Alliance Formation and War

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This article provides a game-theoretic model of alliance formation and war. The model examines how alliances affect the behavior of states. Several aspects of the alliance literature are simultaneously incorporated into the model. The model endogenizes alliance reliability, the effect of alliance on the occurrence of war, and alliance formation. The results explain several empirical phenomena. Although alliance formation has predictable effects on the behavior of nations, the aggregate effect of alliances on the occurrence of war is ambiguous. Under some circumstances alliances increase the probability of war; in others, alliances reduce the likelihood of war. Alliance reliability affects the behavior of nations. Nations with unreliable alliances are more likely to be attacked than those with reliable alliances. I discuss how this sampling affects empirical estimates of alliance reliability.

The study of alliances has been broken up into three lines of research: the relationship between alliance formation and the occurrence of war, the motivations for the formation of alliances, and the reliability of alliances. In this article I argue that these three processes are interconnected, and attempts to theorize about the role of alliances in international relations must consider how these factors interact. Specifically, I maintain that reliability and security issues must be endogenous to any explanation of alliance formation. Empirical investigations correlating alliances and wars that do not consider this are flawed.

The Correlates of War project (COW) has produced a data set containing war and alliance data since 1815 (Singer and Small, 1966a, 1969; Small and Singer, 1982). Using these data, Singer and Small (1966b, 1967, 1968) were among the first to examine the empirical link between alliances and war. Vasquez (1987) provides a comprehensive summary of the extensive empirical work on the alliance–war relationship. He summarizes the two major findings of this work as:

| First, alliances do not prevent war or promote peace; instead they are associated with war, although they are probably not a cause of war. Second, the major consequence of alliances is to expand war once it has started; in the war alliances are important in accounting for the magnitude and severity of war. (Vasquez, 1987, p. 119) |

The inductive method of analysis has not brought us closer to understanding the underlying relationship between alliances and war. Despite numerous studies of the empirical relationship between alliance formation and the occurrence of war, there are few robust findings. The evidence is often contradictory and gener-

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ally inconclusive (Bueno de Mesquita and Singer, 1973; Ostrom and Hoole, 1978; Levy, 1981; Regley and Raymond, 1982; Siverson and Sullivan, 1984; King, 1989; Wayman, 1990). From the sum of these studies it is unclear whether alliances make war more or less likely. Even the inclusion of intervening variables, such as system polarization (Wallace, 1973; Bueno de Mesquita, 1978), has failed to generate conclusive evidence to relate alliance formation with the occurrence of war.

There is convincing evidence that alliances expand conflict (Altfeld and Bueno de Mesquita, 1979; Sabrosky, 1980; Siverson and King, 1980; Siverson and Tennfloss, 1984; King, 1989; Oren, 1990; Kim, 1991). An allied nation is far more likely to intervene in a war than a nonallied nation. If a nation becomes involved in a conflict then it might expect its allies to intervene on its behalf. However, empirically, alliance reliability is low. For every three or four opportunities to intervene on behalf of an ally, only one is used. Although the presence of an alliance increases the probability of allied aid during a war, it does not guarantee it.

Levy (1981) provides a summary of the different hypotheses purported to explain how alliances affect the occurrence of conflict. However, as Levy notes, little systematic evidence can be presented to back these hypotheses.

There are clearly a variety of plausible theoretical linkages between alliances and war (or peace). Recent empirical studies have yet to demonstrate conclusively, however, which of these complex linkages is the most important, in conjunction with which other factors, and under which conditions. (Levy, 1981, p. 584)

I maintain that the lack of conclusive support for a particular set of hypotheses reflects the lack of theoretical understanding about how alliance formation, alliance reliability, and the effect of alliances on the occurrence of conflict interact. To fully explain the role of alliances in the international system, all three theoretical relationships must be considered simultaneously.

Some efforts have been made to endogenize alliance formation into models of war, for example, Niou and Ordeshook (1991, 1992) and Morrow (1986), but these models assume alliances are reliable. In this article, nations choose whether or not to honor their alliance commitments.

Before launching into the model I want to outline the general format I use. To produce a parsimonious model, I restrict attention to three nations: nations A, B, and C. Each of these nations has a role in the game. Nation A is the aggressor. Nation B is the victim, or target, of nation A’s aggression. Nation C is a third-party nation which could intervene on either side of a conflict between A and B. The nations interact as follows: nation A has the opportunity to attack nation B. If attacked, nation B decides whether to retaliate or surrender. A war starts if nation B retaliates when attacked. When a war starts nation C decides whether or not to intervene.

The model captures the basic interaction between nations. There is a sequential nature to the behavior of nations. Nations move in the order described. It is not possible for nation C to intervene in a war that has not yet occurred. However, as I show, the expectation that nation C will intervene will affect the behavior of both A and B.

It appears that the model I present is restrictive precisely because I give each nation a role. However, labels are simply labels; they do not confer special qualities upon nations. The labels and roles of the nations can easily be switched. Rather than ask whether or not nation C will attack under condition X, I ask whether nation A will attack under condition Y. Conditions X and Y are simply permutations of each other. By considering the simple interaction between A, B, and C, I generate general predictions about any permutation of A, B, and C.

A simple example of the possible interactions between A, B, and C is considered
next. This illustrates the incentives nations have to form alliances. Figure 1 is the
game-form which I call the conflict subgame. It shows, diagrammatically, the
sequential order in which nations move. At node \( n = 1 \) nation A decides whether
or not to attack. If nation A does nothing then the game ends at the status quo
(SQ). However, if nation A attacks then nation B must decide whether or not to
retaliate (node \( n = 2 \)). If B surrenders to A’s attack then the outcome is acquies-
cence (ACQ). A war occurs if B decides to retaliate. Once a war occurs nation C
decides whether to intervene or to remain neutral (node \( n = 3 \)). If C, forming a
coalition with B, attacks A, then the outcome is WarA/BC. If C, forming a coalition
with A, attacks B, then the outcome is WarAC/B. A bilateral war, WarA/B, results
if C remains neutral.

