes vote for \(R\) if \(s_r > s_l\), vote for \(L\) if \(s_l > s_r\). \(^7\)

\(^7\) An interesting comparison with Shin (1994) arises. The cutoff \(x\) can be interpreted as the burden of proof: if \(x\) is large the leftists face a large burden of proof, in the sense
Sincere voting does not imply anything for the case that \( s_l = s_r + g(n, p, \tau) \). We assume that independent voters follow the coin throw of the Supreme Court in this particular case.

We conclude this section by defining the voting rules \( f_l, f_i, f_r \in C_2 \) by \( f_l(x) = L \), \( f_r(x) = R \) and \( f_i(x) = L \) if \( s_l > s_r \), \( f_i(x) = R \) if \( s_l > s_r \) and finally if \( s_l = s_r \) then \( f_i(x) = R \) if and only if the publicly thrown coin came up \( R \). In the two preceding Lemmata 1 and 2 we have shown that in any equilibrium profile \( \tau \) the voters will only follow these strategies. In short, we have shown that \( \tau_1(\cdot, \cdot, \cdot, f_{t_1}) = 1 \) for \( t_1 \in \{ l, i, r \} \).

5 Optimal Communication

**Lemma 3**  In equilibrium we have that \( \tau_l(1, \emptyset, \emptyset, f_l) = \tau_r(1, \emptyset, \emptyset, f_r) \).

**Proof** We compare \( EU_{t_1}(\tau, x) \) to \( EU_{t_1}(\tau, x') \) where in \( x \) the voter plays the information strategy \( (1, \emptyset, \emptyset) \) and in \( x' \) the voter plays the information strategy \( (0, \emptyset, \emptyset) \). Remember that a voter’s expected utility can, for any pure strategy \( y \) be expressed as

\[
EU_{t_1}(\tau, y) = -\delta(t_1)Pr(L, r|\tau, y) - (1 - \delta(t_1))Pr(R, l|\tau, y) - cy_1.
\]

So we obtain that \( EU_{t_1}(\tau, x) - EU_{t_1}(\tau, x') = -c \) since the extremists voting strategies never depend on their private signals. Consequently the voter strictly prefers \( x' \) to \( x \).

Observe that the strategies \( (0, \emptyset, \emptyset, f), (0, l, \emptyset, f), (0, \emptyset, r, f), (0, l, r, f) \) are equivalent for any \( f \) in the sense that a voters plan to use information cannot change the outcome if that voter acquires no information. We assume without loss of generality that \( \tau_{t_1}(\{(0, l, \emptyset,), (0, \emptyset, r,), (0, l, r,)\}) = 0 \) for all types \( t_1 \), in words: we assume that a voter that does not acquire information does not plan to send any signals.

Which signals are voters going to pass along in equilibrium? We will show that in equilibrium the extremists will only send the signals that support their case. In equilibrium requirement 3 we assumed that independents that they need to provide more signals for any given amount of right signals to convince an independent that \( L \) is the better candidate. This result is stronger than Shin’s as \( x \) can be completely characterized in terms of \( n, p \) and \( \tau \).
will send any signal that they acquired. We will show in section 8 that this requirement is consistent with the first equilibrium requirement. We will show that given everyone else's strategy profile no independent has an incentive to acquire a signal and to keep it private.

It is convenient to define some more notation for the analysis of information acquisition and broadcasting. In our model voters only care about the probability that candidate L is chosen in state r and the probability that candidate R is chosen in state l. To analyze the optimal broadcasting strategies we need to find out how the broadcasts of l and r-signals change these probabilities. We define $Pr(E|\tau, \omega)$ as the probability that event E occurs in state $\omega$ when all voters follow strategy $\tau$. Let us assume the stance of one particular voter, and let us define the events $R$ and $L$ such that in these events candidates $R$ and $L$ win the election, when the voter under consideration does not send a signal. Additionally we define the events $X + r$ ($X + l$) as the event that the candidate $X$ wins the election given that the voter sent an $r$- ($l$)-signal.

