

Course Description

This course is the first course in the formal theory sequence of PhD courses at NYU. It is designed to provide students with the mathematical knowledge for PhD level formal theory courses such as game theory, advanced game theory, political economy, and formal theory courses taught in the NYU Economics Department. Although an obvious goal is to teach students basic mathematical concepts and techniques, so in some ways it is like any mathematics class, a less obvious goal of the course is to make students comfortable with mathematical models (either theoretical or empirical) as they are used in political science and enhance their ability to use them in their research (either theoretical or empirical). So in some ways it is more than just a math class, but a class that should also be viewed as an important first step in learning how to think mathematically and to use math tools to understand politics. Attached to this syllabus is a set of problems, which illustrate some of the formal modeling questions that taking this class should allow you to solve. As we go through the course we will return to these problems and discuss how the mathematical tools learned help us solve them.

Course Requirements

Course grades will be determined by the following components:

Class Participation	10%
Weekly Assignments	30%
Exams	60%

The exams will not be comprehensive; that is, they will only cover the material presented since the previous exam or the beginning of the semester in the case of the first exam. Weekly assignments will be given at the end of each lecture, to be completed and submitted by the beginning of the next lecture. Students should be prepared to present the assignments to the class if called upon. Because the material is cumulative, i.e. each topic depends on a thorough understanding of previous topics, it is imperative that exercises be done in a timely fashion. Accordingly, late submissions will not receive credit (barring exceptional circumstances). Because attempting problems oneself is a critical condition for learning mathematics, it is best to work on the exercises alone before discussing them with others.

Required Texts

Sheldon M. Ross, Introduction to Probability Models, 6th-8 editions. San Diego: Academic Press, most recent edition recommended.

Rubinstein, Ariel, Lecture Notes in Microeconomic Theory: The Economic Agent, Princeton: Princeton U. Press. A pdf version of this book is available online at <http://www.pupress.princeton.edu/titles/8175.html>

Carl P. Simon and Lawrence Blume, Mathematics for Economists. New York:

W.W. Norton and Company, 1994.

Daniel J. Velleman, *How To Prove It: A Structured Approach*. Cambridge: Cambridge University Press, most recent edition.

In addition to the assigned sections of these books you will also be responsible for additional material covered in class or assigned later in the semester.

Optional Texts

K. G. Binmore, *Mathematical Analysis: A Straightforward Approach*, 2nd ed. Cambridge: Cambridge University Press, 1982.

Samuel Goldberg. *Probability: An Introduction*. New York: Dover, 1960.

Daniel Kleppner and Norman Ramsey, *Quick Calculus: A Self-Teaching Guide*, 2nd ed. New York: John Wiley and Sons, 1985.

Silvanus P. Thompson and Martin Gardner, *Calculus Made Easy*. New York: St. Martin's Press, 1998.

Tentative Listing of Material Covered

The course schedule is still in flux but below is a listing of material that we will attempt to cover. Note that the material (with the exception of the Preliminaries which we will cover the first week of class) is NOT listed in the order in which we will cover it.

Preliminaries:

Scalars, Vectors, Matrices

Unknowns, Solving for Unknowns, Systems of Equations

Sets and Operations on Sets

Functions

Readings: Simon and Blume, pp. 10-21, 82-92, 122-28; Kleppner and Ramsey ch. 1; Thompson and Gardner pp. 10-17; Goldberg, ch. 1

Logic and Preference

Connectives and Quantifiers: Conditional and Biconditional Connectives, Equivalence with Quantifiers

Readings: Velleman, Chs. 1 and 2

We will also discuss sections of Rubinstein here.

Hypotheses, Conclusions, Counterexamples, Rules of Inference

Proof Strategies: Direct Proof, Proof by Contradiction, Exhaustion, Induction

Proofs Using Negations, Conditionals, Quantifiers, Conjunctions and Biconditionals, Disjunctions

Readings: Velleman, Ch. 3, pp. 245-251

Ordered Pairs, Cartesian Products, Binary Relations, Preference, Choice Functions, Maximal Sets, Rationalizable Choice

Readings: Velleman, Ch. 4; Rubinstein sections here.

