G53.1110.001: Mathematics for Political Scientists
Fall 2004, New York University
Mondays & Wednesdays, 2:00pm-3:15pm
726 Broadway, Room 701

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Course Description: Mathematics provide us with a clear language that can often be helpful in describing complicated phenomena and relationships in an understandable way. Several important styles of research in political science, ranging from statistical methodology to formal theory, rely heavily on mathematical representations and reasoning, and as such a basic working knowledge of several topics in mathematics is essential for understanding much of the research carried out in political science today.

The objective of this course is to introduce the basic mathematical concepts that are most relevant to the research techniques used by quantitative political scientists, including both statistical and formal modeling techniques. Gaining a working familiarity with quantitative research techniques early on in graduate school not only gives students access to useful literatures they might otherwise find impenetrable as they learn about their substantive fields of interest, but also provides students with additional standpoints from which to formulate interesting research questions. An early mastery of these techniques also frees students to focus their intellectual energies on what we ultimately care about – developing substantive insight into the political world – in later coursework and in formulating research topics.

The contents of the course are divided roughly into five subject areas: calculus, probability, linear algebra, optimization theory, and techniques in proofs and logic. Familiarity with each of these subject areas is important both to the proper use of statistical methods in political science as well as to the development of useful game theoretic models of political phenomena. Of course, each of these subjects is very rich, and could easily fill a semester or year-long course in themselves; the time we have allows for a rigorous treatment of each, but by necessity an incomplete treatment.

Course Prerequisites: This course is intended primarily for political science Ph.D. students (or others with the permission of the instructor). There are no formal prerequisites for this course. No mathematical knowledge beyond what is typically taught in high school (such as algebra) is required; novel concepts that may be used in class will be introduced and discussed at length as we go along.

Course Requirements: Your course grade will be determined by a combination of the following factors: (1) Problem Sets, which will be assigned every week (50%); (2) and an in-class final exam (50%). Problem sets will be assigned and distributed at Monday lectures, and will be due at the beginning of class one week later, also on Mondays. Because the subject is a cumulative one, it is essential that students do not fall behind; late problem sets will not be accepted unless the circumstances are genuinely exceptional and permission is obtained in advance. Students are permitted to work in small groups on assignments, but only after they have attempted the problems on their own. It is also expected that students only hand in work that, after completing the assignment, they would be able to reproduce on their own.

Course Books: The following books will be available for purchase in the NYU bookstore:

- Carl P. Simon and Lawrence Blume. *Mathematics for Economists.* New York: WW Norton and Company, 1994.
- Daniel J. Velleman. *How to Prove It: A Structured Approach.* Cambridge: Cambridge University Press, 1994.
- Sheldon M. Ross, *Introduction to Probability Models*. San Diego: Academic Press.

In addition to the material covered in the two textbooks, you will of course also be responsible for any material presented in lecture.

Course Outline

The lengths of time to be allocated to each topic are approximate. An effort will be made to coordinate the pace of the subject matter with the simultaneous companion course "Quantitative Research in Political Science I," so that mathematical concepts employed in the latter course are first treated in this course, to the extent possible.

Introduction and Preliminaries (approx. 1 week)
Spaces; Functions; Sets and Operations on Sets; Unknowns and Solving for Variables; Common Mathematical Notation.

Simon and Blume pp. 10-21, 82-92, 122-28.

Calculus (approx. 2 weeks)
Differentiation: Neighborhoods; Limits of Functions; Sequences; Limits of Sequences; Open, Closed, Bounded, Compact, and Convex Sets; Supremeum, Infimum, Maximum, and Minimum; Continuity; Derivatives; Interpreting Derivatives; Higher Order Derivatives; Composite Functions; Chain Rule; L'Hospital's Rule; Partial Derivatives; Derivatives of Special Functions.

Integration: Indefinite Integrals; Definite Integrals; Fundamental Theorem of Calculus; Some Techniques of Integration.

Simon and Blume pp. 75-79, 253-86, 293-99; Chapters 2.3-3.1, 4, 5.5; Appendix A4.

Probability (approx. 2.5 weeks)
Sample Spaces; Events; Probability; Conditional Probability and Independence; Bayes' Rule; Discrete Distributions; Continuous Distributions; Expectation and Conditional Expectation; Joint Distributions; Limit Theorems; Special Distribution Functions.

Ross Chapters 1-2.4 and 3.

Vector and Matrix Algebra (approx. 3.5 weeks)

Vectors and Vector Algebra; Inner Products and Norms; Matrix Algebra; Matrix Multiplication; Transposes and Inverses; Systems of Linear Equations; Rank; Existence of Solutions; Identity and Null Matrices; Determinants; Cramer's Rule.

Simon and Blume , pages 122-146 and 153-173, Chapters 9 and 10.

Optimization (approx. 2 weeks)
Quadratic Forms; Definiteness of Quadratic Forms; Maxima and Minima; First Order and Second Order Conditions of Unconstrained Optimization; Constrained Optimization; Equality Constraints; Lagrange Multipliers; Inequality Constraints; Kuhn-Tucker Theorem; Convexity in Optimization Problems; Quasiconvexity.

Simon and Blume, Chapters 3, 16, 17, 18, 21.

Proofs & Logic (approx. 3.5 weeks)
Hypotheses; Conclusions; Counterexamples; Rules of Inference; Direct Proof; Proof by Contradiction; Exhaustion; Induction; Other Proof Techniques; Existence and Uniqueness Proofs.

Velleman Chapters 1-4.