The probabilities that matter for the expected utility of a voter are $Pr(R, l|\tau, x)$ and $Pr(L, r|\tau, x)$. A voter that have observed a signal, say $l$, can calculate these probabilities as $Pr(R|\tau, l)$ and $Pr(L|\tau, r)(1-p)$, assuming that the voter follows the voting strategy $f_t$ himself. The effect of an additional signal $l$ or $r$ being sent on the expected utility comes through the effect of that additional signal on the probabilities of the various mistakes. We define define $\Delta_{Y,\omega}^z$ as the change in probability that candidate $Y$ is being chosen in state $\omega$ when another $z$ signal is being sent. Formally

$$\Delta_{Y,\omega}^z := Pr(Y + z|\tau, \omega) - Pr(Y|\tau, \omega).$$

We are only concerned with the cases that $Y = L$ and $\omega = r$ and the alternative case $Y = R$ and $\omega = l$ as we are only concerned with the choice of the wrong candidate. Observe that for these cases we have that $\Delta_{Y,\omega}^z > 0$ if $\omega \neq z$ that is the probability of a mistake is increasing if another wrong signal is being sent. Conversely we have that $\Delta_{Y,\omega}^z < 0$ if $\omega = z$, the probability of a mistake is decreasing if another correct signal is being sent.

We are now ready to state and prove a Lemma about the equilibrium information acquisition strategies of extremists.

**Lemma 4** In equilibrium we have that $\tau_l(1, \emptyset, r, f_l) = \tau_l(1, l, r, f_l) = \tau_r(1, l, \emptyset, f_r) = \tau_r(1, l, r, f_r) = 0$. 

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Proof. Conditional on having received an $l$-signal the following holds true for a left wing extremist. $EU_l(\tau, x) - EU_l(\tau, x') = -p(Pr(R + l|\tau, l) - Pr(R|\tau, l)) = -p\Delta^l_{R,l} > 0$ for $x = (1, l, \cdot, f_l)$ and $x' = (1, 0, \cdot, f_l)$. So any left wing extremist strictly prefers sending an $l$ signal strictly to being silent as $\Delta^l_{R,l} < 0$. Let us check next that no leftist extremist will send an $r$ signal in equilibrium. To see this observe that, conditional on having received an $r$, we have that $EU_l(\tau, x) - EU_l(\tau, x') = -p(Pr(R + r|\tau) - Pr(R|\tau, l)) = -(1 - p)\Delta^r_{R,l} < 0$ for $x = (1, \cdot, r, f_l)$ and $x' = (1, \cdot, 0, f_l)$, so a left wing extremist strictly prefers to stay silent. Analogous arguments hold for right wing extremists.

With the assumptions that $\tau_l(0, \cdot, \cdot, \cdot) = \tau_r(0, \cdot, \cdot, \cdot)$ and $\tau_l(0, \cdot, \cdot, \cdot) = 0$ for all types $t_1$ Lemma 4 implies that $\tau_l(1, l, \emptyset, f_l) = \tau_r(1, \emptyset, r, f_r)$ as $\tau_l(0, \emptyset, \emptyset, f_l) = 1 - \tau_l(1, l, \emptyset, f_l)$ and $\tau_r(0, \emptyset, \emptyset, f_r) = 1 - \tau_r(1, \emptyset, r, f_r)$. To save on notation we say from now on that $\lambda$ is the probability that an extremist acquires information in the equilibrium strategy profile $\tau$. Lemmata 3 and 4 imply that the extremists always pass one information that supports their case.

Lemma 4 together with the symmetry assumption also implies that $S_l = S_r$ and consequently the independents equilibrium rule for sincere voting becomes: vote for the candidate for whom more signals have been broadcast, in short, $g(n, p, \tau) = 0$.

The symmetry of the equilibrium also implies that $\Delta^r_{L,r} = \Delta^l_{R,l}$ and $\Delta^l_{L,r} = \Delta^l_{R,l}$. To save on notation we call $\Delta^r_{L,r} = \Delta^l_{R,l} = \Delta^+$ and $\Delta^l_{L,r} = \Delta^r_{R,l} = \Delta^-$. This in turn implies that the left and right wing extremists benefit equally much from the acquisition of information: both value information at $-\frac{1}{2}\Delta^-$. Consequently the symmetry assumption 2 on $\tau$ in our equilibrium concept is consistent with our first assumption on $\tau$ that all players are best responding.