Existence and Uniqueness Proofs, Preference Aggregation, Arrow's Theorem, Sen's Impossibility of a Paretian Liberal Theorem

Readings: Velleman, Ch. 3.6; Austen-Smith and Banks, pp. 25-39; Sen, JPE 1970, sections from Rubinstein

Calculus

More on Functions: 1-to-1, Onto, Inverses, Functions between Euclidean Spaces

Sequences, Limits

Open, Closed, Bounded, Compact and Convex Sets

Supremum, Infimum, Maximum, Minimum

Readings: Velleman, Ch. 5; Simon and Blume, pp. 75-79, 253-86, 293-99; Thompson and Gardner, pp. 18-29; Kleppner and Ramsey, pp. 50-63

Differentiation, Partial Differentiation, Continuity, Interpreting Derivatives

Chain Rule, Derivatives of Special Functions

Readings: Simon and Blume, chs. 2.3-3.1, 4, 5.5; Thompson and Gardner, chs. I-X, XVI; Kleppner and Ramsey, pp. 64-125, 238-240

Integration, Fundamental Theorem of Calculus

Readings: Simon and Blume, Appendix A4; Thompson and Gardner, chs. XVII-XXIII; Kleppner and Ramsey, ch. 3

Probability

Sample Spaces, Events, Probability,

Conditional Probability, Independence, Bayes' Rule

Discrete Random Variables: Expectation, Conditional Expectation

Readings: Ross, Ch. 1, 2.1-2.2, 2.4.1; Goldberg, Ch. 2, 158-97

Continuous Random Variables, Expectation of a Function of a RV, Joint Distribution, Moment-Generating Functions

Limit Theorems: Markov's Inequality, Chebyshev's Inequality, Strong Law of Large Numbers, Central Limit Theorem

Computing Expectation and Probability by Conditioning

Information Aggregation and the Condorcet Jury Theorem

Readings: Ross, Ch. 2.3-3; Goldberg, pp.197-251; Austen-Smith and Banks, APSR 1996

Vector and Matrix Algebra

Euclidean Space

Vectors: Addition, Subtraction, Scalar Multiplication, Inner Product, Norm, Metric

Lines, Planes, and Hyperplanes

Readings: Simon and Blume, ch. 10

Matrix Algebra: Addition, Subtraction, Scalar Multiplication, Matrix Multiplication
Cummulative, Associative, and Distributive Laws
Transpose and Inverse, Identity and Null Matrices
Readings: Simon and Blume, pp. 153-173

Nonsingularity, Determinant, Inverse
Cramer's Rule; Solving Systems of Linear Equations
Readings: Simon and Blume, pp. 122-146, Ch. 9

Optimization
Quadratic Forms, Definite and Semidefinite Matrices
Readings: Simon and Blume, pp. 287-293, Ch. 16

Optimization in R^n , Optimization Problems in Parametric Form
Weierstrass Theorem with Proof and Applications
Readings: Simon and Blume, Ch. 3

Unconstrained Optimization, Critical Points, First- and Second-Order Conditions
Readings: Simon and Blume, Ch. 17; Thompson and Gardner, chs. XI-XII; Kleppner and
Ramsey, pp. 126-150

Constrained Optimization:
Equality Constraints, Theorem of Lagrange, Lagrangean Multipliers
Inequality Constraints, Theorem of Kuhn and Tucker, Kuhn-Tucker Multipliers
Readings: Simon and Blume, Ch. 18

Sufficient Conditions to Guarantee Existence of Optima
Relationship between Concave Functions and Convex Sets, Implications of Convexity in
Optimization Problems
Definition of Quasiconvexity, Implications of Quasiconvexity
Readings: Simon and Blume, Ch. 21

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Math for Political Science Illustrative Problems

The problems below are examples of the types of things we will learn how to solve during the semester by learning the mathematics behind them. They give you a good idea of why the the type of material that you will learn during the semester is useful and what we hope you will be able to do before the end of the semester. Note that NONE of these problems involve knowing game theory. As we go through the material during the semester we will discuss how to answer these problems. I don't expect you to be able to answer these problems now, but if you can, then that is great and maybe you are ready for more advanced formal theory classes!

Problem 1

Define a *weak ordering* as a binary relation that is complete, reflexive, and transitive. Prove that if X is finite and R is a weak ordering $M(R, X) = \{x \in X : xRy \forall y \in X\} \neq \emptyset$. How can we move from this result to using utility functions to represent individuals' preferences? What does it mean to do so? What is the difference between this view of utility and the concept of expected utility?