The general game has exactly the same format as this game. The numbers under
each outcome refer to the utility that nations A, B, and C receive from that
particular outcome. Nations B and C’s most preferred outcome is SQ; they both
receive a payoff of 5. Their least favorite outcome is WarAC/B, a payoff of 1. Nation

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**Fig. 1. Numerical Example: No Alliances**

<table>
<thead>
<tr>
<th>Utility for outcomes</th>
<th>SQ</th>
<th>WarA/B</th>
<th>WarAC/B</th>
<th>WarA/BC</th>
<th>ACQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_A )</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>( U_B )</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>( U_C )</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>
A’s most preferred outcome is ACQ (a payoff of 5), while WarA/BC is A’s least preferred outcome (a payoff of 1).

Backwards induction is used to solve this game. Nation A wants to maximize its expected utility. If it does nothing it receives a payoff of 2. If it attacks then it could receive a payoff between 1 and 5 depending upon the actions of the other nations. Unless the expected behavior of the other nations is predicted first it is not possible to calculate the behavior of nation A. Thus, to solve sequential games the analysis starts at the bottom and works to the top of the tree.

If a war occurs (node n = 3) nation C must decide what to do. It is faced with possible payoffs 4, 1, and 3, which correspond to the outcomes WarA/B, WarAC/B, and WarA/BC. Given this choice, C’s most preferred outcome is WarA/B; nation C remains neutral.

If attacked, nation B must decide how to respond. By surrendering B receives a payoff of 3. In deciding whether or not to retaliate, B considers the likely actions of C. If nation C intervenes on B’s side in a war then B would receive an expected payoff of 4 by retaliating. But nation B knows that unfortunately, C will not intervene: if a war occurs nation B will fight alone. Therefore, if nation B retaliates, it receives the lower payoff of 2. Since 3 is better than 2, nation B surrenders.

Nation A must decide whether to attack or not. It knows that B will surrender if attacked. Its choice is between a payoff of 2, for the status quo, or 5, if it attacks B. Since A prefers 5 to 2, A attacks B. The predicted outcome of this game is ACQ. Nations B and C move after A. Despite this, their anticipated actions affect A’s decision.

Suppose for a moment that nation C, instead of remaining neutral, will attack A if a war occurs: outcome WarA/BC. Since B anticipates a coalition partner in its fight with A it will retaliate if attacked. The decision facing nation A is no longer a choice of SQ or ACQ, but a choice of SQ or WarA/BC. Nation A does not want to fight both B and C. Therefore A does not attack. Nation C does not actually have to fight, but by being prepared to fight, nation C can deter A from aggression. Unfortunately, as the game stands, nation C can never credibly commit to intervening in a war; nation A continues to attack.

Next I examine how the formation of an alliance can affect the outcome of the game. Consider an alliance between B and C. Suppose the cost to both parties of forming the alliance is f and the cost of failing to honor the alliance agreement is h. If an alliance forms then the payoffs differ, as shown in Figure 2.

Initially, suppose that B and C form an alliance and the cost of breaking this alliance is h = 1/2. Nation C’s calculations at node n = 3 will differ. It must choose between 4-f, 1-f, and 2-f. If h = 1/2 then C still chooses not to intervene. Consequently, B surrenders if attacked; so nation A attacks. The outcome remains the same at ACQ. This alliance does not alter the behavior of any of the nations. However, as h increases alliances affect the behavior of every nation.

Suppose that h = 3/2. The cost of failing to meet its alliance commitments is high enough that C is prepared to undergo the costs of fighting in order to support its ally should a war occur. C’s decision is between 4-f, 1-f, and 2-f. If h = 3/2 then C’s preferred outcome is WarA/BC; C intervenes if a war occurs. If allied with C then nation B expects its ally to help it fight should a war occur. Knowing that it receives C’s support in a war, nation B retaliates if attacked. The expected utility of retaliating, 4f, is greater than the expected utility of surrendering, 3-f.

Nation A no longer receives its favorite outcome if it attacks; instead it becomes involved in a multilateral war (WarA/BC). A prefers the status quo (SQ) to this, since 2 > 1. Nation A does not attack and the outcome is SQ. The formation of the alliance alters the behavior of all three nations. However, it should be noted that although nation C would intervene and nation B would retaliate, the effect
of an alliance on the behavior of B and C is never observed because neither nation has the opportunity to act. The strategies of nations B and C are empirically unobservable unless A decides to attack. This, as I show later, has great importance when empirically estimating the effects of alliances.

Given that the effects of forming the alliance are known, we can calculate whether or not nations actually form alliances. Consider the first example, where \( h = 1/2 \). In this particular case the alliance has no effect on the outcome of the game. The outcome is ACQ whether or not an alliance forms. However, the payoffs from this outcome may differ because of the costs of alliance formation (\( f \)). If \( f \) is greater than zero then neither nation wants the alliance; it does not help them achieve a better outcome and it is costly to form. No alliance forms. However, if the cost of breaking an alliance is higher then nations form costly alliances (\( h = 3/2 \)). The effect of the alliance is to alter the outcome, from ACQ to SQ. Nation B receives a payoff of 3 without an alliance and a payoff of 5-f if an alliance forms. Therefore, provided the cost of alliance formation is not more than 2, nation B
wants to form the alliance. Similarly, for nation C, the payoff if an alliance forms is 5-f and the payoff if no alliance forms is 2. Nation C wants an alliance provided that the cost of forming it is not more than 3. This example predicts that if the cost of forming an alliance is less than 2 (f < 2) then the alliance forms and nation A is deterred.

This example shows that the formation of an alliance affects the behavior of nations. However, not every alliance affects behavior in the same way. In the example, if h = 1/2 then the outcome is ACQ, but if h = 3/2 then the outcome is SQ. Not only can different alliances have different effects but also this alters whether or not they form. It may be the case that alliances have a variety of effects. However, empirically these are not all observed. Nations only form those alliances that they expect will affect outcomes in a manner beneficial to them.

In order to theorize about the role of alliances in international relations the game above is generalized. This simple case tells us that in a particular case alliance formation has a particular effect. Unfortunately, this does not explain the effect of alliances under other circumstances. A general model needs to be developed.

Before proceeding further it is important to define exactly what an alliance is. I define an alliance as an agreement between two nations, entered into voluntarily, that represents a nonbinding commitment to help each other in event of armed conflict. This contrasts with a coalition, which is a set of nations that simply fight together in a war, whether or not they had a prior agreement to do so. Alliances are assumed to be nonbinding. Given the anarchic nature of international relations and the evidence that alliances are not always honored, there are both theoretical and empirical justifications behind this assumption (Altfeld and Bueno de Mesquita, 1979; Sabrosky, 1980; Siverson and King, 1980).