Economizing further on notation we say that $\pi$ is the probability that an independent acquires information in the equilibrium strategy profile $\tau$. Our assumption that independents will share any signal they acquired implies that $\tau_l(1, l, r, f_l) = \pi$.

We summarize that up until now we know that in any equilibrium $\tau$ left wing extremists vote for the left candidate and right wing extremists...
vote for the right candidate. Independents vote according to a very simple cutoff rule: if more \( l \)-signals have been published they vote for candidate \( L \), conversely if more \( r \)-signals have been published they vote for candidate \( R \), otherwise they base their vote on the coin of the Supreme Court. We also know that extremists would only broadcast signals in their favor. We show that our assumption that independents broadcast all their signals is consistent with equilibrium in section 8. We are now ready to prove our first main result, namely the fact that in equilibrium that extremists never acquire less information than the independents.

6 Extremists Are More Informed

Theorem 1  In an equilibrium the utility gain from acquiring information for an extremist is always greater than that of an independent.

The intuition for this proof is simple. In any equilibrium \( \tau \) the extremists only broadcast information in their favor. A left wing extremist, for example, would only broadcast an \( l \) signal. This reduces the probability that candidate \( R \) is chosen in state \( l \), at the same time this increases the probability of the alternative mistake namely that candidate \( L \) is chosen in state \( r \). A left wing extremist does not care about the increase in the probability of the second mistake. An independent also broadcasts an \( l \) signal if he receives one. However, differently from the extremists both effects are felt for the independent. The independent appreciates the fact that the probability of an erroneous choice of \( R \) is being reduced. At the same time an independent suffers from the fact that the additional \( l \) signal increases the probability that \( L \) is chosen in state \( r \).

Proof  We compare the expected utility of right wing extremist voter for the pure strategies \( x_r = (1, \emptyset, r, f_r) \) and \( x'_r = (0, \emptyset, \emptyset, f_r) \) to each other.

\[
EU_r(\tau, x_r) = -\frac{1}{2}pPr(L + r|\tau, r) - c \\
EU_l(\tau, x'_r) = -\frac{1}{2}pPr(L|\tau, r) \\
EU_r(\tau, x_r) - EU_r(\tau, x'_r) = -\frac{p}{2}(Pr(L + r|\tau, r) + Pr(L|\tau, r)) - c \\
= -\frac{p}{2} \Delta^- - c.
\]
Analogously we have for a left wing extremist, where \( x_l = (1, l, \emptyset, f_l) \) and \( x'_l = (0, \emptyset, \emptyset, f_l) \)

\[
EU_l(\tau, x_l) - EU_l(\tau, x'_l) = -\frac{p}{2} \Delta^+ - c.
\]

Finally let us investigate the utility difference for an independent, let \( x_i = (1, l, r, f_i) \) and \( x'_i = (0, \emptyset, \emptyset, f_i) \), we have that

\[
EU_i(\tau, x_i) = -\frac{1}{4} p Pr(L + r|\tau, r) - \frac{1}{4} (1 - p) Pr(R + r|\tau, l) - \frac{1}{4} (1 - p) Pr(L + l|\tau, r) - \frac{1}{4} p Pr(R + l|\tau, l) - c
\]

\[
EU_i(\tau, x'_i) = -\frac{1}{4} p Pr(L|\tau, r) - \frac{1}{4} (1 - p) Pr(R|\tau, l) - \frac{1}{4} (1 - p) Pr(L|\tau, r) - \frac{1}{4} p Pr(R|\tau, l)
\]

So the expected utility difference between acquiring and not acquiring becomes:

\[
EU_i(\tau, x_i) - EU_i(\tau, x'_i) = -\frac{1}{2} \left( p \Delta^- + (1 - p) \Delta^+ \right) - c.
\]

Finally observe that \( \Delta^+ > 0 \) so we have that

\[
EU_i(\tau, x_i) - EU_i(\tau, x'_i) + c = -\frac{1}{2} \left( p \Delta^- + (1 - p) \Delta^+ \right) < -\frac{p}{2} \Delta^-
\]

\[
= EU_l(\tau, x_l) - EU_l(\tau, x'_l) + c
\]

We conclude that for any equilibrium strategy profile \( \tau \) information is more valuable for extremists than for independents. \( \square \)

