

# Mathematics and Democracy: Designing Better Voting and Fair-Division Procedures

Politics G53.3200  
New York University  
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## Introduction

Mathematics and Democracy (M&D) is meant to show how mathematics can illuminate two essential features of democracy:

- how individual preferences can be aggregated to give a social choice or election outcome that best reflects the interests of the electorate; and
- how public and private goods can be divided in a way that respects due process and the rule of law.

Whereas questions of aggregation are primarily the focus of *social choice theory*, questions of division are the focus of *fair division*.

*Democracy*, as used in this course, will generally mean representative democracy, in which citizens vote for representatives, from a president on down. Some attention, however, will be devoted to referendums, in which citizens vote directly on propositions, as they did in assemblies in ancient Greece.

M&D will emphasize the procedures, or rules of play, that produce democratic outcomes. By making precise properties that one wishes a voting or fair-division procedure to satisfy and clarifying relationships among these properties, mathematical analysis can strengthen the intellectual foundations on which democratic institutions are based. But because there may be no procedures that satisfy all the properties one might desire, M&D will analyze trade-offs among the properties; practical problems of implementation, and experience with procedures that have been tried out, will also be discussed.

## Institutional Design and Engineering

M&D will spell out how new voting and fair-division procedures may better foster democratic choices. Generally speaking, they do so by giving voters better ways of expressing themselves, by electing officials who are more likely to be responsive to the electorate, and by allocating goods so that players' shares are equitable or preclude envy. Criticisms of current procedures will be tempered with detailed suggestions for improving them.

Designing procedures that satisfy desirable properties, or showing the limits of doing so, is sometimes referred to as *institutional or mechanism design*. M&D will present empirical examples to illustrate this approach, but much of the analysis will be theoretical.

The product of such analysis is normative: The prescription of new procedures that are superior, in terms of specified democratic criteria, to ones that arose more haphazardly. Like engineering in the natural sciences, which translates theory (e.g., from physics) into practical design (e.g., a bridge), democratic engineering translates theory into the design of political-economic-social institutions that better meet the criteria one deems important.

While a good mathematical background will facilitate understanding some topics in the course, many topics will be accessible to those with little mathematical training. Students will be encouraged to take normative positions in support of procedures they find appealing, including those that they may develop themselves. They will also be challenged to try to test whether the advantages they see in theory translate into advantages in practice. To the extent that this is not the case, they will be encouraged to revise the procedures, restarting the process of both discovery and testing.

Not all institutions in the public sphere will be covered. For example, there is now a large literature on redistricting, or the drawing of district boundaries after a census; on auctions, which governments employ to sell such things as oil leases and parts of the electronic spectrum; and on matching algorithms, which are used in the selection of schools by children and parents and hospital residencies by doctors. There is also a substantial qualitative literature on problems of implementing and evaluating democratic reforms.

The course will be divided into two parts:

### **Part 1: Voting Procedures**

One cornerstone of democracy is honest and periodic elections, in which there is meaningful competition among parties, interest groups, or individuals for political office. Several of the voting procedures to be analyzed are relatively new and not well-known, but they offer significant advantages over extant procedures. Common to many of them is approval balloting, whereby voters can approve of as many candidates or alternatives as they like.

Approval balloting may take different forms. Under approval voting, the candidate or alternative with the most votes wins. Under other methods of aggregating approval votes, different candidates or alternatives may win. These methods maximize different objective functions, or constrain outcomes in certain ways, in order to achieve different goals, such as the proportional representation of different interests in the electorate.

Most social-choice analysis assumes rational individuals, who choose the most effective or efficient means to satisfy their goals. *Game theory* is an important tool in rational-choice analysis, especially in identifying outcomes that are stable or in equilibrium, and institutions that support the equilibria one finds. These institutions do not necessarily dispel conflict but manage it so that political life continues, with politicians never being certain that they will continue to hold office but quite certain that the institutions will stay in place.

## **Part 2: Fair-Division Procedures**

As central as elections are to the performance of a democracy, a democracy would be a sham if the politicians elected were not restricted by due process and the rule of law. Ideally, democracies treat all citizens the same way—at least when governed by a constitution or other laws—particularly with respect to their civil rights and certain freedoms, such as freedom of association and freedom of religion.

The equal treatment of citizens depends in part on their receiving fair shares of things that must be divided among them. Accordingly, different procedures of fair division—applicable to both divisible and indivisible goods—will be studied.

Fairness requires that one take into account the different preferences or claims of players who have a stake in an outcome. Step-by-step rules to implement the fair division of goods, which may be homogenous (like money) or heterogeneous (like land with different objects on it), will be analyzed. Questions that relate to the fair division of people or groups include: What political parties are best suited to form a government? Which parties should get what cabinet ministries in the government?

### **Required Books and Assignments**

The main text will be *Mathematics and Democracy*, which is currently available only in manuscript form. Five other books (all paperback) that complement this text, or provide useful background, are required.

1. Steven J. Brams, *Mathematics and Democracy: Designing Better Voting and Fair-Division Procedures*. Princeton University Press, 2007 (forthcoming).
2. Robert A. Dahl, *On Democracy*. Yale University Press, 1998.
3. Larry Diamond and Marc F. Plattner (eds.), *Electoral Systems and Democracy*. Johns Hopkins University Press, 2006.
4. Samuel Fleischacker, *A Short History of Distributive Justice*. Harvard University Press, 2004.
5. Hannu Nurmi, *Models of Political Economy*. Routledge, 2006.
6. Paul Woodruff, *First Democracy: The Challenge of an Ancient Idea*. Oxford University Press, 2005.

The requirements of the course include a midterm exam, a final exam, and a short paper on the design and analysis of a democratic institution. Readings from the books

listed in the Bibliography—particularly from the “voting and fair division” section—will be recommended throughout the course.

- Jan. 22: Introduction  
 Jan. 29: Brams, chs. 1-2; Dahl, chs. 1-3; Nurmi, chs. 1-2; Woodruff, chs. 1-2.  
 Feb. 5: Brams, ch. 3; Dahl, chs. 4-5; Nurmi, ch. 3; Woodruff, ch. 3.  
 Feb. 12: Brams, ch. 4; Dahl, chs. 6-7; Nurmi, ch. 4; Woodruff, ch. 4.  
 Feb. 19: Brams, ch. 5; Dahl, chs. 8-9; Nurmi, ch. 5; Woodruff, ch. 5.  
 Feb. 26: Brams, ch. 6; Diamond and Plattner, chs. 1-10.  
 March 5: Brams, ch. 7; Diamond and Plattner, chs. 11-14.  
 March 19: Midterm exam.  
 March 26: Brams, ch. 8; Diamond and Plattner, chs. 15-19.  
 April 2: Brams, chs. 9-10; Fleischacker, chs. 1-2; Nurmi, ch. 6; Woodruff, ch. 6.  
 April 9: Brams, chs. 11-12; Fleischacker, ch. 3; Nurmi, ch. 7; Woodruff, ch. 7.  
 April 16: Brams, ch. 13; Dahl, chs. 10-11, Nurmi, ch. 8; Woodruff, ch. 8.  
 April 23: Brams, ch. 14; Dahl, chs. 12-13; Nurmi, ch. 9; Woodruff, ch. 9.  
 April 30: Brams, ch. 15; Dahl, chs. 14-15; Nurmi, chs. 10-11; Woodruff, ch. 10.  
 May 7(?): Final exam.

## Bibliography

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- Voting and Fair Division**
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