Given this characterization, why do nations sign alliances? For the purposes of this article I assume that, although nonbinding, alliances impose costs on nations. Whether these costs are generated domestically (Bueno de Mesquita et al., 1992, 1994) or reputationally (Snyder and Diesing, 1977; Alt, Calvert, and Humes, 1988) is not relevant here. All that matters for the arguments here is that there are costs associated with making and breaking alliances. Specifically, if nation i attacks an ally, nation j, or fails to support an ally, nation j, that is involved in a war, then nation i pays an alliance failure cost, h_i > 0. In addition, nation j pays a cost, g_j, associated with being forsaken by its ally. Alliances are also costly to form. If nation i forms an alliance with nation j, they both pay alliance formation costs, f_i and f_j. In some cases there are nonsecurity externalities associated with alliance formation (Gowa, 1989). Therefore, the cost of alliance formation can be either positive or negative. However, I assume that any benefit from alliance formation does not completely offset the cost of alliance failure (−h_i − f_i < 0 and −g_j − f_j < 0).

A General Model

The game tree is identical to that shown in Figure 1. In order to generalize, I consider a more extensive set of preferences (shown in Figure 3). Nations A and B are in competition over some international policy. Nation A wants to move the SQ in its favor and nation B opposes these changes.1 The nature of the international policy does not matter for what follows; all that is important is that A and B want to move the policy in opposite directions. In terms of international policy, A's most preferred outcome is ACQ. Since B has the opposite preferences, its most preferred outcome is SQ.

If B acquiesces then A gets to change the international policy. However, if a war

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1 Presumably, if both nations agree on how to change the SQ there is no conflict of interests and hence no reason to fight.
occurs then the resulting international policy will depend upon which nation wins. War produces an expected international policy that reflects the likelihood of A or B winning and the policy these nations will impose if they do win. Let x represent the expected outcome of a bilateral war. y and z represent the expected policy outcomes associated with WarAC/B and WarA/BC. I assume that the larger a nation’s military force, the more it likes the expected international policy that the war produces. For example, if nation C forms a coalition with nation B then this coalition is more likely to win than if nation B fights alone. Therefore, nation B prefers the expected policy outcome from the multilateral war, WarA/BC, to the expected policy outcome from the bilateral war, WarA/B (zB > xB). Correspondingly, nation A prefers those wars that it expects to win since this means that the expected policy outcome is moved in its favor (xAA > zA). In terms of international policy, A’s preference ordering is as follows: acqA > yA > xA > zA > sqA. B’s preferences over expected international policy are sqB > zB > xB > yB > acqB.

In addition to international policy, nations also care about the costs of violence and the costs of alliances. If nation i is involved in a war then it incurs costs, wi. Unreciprocated violence is also costly to both the aggressor and the victim. If the outcome is ACQ then A and B incur costs vA and vB, respectively. Alliance costs were considered above. Figures 3–6 show the preferences of each nation over outcomes for each alliance state.

Nation C also has international policy preferences. Although, generally, nation C does not have the same international policy preferences as A or B, it usually has a leaning toward one side. If C’s policy preferences are closer to nation A’s than to nation B’s then nation C is friends with nation A. Correspondingly, if nation C is closer in policy terms to B than A, then nation C is friends with B. If nation C is friends with A then it wants the policies that A will support rather than those policies that B supports. Therefore, acqC > yC > xC > zC > sqC. If C is friends with B then it wants policies that B supports; sqC > zC > xC > yC > acqC. In addition to expected policy outcomes, nation C is also concerned with the cost of fighting and the costs of alliances. So when nation C decides whether or not to intervene it considers not only expected policy but also the costs of fighting and the costs of its alliance commitments.

The strategy for each nation depends upon the alliance state. A strategy profile is a triple s = (sA, sB, sC) where sA is the action A takes at node 1 (attack or φ), sB is the action B takes at node 2 (retaliate or φ), and sC is the action C takes at node 3 (φ, attack B (form a coalition with A), or attack A (form a coalition with B)). Let θ be the mapping from strategy profiles, S = {sA, sB, sC}, into outcomes as described by the game tree, θ:S → {SQ, ..., ACQ}. Thus, if the alliance state is (AB) and the strategy profile is s*(AB), then the outcome of the conflict subgame is θ(s*(AB)). At the end of the game each nation is rewarded according to the outcome.

The solution concept is subgame perfect Nash equilibrium (SPE). At any information set, neN, define Γn to be all the nodes/information sets that follow n sequentially in the game tree (Γn is a subgame). sj is a best response to sj in some subgame Γn if and only if Uj(θ(sj, sj)) ≥ Uj(θ(sj*, sj)) ∀sj*. If (sA*, sB*, sC*) is a subgame perfect Nash equilibrium then sA*, sB*, and sC* are mutually best responses in Γn, ∀nE N.

Results

The subgame perfect Nash equilibria for each alliance state are characterized in Smith (1995). This section highlights how alliances affect the behavior of nations. Alliance can have a variety of effects. The presence of an alliance discourages a nation from attacking its ally. If a nation attacks its ally then it incurs the costs of
alliance failure. By not attacking, a nation can avoid these costs. The prospect of paying for failed alliances also encourages a nation to honor its alliance commitments. A nation is more likely to intervene if it has an alliance.

Nations A and B’s decisions to attack and retaliate are conditional on the expected behavior of nation C. Since an alliance affects the probability of a nation intervening, it also affects the probability of an attack. The effect of an alliance on A’s and B’s behavior depends upon whether or not the alliance is reliable. Suppose that B has a reliable alliance with C. The presence of the alliance means that B is more likely to retaliate because B expects allied support. Since B is likely to retaliate and C is likely to support B, a reliable alliance deters A from attacking. However, if the (BC) alliance is unreliable then B faces the prospect of fighting without allied help. In addition, if B retaliates it incurs costs associated with alliance failure, gb. Thus, an unreliable (BC) alliance discourages B from retaliating. Since B is likely to surrender, an unreliable alliance encourages A to attack. Therefore, the effects of an alliance depend upon whether or not the alliance is reliable.