**Corollary 1** If the equilibrium probability that independents acquire information is positive then all extremists acquire information in equilibrium. If the equilibrium probability that an extremist acquires no information is positive, then no independent acquires any information in equilibrium.
Proof Mathematically we can express Corollary 1 as:

\[ \pi > 0 \implies \lambda = 1 \quad \text{and} \quad \lambda < 1 \implies \pi = 0. \quad (7) \]

Using the results derived in Theorem 1:

\[ 0 < \pi < 1 \implies EU_i(\tau, x_i) - EU_i(\tau, x_i') = 0 \]
\[ \implies -\frac{1}{2}(p\Delta^- + (1 - p)\Delta^+) = c \]
\[ \implies -\frac{p}{2}\Delta^- > c \]
\[ \implies EU_i(\tau, x_i) - EU_i(\tau, x_i') > c \implies \lambda = 1 \]

and

\[ 0 < \lambda < 1 \implies EU_l(\tau, x_l) - EU_l(\tau, x_l') = 0 \]
\[ \implies -\frac{p}{2}\Delta^- = c \]
\[ \implies -\frac{1}{2}(p\Delta^- + (1 - p)\Delta^+) < c \]
\[ \implies EU_l(\tau, x_l) - EU_l(\tau, x_l') < 0 \implies \pi = 0 \]

\[ \square \]

7 Sincere Voting is Strategic Voting

Equilibrium requirement 4 imposes that in any equilibrium \( \tau \) all voters follow \( \tau \). In section 4 we described the sincere voting strategies of the three voter types. It remains to be shown that no voter has an incentive to deviate from the sincere strategies given that all other voters vote sincerely. We need to show that a voter that votes for \( L \) following the sincere strategy, would like to vote for \( L \) if he knew that he was the pivotal voter. Clearly for the extremists this is very easy to see.

It is somewhat more difficult to see that an independent would like to vote sincerely. Consider the case of an equilibrium strategy profile \( \tau \) and a signal structure \( \vec{s} \) such that \( \tau \) prescribes that independents votes \( L \). We need to find out if any independent could improve their utility by voting for \( R \) instead. Would an independent who knows that he is pivotal think that the state \( r \) is more likely than the state \( l \)? We will show that the information
contained in the pivotality event never overturns the information in the public signal \( \tilde{s} \), in fact in most cases the information in the event of being pivotal strengthens the information contained in the public signal \( \tilde{s} \).

The intuition goes as follows: Suppose we have that \( s_l \geq s_r \), and suppose that the coin of the Supreme Court came up \( L \) (which is relevant only in the case that \( s_l = s_r \)). So all independents will vote \( L \) according to the strategy profile \( \tau \). An independent is pivotal if \( L \) and \( R \) received equally many votes. The fact that all independents vote for \( L \) implies that there must be more right wing extremists in the electorate than there are left wing extremists. Now consider the fact that more \( l \) signals have been sent. Taken together this implies it is likely that more right wing extremists decided to hide an \( l \) signal than there are a left wing extremist who hid an \( r \) signal.

The following Lemma will prove useful int his context.

**Lemma 5** Suppose that \( \text{Pr}(l|\bar{n},\bar{s}) \geq \text{Pr}(r|\bar{n},\bar{s}) \). Let \( \bar{n}', \bar{s}' \) be such that \( n'_r \geq n_r, n'_l \leq n_l, n'_l = n_i, s'_r \leq s_r, s'_l \geq s_l \) and either \( \bar{n}' \neq \bar{n} \) or \( \bar{s}' \neq \bar{s} \) or both, then we have that \( \text{Pr}(l|\bar{n}',\bar{s}') > \text{Pr}(r|\bar{n}',\bar{s}') \).

**Proof** Observe that \( \text{Pr}(l|\bar{n},\bar{s}) \geq \text{Pr}(r|\bar{n},\bar{s}) \) holds if and only if \( \text{Pr}(\bar{n},\bar{s}|l) \geq \text{Pr}(\bar{n},\bar{s}|r) \). We proceed by distinguishing two cases: 1. \( \pi = 0 \) and 2. \( \pi > 0 \).