Problem 2

Suppose that you are a voter on a five member committee in a legislature that has to vote whether to send a particular bill to the entire legislature for further consideration. If the majority of the committee votes yes the bill will be sent to the legislature and if the majority votes no the bill will not be sent. Committee members are not allowed to abstain. Suppose you think that there are two types of fellow committee members, pro and con, and you think that the probability that a randomly selected fellow committee member is pro is given by $p < 0.5$. Assume further that you think that the probability that a pro member will vote yes is given by $q > 0.5$. You assume that these probabilities are independent and that each individual's probability of being a pro type or voting is independent of the other committee members. You think that con members always vote no. You receive 1 unit of utility if you vote yes and the bill is approved by the committee; 0 if you vote no and the bill is approved by the committee; x such that $0.5 < x < 1$ if you vote no and the bill is not approved by the committee; and $0.5x$ if you vote no and the bill is approved by the committee. How should you vote if you maximize expected utility? Justify your answer.

Problem 3

Suppose candidate i promises to provide a level of services s_i to an interest group that contributes campaign contribution c_i . An interest group is identified by a type, $\theta_i \in [0, \theta_i^+]$, which represents a parameter of its private return, $R(s_i, \theta_i)$, from the service to be provided by candidate i if she wins the election. R is assumed to be a strictly increasing function of both s_i & θ_i and R is assumed to have diminishing marginal returns to s_i beyond some point s_i^* . Groups are assumed to be risk neutral. Defining p_i as the probability that candidate i wins which the interest group treats as exogenous, then the expected gain to contributing is given by:

$$G_i(\theta_i) = p_i R(s_i, \theta_i) - c_i$$

Define $\hat{\theta}_i = \hat{\theta}_i(s_i, c_i; p_i)$ as the value of θ_i such that $p_i R(s_i, \hat{\theta}_i(s_i, c_i; p_i)) - c_i \equiv 0$.

1. What can we say about the signs and values of $\frac{\partial \hat{\theta}_i}{\partial s_i}$, $\frac{\partial \hat{\theta}_i}{\partial c_i}$, and $\frac{\partial \hat{\theta}_i}{\partial p_i}$ with only the information above?
2. Suppose that candidate i does not know an interest group's particular value of θ_i but knows the probability distribution function of types, $F_i(\theta_i)$ and that the number of groups is normalized to 1. Give expressions for the total contributions, X_i , the candidate will receive and the total amount of services, S_i , that the candidate will give assuming that groups with $\theta_i \geq \hat{\theta}_i$ contribute to candidate i .
3. Suppose that candidate i is assumed to maximize her expected gain from the opportunity to seek office. Define V_i as the value from holding office, b_i is per unit cost of providing services to interest groups, & S_i is the total amount of services candidate i provides. Assume that the candidate receives 0 value from losing.
 - (a) Provide an expression of the expected gain the candidate will receive from running for office.
 - (b) Prove that the service contribution pair chosen by candidate i who wishes to maximize his expected value from running for office (assuming that he takes other candidates' behavior as exogenous) is such that the group that is indifferent between contributing or not to candidate i has diminishing marginal returns from services promised by the candidate.
 - (c) Assume that there are two candidates, $i = 1, 2$ and $p_1 = P = \frac{X_1}{X_1 + X_2}$, the $f_i(\theta_i)$ are identical and have a standard uniform dist.; i.e. $F_i(\theta_i) = \theta_i$, and $R(s_i, \theta_i) = s_i^\alpha \theta_i$. Assuming candidate 1 takes candidate 2's behavior as exogenous, demonstrate that in order to maximize her expected value of running for office, candidate 1 will choose an s_i and c_i combination such that $\hat{\theta}_i = \frac{1}{2-\alpha}$.

Problem 4

Explain, using probability theory and expected utility theory why a president who has to make a forthcoming foreign policy decision might choose to consult a foreign policy advisor who has more information than the president, but who sometimes lies about the truth because the advisor would like the president to follow a particular action independent of the advisor's information. Explain further why the president who has a choice of two biased advisors, might prefer one who is biased in favor of the choice the president would make ex ante (before seeking advice) over one who is biased against the choice the president would make ex ante.