Alliances affect the outcome of the conflict subgame. Since nations care about
outcomes, they care about which alliances form. Thus far I have modeled alliances as already formed. However, nations do not randomly form alliances. They choose those alliances that are most advantageous to them.

Alliance Formation Game

The conflict subgame equilibria predict the outcome for each alliance. In the alliance formation game nations decide which alliances they want to form. Just as they did in the earlier example, nations decide whether or not to form an alliance knowing how alliances affect outcomes. There are four possible alliance states,

\[ U_A \]

\[ U_B \]

\[ U_C \]

\[ SQ \]

\[ WarA/B \]

\[ WarAC/B \]

\[ WarA/BC \]

\[ ACQ \]

Fig. 4. General Case: (AB) Alliance

2 Li and Thompson (1978), McGowan and Rood (1975), and Midlarsky (1981, 1983) argue that alliance formation patterns cannot be distinguished from a random process. However, Siverson and McCarthy's (1982) findings contradict this. Lalman and Newman (1990) show that, far from choosing randomly, nations form those alliances that most increase their security.
AL = \{ (\phi), (AB), (AC), (BC) \}, which correspond to no alliance, an alliance between A and B, and so forth. Each nation is allowed to sign one alliance agreement, subject to being a member of that alliance. Formally, the pure strategies for each nation are \( \sigma_A \in \{ \phi, AB, AC \} \), \( \sigma_B \in \{ \phi, AB, BC \} \), and \( \sigma_C \in \{ \phi, AC, BC \} \).

If both parties sign an alliance, then the alliance forms. The payoffs from the alliance formation game are those payoffs that a nation would receive in the conflict subgame given the particular alliance state. Thus, when nations decide to form alliances they do so knowing how those alliances are going to affect behavior. Generically, the conflict subgame outcome is unique. Thus, if an (AB) alliance forms then A's expected payoff is \( U_A((AB), \theta(s^*(AB))) \), where \( s^*(AB) \) is the subgame perfect Nash equilibrium strategy in the presence of an (AB) alliance.

The solution concept used is strong equilibrium. This eliminates the possibility of a nation forming one alliance when there is another alliance that is better for both it and its new potential alliance partner. The Nash equilibrium conditions
constitute necessary conditions for alliance formation. A strategy profile \((\sigma_A, \sigma_B, \sigma_C)\) is a strong equilibrium if for all subsets, \(k \in K\) of nations, \((K = 2^{\{A, B, C\}})\), 
\(U_j(\sigma_A, \sigma_B, \sigma_C) \geq U_j(\sigma'_A, \sigma'_B, \sigma'_C), \forall \sigma'_k, \text{ and } \forall j \in k.\) According to the definition of a strong equilibrium, multilateral deviations must be considered.

**Results of the Alliance Formation Game**

Given the definition of a strong equilibrium, a necessary condition for nations to want to form an alliance is that the alliance does not make them worse off.

**Proposition 1. (AB) alliance**

Necessary conditions for the formation of an (AB) alliance are:

(i) the outcome is the SQ;
(ii) \(f_A \geq 0;\)
(iii) \(f_B \geq s_B - U_B((\phi),\theta(s^*(\phi))).\)
If an (AB) alliance forms then the outcome is the SQ. Nation A does not want to form an alliance and then attack its ally; doing so is costly, and A can avoid these costs by not forming an alliance. Empirically, nations that recently formed an alliance should not attack each other. For example, we should not expect Nazi Germany to have invaded Yugoslavia on 6th April 1941 since Yugoslavia had just joined the Tripartite alliance with Germany on 25th March 1941. The Tripartite alliance included agreements to allow Germany to transport troops across Yugoslavia to Greece. The alliance was beneficial to the Germans, so they were prepared to give up the opportunity to attack Yugoslavia. Correspondingly, the Yugoslav government was prepared to bear the costs of the alliance, having German troops transported across its territory, in order to avoid being attacked. Given that both Germany and Yugoslavia preferred to form an alliance rather than have Germany attack Yugoslavia, why did the Germans attack on 6th April? Although the Yugoslav government was prepared to accept the terms of the alliance, the Yugoslav air force was not. The air force overthrew the government in a bloodless coup d’etat and immediately denounced the alliance. Since the Germans were no longer receiving the benefits of the alliance they attacked Yugoslavia in order to reach Greece.

**Proposition 2. Offensive (AC) alliance**

Necessary conditions for the formation of an (AC) alliance are:

(i) the outcome is either SQ, WarAC/B, or ACQ;

(ii) B is weakly less likely to retaliate if an alliance forms;

(iii) A is weakly more likely to attack if an alliance forms;

(iv) if the outcome is SQ then \( f_A \leq 0 \) and \( f_C \leq 0 \);

(v) if alliance formation is costly then C is friends with A (i.e., if \( f_C > 0 \) then \( acq_C > y_C > x_C > z_C > sq_C \)).

An alliance between A and C means that C will support A should a war occur. Alliances never form if they result in alliance failure. An unreliable alliance does not help A in its competition with B. In addition, if A anticipates being deserted by its ally, then attacking B becomes less attractive. This does not mean that unreliable alliances cannot form. Condition (iv) states that, providing alliance formation is beneficial to both parties, an alliance can form if the outcome is SQ. In this situation it does not matter whether or not an alliance is reliable because A never attacks so alliance failure can never occur. However, in circumstances where wars occur then all alliances that form are reliable.

Alliance formation increases the probability that nation C intervenes if a war occurs. Following alliance formation, nation B faces the prospect of fighting the coalition of A and C if it retaliates. Since fighting both A and C makes victory unlikely, nation B often surrenders. Alliance formation increases the probability that B surrenders when attacked.

Alliance formation increases the probability that A attacks. The presence of an (AC) alliance reduces the probability of B resisting an attack. Therefore, A is likely to achieve its aims without having to fight. Even if B does retaliate, nation A anticipates C’s help in any war. Having a coalition partner increases A’s probability of victory in a war.

Offensive alliances form between friends, those nations that have common preferences across international issues. The formation of an (AC) alliance in-

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5 When I refer to an increase in a probability, what I actually mean is that the probability either increases or remains the same; the probability cannot go down.
creases the prospect of A attaining its wishes on international policy. If C prefers B’s international policy position compared to A’s, then C does not want to help A. Nations must have common goals in order to form costly alliances.