**Case 1.** The inequality \( \text{Pr}(\bar{n},\bar{s}|l) \geq \text{Pr}(\bar{n},\bar{s}|r) \) holds if and only if

\[
\left( \frac{n_l}{s_l} \right) (\lambda p)^{s_l} (1-\lambda p)^{n_l-s_l} \left( \frac{n_r}{s_r} \right) (\lambda(1-p))^{s_r} (1-\lambda(1-p))^{n_r-s_r} \geq \left( \frac{n_l}{s_l} \right) (\lambda(1-p))^{s_l} (1-\lambda(1-p))^{n_l-s_l} \left( \frac{n_r}{s_r} \right) (\lambda p)^{s_r} (1-\lambda p)^{n_r-s_r} \iff \\
\left( \frac{p}{1-p} \right)^{s_l-s_r} \geq \left( \frac{1-\lambda p}{1-\lambda(1-p)} \right)^{n_r-n_l+s_l-s_r}
\]

As \( 1-\lambda p > 1-\lambda(1-p) \) and \( p > (1-p) \) we have that

\[
\left( \frac{p}{1-p} \right)^{s'_l-s'_r} > \left( \frac{1-\lambda p}{1-\lambda(1-p)} \right)^{(n'_r-n'_l)+(s'_l-s'_r)}.
\]

for \( s'_l-s'_r \geq s_l-s_r \) and \( n'_r-n'_l \geq n_r-n_l \) with at least one of the inequalities strict. We conclude that \( \text{Pr}(l|\bar{n}',\bar{s}') > \text{Pr}(r|\bar{n}',\bar{s}') \) for \( \bar{s}', \bar{n}' \) described in the statement of the Lemma.
Case 2. The state $l$ is more likely if more signals have been sent in its favor. So we need to compare the expected total number of signals in favor of $l$, $s^*_l$, with the expected total number of signals in favor of $r$, $s^*_r$, for a fixed $\bar{s}, \bar{n}$: $Pr(l|\bar{n}, \bar{s}) \geq Pr(r|\bar{n}, \bar{s})$ holds if and only if $E(s^*_l - s^*_r|\bar{s}, \bar{n}) \geq 0$.

Define $s^*_l, s^*_r$ as the number of $l$ and $r$ signals sent by the independents. Since $\pi > 0$, we have by Corollary 1 that all extremists acquire information. Consequently every silent extremist hides a signal in favor of the opposite candidate. We can calculate $s^*_l, s^*_r$ as:

$$s^*_l = s_l + n_r - (s_r - s^*_r) \quad s^*_r = s_r + n_l - (s_l - s^*_l)$$

The expected difference becomes:

$$E(s^*_l - s^*_r|\bar{s}, \bar{n}) = E(2(s_l - s_r) + (n_r - n_l) + (s^*_l - s^*_r)|\bar{s}, \bar{n}) = 2(s_l - s_r) + (n_r - n_l) + E(s^*_l - s^*_r|\bar{s}, \bar{n})$$

It is easy to see that $E(s^*_l - s^*_r|\bar{s}, \bar{n})$ is increasing in $n_r - n_l$. The expression is also increasing in $s_l - s_r$ as an increase of $s_l - s_r$ by 1 increases $s_l - s_r$ by 2 while it decreases $E(s^*_l - s^*_r|\bar{s}, \bar{n})$ by at most 1.

**Theorem 2** Equilibrium requirement 4 and 1 are consistent.

**Proof** Suppose that $\bar{s}$ is such that $\tau$ prescribes for an independent to vote $L$. Would this independent want to vote $L$ if he knew that he was pivotal? In other words: is it true that $Pr(l|\bar{s}, T) \geq Pr(r|\bar{s}, T)$ where $T$ stands for the event that voter $i$ is pivotal, $T = \{|\bar{n}|n_r = n_l + n_i, n_r = n_l + n_i + 1\}$. We know from Lemma 5 that $Pr(l|\bar{s}, T) \geq Pr(r|\bar{s}, T)$ holds for all cases in which independents are supposed to pick $L$ if it holds for the case in which $s_l - s_r$ and $n_r - n_l$ are being minimized given that $s_l \geq s_r$ and given that the independent is pivotal, namely $s_l = s_r$ and $n_r - n_l = n_i$. In the sequel we will only investigate this case and show that the independent has an incentive to vote for $L$ even in this worst case scenario.

