Fall 2003
MW 11:00-12:15
Catherine Hafer

Mathematics for Political Scientists

This course will provide students with a rigorous, if partial, introduction to a variety of mathematical concepts that have been selected for inclusion because of their relevance to advanced topics in methodology and formal theory. It is intended primarily for students who will be pursuing advanced technical training in preparation for an academic research career. Familiarity with this material will also allow students to better understand the application of mathematical tools to problems in political science and to concentrate on the substantive, rather than technical, aspects of such work as it arises in later coursework.

Course Requirements:
Course grades will be determined by graded exercises (50%), class participation (10%), and the final exam (40%).

Exercises will be assigned at the end of each lecture, to be completed and submitted by the beginning of the next lecture. Because the material is cumulative, i.e. each topic depends upon a thorough understanding of previous topics, it is imperative that exercises be done in a timely fashion. Accordingly, late submissions will not receive credit (barring exceptional circumstances). Because attempting problems oneself is a critical condition for learning mathematics, it is best to work on the exercises alone before discussing them with others.

Students are urged to ask questions, answer questions, and tackle problems in the classroom. Active and constructive behavior in the classroom enhances everyone's learning experience and develops skills that are requisite in a scholarly community.

An in-class final exam will be administered Monday, December 12, 10:00-11:50 am.

Required Texts:

Sheldon M. Ross, *Introduction to Probability Models,* 6th-8th editions. San Diego: Academic Press, 1997-2003.

Carl P. Simon and Lawrence Blume, *Mathematics for Economists.* New York: W.W. Norton and Company, 1994.

Daniel J. Velleman, *How To Prove It: A Structured Approach.* Cambridge: Cambridge University Press, 1994.

Additional texts:

K. G. Binmore, Mathematical Analysis: A Straightforward Approach, 2nd ed. Cambridge: Cambridge University Press, 1982.

Samuel Goldberg. *Probability: An Introduction*. New York: Dover, 1960.

Daniel Kleppner and Norman Ramsey, *Quick Calculus: A Self-Teaching Guide,* 2nd ed. New York: John Wiley and Sons, 1985.

Silvanus P. Thompson and Martin Gardner, *Calculus Made Easy*. New York: St. Martin's Press, 1998.

Course outline:

Preliminaries:

Scalars, Vectors, Matrices
Unknowns, Solving for Unknowns, Systems of Equations
Sets and Operations on Sets
Functions
Readings: Simon and Blume, pp. 10-21, 82-92, 122-28; Kleppner and Ramsey ch. 1; Thompson and Gardner pp. 10-17; Goldberg, ch. 1

Logic and Preference

Connectives and Quantifiers: Conditional and Biconditional Connectives, Equivalence with Quantifiers
Readings: Velleman, Chs. 1 and 2

Hypotheses, Conclusions, Counterexamples, Rules of Inference
Proof Strategies: Direct Proof, Proof by Contradiction, Exhaustion, Induction
Proofs Using Negations, Conditionals, Quantifiers, Conjunctions and Biconditionals, Disjunctions
Readings: Velleman, Ch. 3, pp. 245-251

Ordered Pairs, Cartesian Products, Binary Relations, Preference, Choice Functions, Maximal Sets, Rationalizable Choice
Readings: Velleman, Ch. 4; Austen-Smith and Banks, Ch. 1

Existence and Uniqueness Proofs, Preference Aggregation, Arrow's Theorem, Sen's Impossibility of a Paretian Liberal Theorem
Readings: Velleman, Ch. 3.6; Austen-Smith and Banks, pp. 25-39; Sen, JPE 1970

Calculus

More on Functions: 1-to-1, Onto, Inverses, Functions between Euclidean Spaces
Sequences, Limits
Open, Closed, Bounded, Compact and Convex Sets
Supremum, Infimum, Maximum, Minimum
Readings: Velleman, Ch. 5; Simon and Blume, pp. 75-79, 253-86, 293-99; Thompson and Gardner, pp. 18-29; Kleppner and Ramsey, pp. 50-63

Differentiation, Partial Differentiation, Continuity, Interpreting Derivatives
Chain Rule, Derivatives of Special Functions
Readings: Simon and Blume, chs. 2.3-3.1, 4, 5.5; Thompson and Gardner, chs. I-X, XVI; Kleppner and Ramsey, pp. 64-125, 238-240

Integration, Fundamental Theorem of Calculus

Readings: Simon and Blume, Appendix A4; Thompson and Gardner, chs. XVII-XXIII; Kleppner and Ramsey, ch. 3

Probability

Sample Spaces, Events, Probability,
Conditional Probability, Independence, Bayes' Rule
Discrete Random Variables: Expectation, Conditional Expectation
Readings: Ross, Ch. 1, 2.1-2.2, 2.4.1; Goldberg, Ch. 2, 158-97

Continuous Random Variables, Expection of a Function of a RV, Joint Distribution, Moment-Generating Functions
Limit Theorems: Markov's Inequality, Chebyshev's Inequality, Strong Law of Large Numbers, Central Limit Theorem
Computing Expectation and Probability by Conditioning
Information Aggregation and the Condorcet Jury Theorem
Readings: Ross, Ch. 2.3-3; Goldberg, pp.197-251; Austen-Smith and Banks, APSR 1996

Vector and Matrix Algebra

Euclidean Space
Vectors: Addition, Subtraction, Scalar Multiplicaton, Inner Product, Norm, Metric
Lines, Planes, and Hyperplanes
Readings: Simon and Blume, ch. 10

Matrix Algebra: Addition, Subtraction, Scalar Multiplication, Matrix Multiplication
Cummulative, Associative, and Distributive Laws
Transpose and Inverse, Identity and Null Matrices
Readings: Simon and Blume, pp. 153-173

Nonsingularity, Determinant, Inverse
Cramer's Rule; Solving Systems of Linear Equations
Readings: Simon and Blume, pp. 122-146, Ch. 9

Optimization

Quadratic Forms, Definite and Semidefinite Matrices
Readings: Simon and Blume, pp. 287-293, Ch. 16

Optimization in R^n, Optimization Problems in Parametric Form
Weierstrass Theorem with Proof and Applications
Readings: Simon and Blume, Ch. 3

Unconstrained Optimization, Critical Points, First- and Second-Order Conditions
Readings: Simon and Blume, Ch. 17; Thompson and Gardner, chs. XI-XII; Kleppner and Ramsey, pp. 126-150

Constrained Optimization:

Equality Constraints, Theorem of Lagrange, Lagrangean Multipliers
Inequality Constraints, Theorem of Kuhn and Tucker, Kuhn-Tucker Multipliers
Readings: Simon and Blume, Ch. 18

Sufficient Conditions to Guarantee Existence of Optima
Relationship between Concave Functions and Convex Sets, Implications of Convexity in Optimization Problems
Definition of Quasiconvexity, Implications of Quasiconvexity
Readings: Simon and Blume, Ch. 21