Proposition 3. (BC) alliance
Necessary conditions for the formation of a (BC) alliance are:
(i) the outcome is either SQ, WarA/BC, or ACQ;
(ii) B is weakly more likely to retaliate if a costly alliance forms;
(iii) A is weakly less likely to attack if a costly alliance forms;
(iv) if alliance formation is costly then C is friends with B (i.e., if $f_c > 0$ then $s_q e > z_c > x_c > y_c > acq_c$).

If a (BC) alliance forms, then, in the event of a war, nation C supports nation B. With complete information, alliance formation never results in alliance failure. If C is not prepared to intervene on behalf of its ally, then it does not form an alliance in those situations that result in war. The outcome following alliance formation is either SQ, ACQ, or WarA/BC.

Forming an alliance increases the probability of C forming a coalition with B if a war occurs. This means that B anticipates allied support if it resists A’s aggression. Retaliating is a better prospect for B if it has an alliance partner because it expects allied support. (BC) alliances reassure target nations that they will receive allied assistance if they resist. The probability of retaliation increases upon the formation of a defensive alliance.

If A attacks in the face of a defensive alliance then it knows that B is likely to retaliate and that C is likely to intervene against it. This makes attacking an unattractive alternative. Nation A behaves less aggressively following defensive (BC) alliance formation. Defensive alliances deter aggression.

If defensive alliances are costly to form then they only form between friends. A nation that does not support the policy position of another nation does not form an alliance with it. By forming an alliance a nation enhances the ability of its ally to alter international policy. If a nation does not support these changes then it does not want to form the alliance. Rather than further elaborate these results, I consider their implications for the study of alliances.

Implications
The results provide explanations for the empirically observed properties of alliances. Looking at offensive and defensive alliances, several generalizations can be drawn. I start by considering how alliances affect the occurrence of war.

Empirical studies have failed to identify a robust relationship between alliance formation and the occurrence of war. The propositions above reveal that alliance formation affects both an aggressor’s decision to attack and a target’s decision to retaliate. An offensive (AC) alliance makes A more likely to attack and B less likely to retaliate. A defensive (BC) alliance has the opposite effect, decreasing the probability of A attacking and increasing the probability of B retaliating. However, in both cases the effect of alliance formation on the probability of war is ambiguous.

For a war to occur, A must attack B, and B must retaliate. A defensive alliance makes A less likely to attack. This should reduce the number of wars occurring after alliance formation. However, (BC) alliances also reassure target nations. This makes war more likely because any attack is likely to be resisted. Defensive alliances both deter aggression and encourage resistance. The aggregate effect of alliance
formation on the occurrence of war is ambiguous. Offensive alliances also have an ambiguous effect on the occurrence of war. Offensive alliances make A more likely to attack, which increases the probability of war. However, in the presence of an offensive alliance, B is less likely to resist an attack, which decreases the probability of war. These two factors work in opposite directions. Although the aggregate effect is ambiguous, alliances have predictable effects on the behavior of aggressors and targets.

The propositions suggest that with complete and perfect information there are never any instances of alliance failure. Empirically, we observe that nations intervene on behalf of their allies only about 25 percent of the time (Sabrosky, 1980; Siverson and King, 1980; Kim, 1991). Because there is no complete information it is tempting to suggest that this accounts for these differences. However, given the huge disparity between the prediction and the evidence, this alone is not a satisfactory explanation. Another explanation is that the reliability of an alliance changes over time. Over time situations change and an alliance that was perhaps reliable when it was formed could now be unreliable. Indeed, both Siverson and King (1980) and Sabrosky (1980) find that alliance reliability declines over time. Although incomplete information and changes in environmental factors are both important, I argue that the empirical observation that alliances are often unreliable is a result of sampling.

Empirically, we can only measure alliance reliability by observing whether or not an ally intervenes in the event of a war. Consider defensive alliances. The characterization of the conflict subgame equilibria revealed that while reliable alliances reassure target nations, unreliable alliances encourage targets to surrender. An unreliable (BC) alliance encourages A to attack. It does so because B is likely to surrender, and even if a war does occur A only has to fight nation B since C remains neutral. Nation A prefers to attack unreliable defensive alliances. If an alliance, designed to deter A, is reliable, then the reliability of the alliance is never revealed, because A never attacks. If A attacks an alliance partner it is likely that the alliance is unreliable.

Incomplete information and the effects of change over time imply that some alliances are unreliable. It is these unreliable alliances that nation A attacks. Only a small proportion of alliances need to be unreliable to generate the empirical observation that alliances are on average unreliable. Unreliable alliances are selected into the sample, alliance partners involved in wars, at a greater rate than reliable alliances. Recent empirical studies reveal exactly this pattern of alliance reliability (Smith, 1994, 1995).

Alliances form between friends. If an alliance is costly to form then only nations that support each other’s policies form alliances. This result is not particularly novel, but it is important. The assumption that alliances form between nations that share international policy preferences is at the core of the expected utility literature. This literature, beginning with Bueno de Mesquita (1981), uses the correlation between the alliance profiles of nations as a proxy for their commonality of

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4 This figure is from studies that include ententes and nonaggression pacts. These types of treaties do not explicitly state that nations must fight on behalf of their ally. Thus, in these alliances, failing to intervene need not be classified as alliance failure.

5 Empirical tests of offensive alliances are also biased toward unreliable alliances. Singer and Small (1966a, 1969) assume that alliances formed within one month of a war are part of the war and as such they exclude these events from their data set. Suppose that an alliance is reliable and results in war, WarAC/B. Immediately after the signing of the alliance A is likely to attack. Therefore, these reliable alliances are excluded from the sample because a war occurs immediately. Should a nation be called upon to intervene on behalf of an aggressive ally then this is not the situation for which the alliance was designed. The alliance may very well be reliable in the situation for which it was intended but this is not the situation being sampled. Therefore, the observed reliability of offensive alliances is low.
international preferences. The model confirms that if alliances are costly to form then a commonality of international preferences is a necessary condition for alliance formation.