**Case 1:** By the argument given in the proof of Lemma 5 we know that $Pr(l|\bar{n}, \bar{s}) \geq Pr(r|\bar{n}, \bar{s})$ if and only if

$$\left(\frac{p}{1-p}\right)^{s_l - s_r} \geq \left(\frac{1 - \lambda p}{1 - \lambda(1 - p)}\right)^{n_r - n_l + s_l - s_r}$$
Observe that for \( s_l = s_r \) and \( n_r - n_l = n_i \geq 0 \) the above inequality always holds so we are done.

**Case 2:** By Lemma 5 we need to show that \( E(s^*_l - s^*_r | s_r = s_l, n_r - n_l = n_i) \geq 0 \). Following the arguments given in the proof of Lemma 5 we can calculate \( E(s^*_l - s^*_r | s_r = s_l, n_r - n_l = n_i) \) as \( E(n_i + s^*_i - s^*_l | s_r = s_l, n_r - n_l = n_i) \).

If \( k \) out of the \( n_i \) independents acquired information then we can calculate a lower bound on \( E(s^*_l - s^*_r | s_r = s_l, n_r - n_l = n_i) \) as \( E(n_i + s^*_i - s^*_l | s_r = s_l, n_r - n_l = n_i) \).

Equilibrium requirement 3 says that no independent would acquire a signal and not send it. The requirement states that \( \tau_i(1, l, \emptyset, f) = \tau_i(1, \emptyset, r, f) = \tau_i(1, \emptyset, \emptyset, f) = 0 \) for all \( f \in C_2 \). In our solution we used this requirement. We now need to show that no strategic voter would like to deviate from this behavior. We need to show that \( EU_i(\tau, (1, \emptyset, \cdot, \cdot)) \leq EU_i(\tau) \).

**Lemma 6** Equilibrium requirement 1 and 3 are consistent.

**Proof** We will show that if \( EU_i(\tau, (1, \emptyset, \cdot, \cdot)) \geq EU_i(\tau, (0, \cdot, \cdot, \cdot)) \) then \( EU_i(\tau, (1, 1, \cdot, \cdot, \cdot)) \geq EU_i(\tau, (1, \emptyset, \cdot, \cdot, \cdot)) \). So \( (1, \emptyset, \cdot, \cdot, \cdot) \) is never a best reply for an independent. For \( EU_i(\tau, (1, \emptyset, \cdot, \cdot, \cdot)) \geq EU_i(\tau, (0, \cdot, \cdot, \cdot, \cdot)) \) to hold it must be true that the voting strategy of the privately informed independent does not coincide with the voting strategy of the independents that follows the public signal. If these two would coincide then the independent would be strictly better off not to incur any cost for the private signal which he is never going to use.

We know from Theorem 2 that sincere voting is strategic voting: If the public signal prescribes to vote \( X \) and if the public signal together with
the \( l \) signal of the independent would still prescribe to vote \( X \), then the independent does not have an incentive to deviate. So we need to only consider the cases in which the signal of the independent could possibly sway the vote of all independents. This happens in case 1. \( s_l = s_r \)\(^8\) and case 2. \( s_l = s_r - 1 \).

In the case 2. the publicly informed independents will all vote \( R \). Together with the signal of the independent under consideration there would be equally many signals in favor of each candidate. Observe that the same voting behavior is prescribed in the case of the hidden signal and in the case that the independent publicizes his signal and the supreme court issues a recommendation to vote \( R \). By Theorem 2 an independent prefer to vote \( R \) in the latter case. So the independent would prefer to vote \( R \) in the former case. Consequently, if an independent has an incentive to vote according to his hidden signal this must happen in case 1. above.

Now let us look at the remaining case \( s_l = s_r \).\(^9\) In case the supreme court’s coin shows \( L \) the voter under consideration would not want to deviate from the public recommendation to vote \( L \). So we only need to consider the case in which the Court’s coin shows \( R \).