Related to the prediction that costly alliances are between friends, the model predicts that alliances only form when they improve outcomes for both parties. If there are nonsecurity benefits from forming alliances, \( f_i < 0 \), then nations can form alliances that produce less favorable outcomes for them. If the benefits they receive from an alliance offset the costs of an inferior outcome, then nations may form alliances that do not enhance the outcomes they receive. However, if alliances are costly to form then the alliance improves the outcome for the alliance partners. Lalman and Newman (1990) show that when nations form alliances the alliances they form are optimal in terms of improving the outcomes they receive.

Conclusions

In order to theorize about the role of alliances in international relations, this article models the interactions between three nations. The behavior of a third party affects the behavior of the other states. Alliances impose costs upon nations. Because of these costs, alliances can change the behavior of states. Because the behavior of one nation affects the behavior of other nations, alliances can change outcomes in the international system. Knowing how alliances affect outcomes, nations can decide whether or not it is in their interests to form an alliance.

The model addresses the study of alliances in several ways. First, the theory suggests that studies looking at the correlation between alliance formation and war are simply looking at the wrong question. The analysis suggests that alliance formation affects the behavior of aggressors and targets in a predictable manner. For example, a defensive alliance encourages a victim to resist an attack. This increases the probability of war. However, a defensive alliance also deters aggression in the first place, which reduces the probability of war. Although alliance formation has predictable effects on behavior, the aggregate effect is ambiguous. The analysis suggests that we should be examining the effects of alliances on the behavior of nations rather than looking at the incidence of war.

Second, the reliability of alliances affects the behavior of states. For example, a potential aggressor is more likely to attack an unreliable defensive alliance than a reliable one. This produces sampling bias in estimating the reliability of alliances. Hence, low observed reliability does not necessarily imply that the average reliability of alliances is low.

Finally, the model suggests that costly alliances form between nations that have a commonality of interests. Nations form alliances because it improves the outcomes they expect to receive.

The value of modeling alliance formation is that it gives us a framework on which to interpret empirical evidence. The model clearly predicts the relationship between variables. It also shows us which relationships are important and which are not. The final advantage of a theoretical model is that it suggests appropriate testing procedures. For example, the model shows that sampling effects mean that conventional estimation techniques are inappropriate for estimating the reliability of alliances (Smith, 1994, 1995).

The model integrates several aspects of the alliance literature. Alliance reliability affects the behavior of nations in the international system. When nations form alliances they do so knowing how alliances affect behavior. Alliance reliability, alliance formation, and the effect of alliances on the occurrence of conflict are interrelated.
Appendix

Proof of Proposition 1: (AB) alliance
If \( s^*(AB) \) is an equilibrium strategy profile then
\[
U_A((AB),\theta(s^*(AB))) \geq U_A((\phi),\theta(s^*(\phi))) \text{ and } U_B((AB),\theta(s^*(AB))) \geq U_B((\phi),\theta(s^*(\phi))).
\]
The formation of an (AB) alliance does not alter C's payoffs for any outcome. Since nation C is the last player to move it is faced with the same decision whether or not an alliance is formed. Therefore, \( s_c^*(\phi) = s_C^*(AB) \). Similarly, at node \( n = 2 \) nation B faces the same decision whether or not an (AB) alliance forms. If an alliance is formed then B pays the cost \( f_B \) associated with forming the alliance and pays the cost \( g_B \) associated with alliance failure. Since B pays these costs whether or not it retaliates and C's decision is unchanged by the formation of an (AB) alliance then \( s_B^*(\phi) = s_B^*(AB) \).

(i) \( \theta(s^*(AB)) = SQ \). B and C's strategies do not change as a result of alliance formation. If A attacks then \( \theta(attack, s_B^*(AB), s_C^*(AB)) = \theta(attack, s_B^*(\phi), s_C^*(\phi)) \). A pays the cost \( g_A \) for attacking its ally. \( U_A((AB), \theta(attack, s_B^*(AB)), s_C^*(AB)) - U_A((\phi), \theta(attack, s_B^*(\phi)), s_C^*(\phi)) = -f_A - g_A < 0 \). Therefore, \( s_A^*(AB) \neq attack \). This implies that \( s_A^*(AB) = \phi \) and hence that \( \theta(s_A^*(AB)) = SQ \).

(ii) \( f_A \leq 0 \). Given (i), \( U_A((AB), \theta(s^*(AB))) = sqa_f - f_A \). If A does not form an alliance then it can ensure itself a payoff of \( sqa_f \) by not attacking: \( U_A((\phi), \theta(s_B^*(\phi)), s_A^*(\phi)) = sqa_f \). \( U_A((AB), \theta(s^*(AB))) \geq U_A((\phi), \theta(s^*(\phi))) \) implies that \( sqa_f - f_A \leq sqa_f \). Therefore, \( f_A \leq 0 \).

(iii) \( f_B \leq sqB - U_B((\phi), \theta(s^*(\phi))) \). If no alliance forms then B's payoff is \( U_B((\phi), \theta(s^*(\phi))) \). Given (i), if an alliance forms then B's payoff is \( U_B((AB), SQ) = sqB - f_B \). Therefore, \( U_B((AB), \theta(s^*(AB))) \geq U_B((\phi), \theta(s^*(\phi))) \) implies \( sqB - f_B \geq U_B((\phi), \theta(s^*(\phi))) \). QED

Proof of Proposition 2: (AC) alliance
If \( s^*(AC) \) is an equilibrium strategy profile then
\[
U_A((AC), \theta(s^*(AC))) \geq U_A((\phi), \theta(s^*(\phi))) \text{ and } U_C((AC), \theta(s^*(AC))) \geq U_C((\phi), \theta(s^*(\phi))).
\]
(i) \( \theta(s^*(AC)) \in \{SQ, WarAC/B, ACQ\} \). Suppose not. Suppose that \( \theta(s^*(AC)) = WarA/B \). First, show that if the outcome is WarA/B then the outcome is also WarA/B regardless of any alliance forms. Second, show that the cost of alliance failure deters C from forming an alliance given that it does not alter the outcome.

\( \theta(s^*(AC)) = WarA/B \) implies that C remains neutral in the event of war. If \( s_c^*(AC) = \phi \) then \( x_c - f_c - h_c \geq \max[y_c - w_c - z_c - h_c - f_c] \). Therefore, \( x_c \geq \max[y_c - w_c - z_c - h_c] \). This implies that \( s_c^*(\phi) = \phi \). If C remains neutral when an (AC) alliance forms then C also remains neutral without an alliance. C's strategy does not change with the formation of an (AC) alliance.

B's payoffs are unaffected by the presence of an (AC) alliance. Therefore, \( s_B^*(AC) = retaliate \) implies that \( y_B - w_B \geq acc_B - v_B \). Since \( s_c^*(\phi) = \phi \) then \( s_B^*(\phi) = retaliate \).

Similarly, \( \theta(s^*(AC)) = WarA/B \) implies that \( x_A - w_A - f_A - g_A > sq_A - f_A \Rightarrow x_A - w_A > sq_A \). This implies that \( s_A^*(\phi) = attack \). A would attack if no alliance formed. Therefore, if \( \theta(s^*(AC)) = WarA/B \) then \( \theta(s^*(\phi)) = WarA/B \).
There are costs associated with alliance formation and alliance failure. \( U_c((AC), \text{War}A/B) - U_c((\phi), \text{War}A/B) = -f_c - h_c < 0 \). If \( \theta(s*(AC)) = \text{War}A/B \) then \( U_c((AC), \theta(s*(AC))) < U_c((\phi), \theta(s*(\phi))) \). Hence the outcome is not WarA/B.

A similar argument shows that the outcome cannot be WarA/BC. Suppose that \( \theta(s*(AC)) = \text{War}A/BC \). This implies that C intervenes on behalf of B should a war occur. If \( s_c*(AC) = \text{attack} \) then \( z_c-w_c-h_c-f_c \geq \max[X_c-f_c-h_c-y_c-w_c-f_c] \). This implies that \( z_c-w_c \geq \max[X_c-y_c-w_c] \) and hence \( s_c*(\phi) = \text{attack} \).

If \( \theta(s*(AC)) = \text{War}A/BC \) then \( s_B*(AC) = \text{retaliate} \). \( s_B*(AC) = \text{retaliate} \Rightarrow z_B-w_B \geq \text{acc}B \Rightarrow s_B*(\phi) = \text{retaliate} \). B and C's strategies remain constant. Therefore, \( \theta(s*(AC)) = \text{War}A/BC \Rightarrow s_A*(AC) = \text{attack} \Rightarrow z_A-w_A-f_A-g_A \geq s_A-f_A \Rightarrow z_A-w_A > s_A \Rightarrow s_A*(\phi) = \text{attack} \). Therefore, if \( \theta(s*(AC)) = \text{War}A/BC \) then \( \theta(s*(\phi)) = \text{War}A/BC \).

There are costs associated with alliance failure. \( U_c((AC), \text{War}A/BC) - U_c((\phi), \text{War}A/BC) = -f_c - g_c < 0 \). Nation C can avoid these costs by not forming an alliance. Therefore, \( \theta(s*(AC)) \neq \text{War}A/BC \).

If an (AC) alliance forms then the outcome must be either SQ, ACQ, or WarA/BC.

(ii) B is weakly less likely to retaliate if an alliance forms. This means that if \( s_B*(\phi) = \phi \) then \( s_B*(AC) = \phi \) and if \( s_B*(AC) = \text{retaliate} \) then \( s_B*(\phi) = \text{retaliate} \).

Suppose that \( s_B*(\phi) = \phi \). This implies that \( U_B((\phi), \theta(\text{attack}, \text{retaliate}, s_c*(\phi))) \in [y_B-w_B, y_B-w_B, z_B-w_B] \leq U_B((\phi), \theta(\text{attack}, s_c*(\phi))) = \text{acq}B-v_B \). Condition (i) implies that if the alliance state is (AC) and a war occurs then C attacks B: \( s_c*(AC) = \text{attack} \). B. Therefore, \( U_B((AC), \theta(\text{attack}, \text{retaliate}, s_c*(AC))) \leq y_B-w_B \leq U_B((AC), \theta(\text{attack}, s_c*(AC))) = \text{acq}B-v_B \). This implies that \( s_B*(\phi) = \phi \).

Now consider the latter case. Suppose \( s_B*(AC) = \text{retaliate} \). This implies that \( y_B-w_B \geq \text{acq}B-v_B \). Note that due to condition (i) if a war occurs then nation C attacks B. If there is no alliance then \( U_B((\phi), \theta(\text{attack}, \text{retaliate}, s_c*(\phi))) \in [y_B-w_B, y_B-w_B, z_B-w_B] \geq y_B-w_B \geq \text{acq}B-v_B = U_B((\phi), \theta(\text{attack}, s_c*(\phi))) \). Therefore, \( s_B*(\phi) = \text{retaliate} \).

(iii) A is weakly more likely to attack if an alliance forms. This means that if \( s_A*(\phi) = \text{attack} \) then \( s_A*(AC) = \text{attack} \) and if \( s_A*(AC) = \phi \) then \( s_A*(\phi) = \phi \).

First, suppose \( s_A*(\phi) = \text{attack} \). If \( s_B*(\phi) = \text{retaliate} \) then \( s_A*(\phi) = \text{attack} \) implies \( U_A((\phi), \theta(s*(\phi))) \in [y_A-w_A, x_A-w_A, z_A-w_A] \geq s_A \). Since \( y_A > x_A > z_A, y_A-w_A \geq s_A \). Given (i), \( U_A((AC), \theta(s*(AC))) \in [y_A-w_A-f_A, x_A-w_A-s_A-s_A-f_A] \). Since \( x_A > y_A-w_A > s_A \) then \( s_A*(AC) = \text{attack} \). Now consider \( s_B*(\phi) = \phi, s_A*(\phi) = \text{attack} \) implies \( U_A((\phi), \theta(s*(\phi))) = \text{acq}A \geq s_A \). By (ii), \( s_B*(\phi) = \phi \Rightarrow s_B*(AC) = \phi \). Therefore, \( s_A*(AC) = \text{attack} \) because \( U_A((AC), \theta(s*(AC))) = \text{acq}A - f_A \geq s_A - f_A \).

The second statement can be proved in a similar manner. With an (AC) alliance nation B is likely to surrender and nation C is likely to intervene to help A should a war occur. If A does not want to attack under these circumstances then it does not want to attack when B is more likely to resist and when C is less likely to intervene on its behalf.