To fully understand the incentives of the privately informed voter we need to find out when this voter would be pivotal. The question for the voter is: Conditional on the information contained in the event that I am pivotal and conditional on the public signal and my own private signal, should I prefer \( R \) or \( L \)? In the case under study all publicly informed independents vote \( R \). So \( R \) receives \( n_r + n_i \) votes whereas \( l \) receives \( n_l \) votes\(^10\). The privately informed independent is pivotal in exactly two cases namely \( n_r + n_i = n_l \) and \( n_r + n_i = n_l - 1 \) (remember that in case of a tie \( R \) wins as this was the public randomization outcome). So we have found a necessary condition for \( EU_i(\tau, (1, \emptyset, \ldots)) \geq EU_i(\tau, (0, \ldots, \ldots)) \). An independent who keeps an \( l \) signal private must prefer \( L \) to \( R \) in the case

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\(^8\)remember the convention that \( s_l, s_r \) stands for all public signals, so \( s_l, s_r \) does not contain the signal of the independent under consideration.

\(^9\)A plausible case in which an independent would have an incentive to vote according to the hidden signal is that \( n \) is very small and \( s_l = s_r = 0 \). In this case it is very unlikely there are any other voters out there, so pivotally concerns do not matter much, the privately informed independent should follow his signal

\(^10\)\( n_i \) denotes the number of all independents without the independent under consideration.
that \( s_l = s_r \) and \( n_l - n_r \in \{n_i, n_i + 1\} \). Using Lemma 5 we obtain that

\[
Pr(l|\vec{s}, \vec{n}', l) = Pr(r|\vec{s}, \vec{n}', l)
\] (8)

for all \( \vec{n}' \) with \( n_l' - n_r' < n_i \).

We need to show next that if a voter prefers \( L \) to \( R \) under the condition named above, then he will prefer sending his signal \( l \) to keeping it secret. To show this we need to first identify the range of cases in which a switch from silence to sending changes the outcome. As above for all signals \( \vec{s} \) with either \( s_l > s_r \) or \( s_l < s_r - 1 \) the outcome of both strategies remains the same. So let us investigate how the outcome changes in the remaining cases that \( s_l = s_r - 1 \) and \( s_l = s_r \). As before the case that \( s_l = s_r - 1 \) is easier to deal with, and we attack this case first.

If \( s_l = s_r - 1 \) and the privately informed individual remains silent then all independents will vote for \( R \), by our arguments above we know that this includes the privately informed independent. To the contrary if the independent publishes his signal a publicly thrown coin will decide whom the independents will vote for. Under the assumption that \( s_l = s_r - 1 \) and that the independent under study holds an additional \( l \) signal, the signals do not reveal anything about the state of the world. The symmetry of the problem implies that the decision whether to reveal the signal or not does not entail a utility change for the privately informed independent.

So let us now have a look at the alternative case \( s_l = s_r \). If the public lottery directs the independents to vote for \( L \), it does not matter whether the independent sends his signal or keeps it secret. So we only need to look at the case that the public lottery directs all independents to vote \( R \). In this case the independent might sway the vote of the independents by sending his \( l \) signal. To evaluate whether he should send his signal the independent has to come up with a list of cases in which his sending of the signal changes the outcome of the election (switch from \( R \) to \( L \)). If the independent does not send his signal the right wing candidate receives \( n_i + n_r \) votes whereas the left wing candidate receives \( n_l + 1 \) votes. On the other hand if he sends the signal the right wing candidate receives \( n_r \) votes whereas the left wing candidate receives \( n_l + n_i + 1 \) votes. The signal of the independent is pivotal if \( n_l - n_r \leq n_i - 1 \) and \( n_l - n_r \geq -n_i - 1 \). In other words the voter signal is pivotal if \( n_l - n_r \in -n_i - 1, \ldots, n_i - 1 \). By equation 8 and Lemma 5 we have that the voter strictly prefers \( L \) to \( R \) for any single one of these cases.
We conclude that $EU_i(\tau, (1, 1, \ldots)) > EU_i(\tau, (1, 0, \ldots))$ if $EU_i(\tau, (1, 0, \ldots)) > EU_i(\tau, (0, \ldots))$. \hfill \Box

9 A Unique Equilibrium Exists

We have now reduced the problem of showing that an equilibrium exists to a problem of showing that there exist probabilities of information acquisition $\lambda, \pi$ such that neither the extremists nor the independents would change their information acquisition behavior given every one else’s information acquisition behavior, and given that all voters follow $\tau$ after the information has been acquired. For the proofs in this section it is convenient to define $\Delta^+$ and $\Delta^-$ as functions of the information acquisition probabilities. We write $\Delta^+(\pi, \lambda)$ and $\Delta^-(\pi, \lambda)$ for the probability increases of the wrong choice being made when one more wrong or right signal is being sent given that the information acquisition probabilities of the independents and extremists are $\pi$ and $\lambda$.

**Theorem 3** An equilibrium $\tau$ exists.

**Proof** Define a correspondence $f : [0, 1] \times [0, 1] \rightarrow [0, 1] \times [0, 1]$, by $f = h \circ g$ with $g$ being a function and $g : [0, 1] \times [0, 1] \rightarrow \mathbb{R} \times \mathbb{R}$ such that

$$g(\pi, \lambda) = \begin{bmatrix} -p\Delta^-(\pi, \lambda) \\ -p\Delta^-(\pi, \lambda) - (1 - p)\Delta^+(\pi, \lambda) \end{bmatrix}$$

and $h$ being a correspondence $h : \mathbb{R} \times \mathbb{R} \rightarrow [0, 1] \times [0, 1]$ with

$$h_i(x, y) = \begin{cases} 1 & \text{if } g_i(x, y) > c \\ [0, 1] & \text{if } g_i(x, y) = c \\ 1 & \text{if } g_i(x, y) < c \end{cases}$$

Observe that $g$ is continuous and $h$ is upperhemicontinuous. So $f$ is upperhemicontinuous. We conclude by Kakutani’s fixed point theorem that a fixed point exists. \hfill \Box
The reason why we could not apply Myerson (1997) existence result is that strategy spaces in our game are not finite: the set of voting rules is infinite. However, we reduce the our existence problem early on to a problem that would fit Myerson’s existence result. As soon as we know that players will play $f_1$, in equilibrium, we could apply Myerson’s result. The reason why we proved existence here is that this proof introduces the necessary terminology for the proof of our uniqueness result.

**Theorem 4** The equilibrium is unique.

**Proof** Define a continuous correspondence $f^* : [0, 2] \rightarrow \mathbb{R} \times \mathbb{R}$, by $f^* = h \circ g \circ g^*$. Let $g^* : [0, 2] \rightarrow [0, 1] \times [0, 1]$ as

$$g^*(x) = \begin{cases} (x, 0) & \text{if } 0 \leq x \leq 1 \\ (1, x - 1) & \text{if } 1 < x \leq 2 \end{cases}$$

The function $f^*$ transforms the probabilities of acquiring and broadcasting information and to expected marginal utilities from broadcasting for extremists and independents. We have that $f^*(\lambda + \pi) = [(-p\Delta^- (\pi, \lambda)), (-p\Delta^- (\pi, \lambda) - (1 - p)\Delta^+ (\pi, \lambda))]$ for all combinations $\lambda, \pi$ that satisfy Corollary 1. Both components of $f^*$ are strictly decreasing in $(\lambda + \pi)$ and the first is always greater than the second. Figure (1) illustrates the function $f^*$. Now define a correspondence $h^* : [0, 2] \rightarrow [0, 1]$ by

$$h^*(x) = \begin{cases} f_1^* (x) & \text{if } 0 \leq x < 1 \\ [f_2^*(x), f_1^* (x)] & \text{if } x = 1 \\ f_2^* (x) & \text{if } 1 < x \leq 2 \end{cases}$$

Figure (2) illustrates the correspondence $h^*$. Observe that for any $c \in \mathbb{R}$ the constant function $c : [0, 2] \rightarrow \mathbb{R}$ mapping every value in $[0, 2]$ to $c$ intersects $h^*$ at most once. If $c$ lies always above then the unique equilibrium is that $\lambda = \pi = 0$ if it always lies below then the unique equilibrium is $\lambda = \pi = 1$. If $c$ intersects $h^*$ then $(\pi, \lambda)$ is an equilibrium for if and only if $c$ intersects $h^*$ at $\pi + \lambda$. Since $c$ intersects $h^*$ at most once any equilibrium is unique.

$\square$
Figure 1: $f^*$

Figure 2: $h^*$
References