(iv) If the outcome is SQ then \( f_A \leq 0 \) and \( f_c \leq 0 \). If \( \theta(s*(AC)) = \text{SQ} \) then \( U_A((AC), \theta(s, s_B*(AC), s_c*(AC))) \geq U_A((AC), \theta(\text{attack}, s_B*(AC), s_c*(AC))) \). Condition (iii) implies that \( U_A((\phi), \theta(s, s_B*(\phi), s_c*(\phi))) \geq U_A((\phi), \theta(\text{attack}, s_B*(\phi), s_c*(\phi))) \). Therefore, \( \theta(s*(AC)) = \text{SQ} \) implies that \( \theta(s*(\phi)) = \text{SQ} \). For an alliance to form, \( U_A((AC), \text{SQ}) - U_A((\phi), \text{SQ}) = -f_A \geq 0 \) and \( U_c((AC), \text{SQ}) - U_c((\phi), \text{SQ}) = -f_c \geq 0 \). Therefore, \( f_A \leq 0 \) and \( f_c \leq 0 \).
(v) If alliance formation is costly then C is friends with A. Suppose A and C are not friends and that alliance formation is costly: \(ac,< y_c< x_c < z_c< sq_c \) and \(f_c>0\). If an (AC) alliance forms then \(\theta(s^*(AC)) \in \{SQ,ACQ,WarAC/B\}\). Condition (iv) implies that if \(f_c>0\) then \(\theta(s^*(AC)) \neq SQ\).

Suppose \(\theta(s^*(AC)) = ACQ\). \(U_c((AC),ACQ) = ac_c-f_c < U_c((\phi),\theta(s^*(\phi))) \in \{ac_c,qc,MAX[x_c,z_c-w_c]\}\). C prefers the outcome reached if there is no alliance. Therefore, \(\theta(s^*(AC)) \neq ACQ\).

Suppose \(\theta(s^*(AC)) = WarAC/B\). \(U_c((AC),WarAC/B) = y_c-w_c-f_c\). By (ii) \(\theta(s^*(\phi)) \neq ACQ\). This implies that \(U_c((\phi),\theta(s^*(\phi)) \in \{qc,MAX[x_c,z_c-w_c]\}> y_c-w_c-f_c\). Therefore, \(\theta(s^*(AC)) \neq WarAC/B\). Therefore, either \(f_c \leq 0\) or \(ac_c > y_c > x_c > z_c > sq_c\). QED

Proof of Proposition 3: (BC) alliance

The formation of a (BC) alliance implies that \(U_B((BC),\theta(s^*(BC))) \geq U_B((\phi),\theta(s^*(\phi)))\) and \(U_c((BC),\theta(s^*(BC))) \geq U_c((\phi),\theta(s^*(\phi)))\).

(i) The outcome is either SQ, WarA/BC, or ACQ. \(\theta(s^*(BC)) \in \{SQ, WarA/BC, ACQ\}\).

To prove this, suppose that the outcome is neither WarA/B nor WarAC/B. First suppose that the outcome is WarA/B. This implies that \(s_*(BC) = \phi\). Therefore, \(U_c((BC),WarA/B) = x_c-h_c-f_c \geq MAX[x_c-w_c-x_c-h_c-f_c]\). Which implies \(s_*(\phi) = \phi\). If C remains neutral when an alliance forms then C also remains neutral when there is no alliance. B's strategy is the same whether an alliance exists or not. \(s_*(BC) = retaliate\) implies that \(x_B-w_B-f_B-g_B \geq ac_{qB}-v_{B-B}\). This implies that \(x_B-w_B > ac_{qB}-v_{B-B}\). Hence \(s_*(\phi) = retaliate\). Since B and C both behave identically whether or not the alliance forms, nation A also behaves identically. \(s_A*(BC) = attack\) \(\Rightarrow x_A-w_A \geq sq_A \Rightarrow s_A*(\phi) = attack\). Therefore if \(\theta(s^*(BC)) = WarA/B\) then \(\theta(s^*(\phi)) = WarA/B\).

There are costs associated with alliance failure. Nation C avoids these costs if no alliance forms. \(U_c((BC),\theta(s^*(BC))) - U_c((\phi),\theta(s^*(\phi))) = -f_c < 0\). Therefore C does not want to form an alliance. The outcome cannot be WarA/B.

Now consider the second case. Suppose the outcome is WarAC/B. \(s_*(BC) = attack\). Then \(U_c((BC),WarAC/B) = y_c-w_c-h_c-f_c \geq MAX[y_c-w_c-x_c-h_c-f_c]\). Therefore, \(y_c-w_c \geq MAX[y_c-w_c-x_c]\) which implies \(s_*(\phi) = attack\). If nation C attacks its ally B then it would also attack B if it were nonallied. B's strategy is the same whether an alliance exists or not. \(s_*(BC) = retaliate\) implies that \(y_B-w_B-f_B-g_B \geq ac_{qB}-v_{B-B}\). This implies that \(y_B-w_B > ac_{qB}-v_{B-B}\). Hence \(s_*(\phi) = retaliate\). Since B and C both behave identically whether or not the alliance forms, nation A also behaves identically. \(s_A*(BC) = attack\) \(\Rightarrow y_A-w_A \geq sq_A \Rightarrow s_A*(\phi) = attack\). Therefore, if \(\theta(s^*(BC)) = WarAC/B\) then \(\theta(s^*(\phi)) = WarAC/B\).

There are costs associated with alliance failure. Nation C avoids these costs if no alliance forms. \(U_c((BC),\theta(s^*(BC))) - U_c((\phi),\theta(s^*(\phi))) = -f_c < 0\). Therefore C does not want to form an alliance. The outcome cannot be WarAC/B.

Since the outcome is neither WarA/B nor WarAC/B then the outcome must be either SQ, WarA/BC, or ACQ.

(ii) B is weakly more likely to retaliate if a costly alliance forms. If \(s_B^*(\phi) = retaliate\) then \(s_B^*(BC) = retaliate\) if \(f_B > 0\) and \(s_B^*(BC) = \phi\) then \(s_B^*(\phi) = \phi\).

First, show that if \(s_B^*(\phi) = retaliate\) then \(s_B^*(BC) = retaliate\). Second, show that if \(s_B^*(BC) = \phi\) then \(s_B^*(\phi) = \phi\).

Suppose that \(s_B^*(\phi) = retaliate\). This implies that \(U_B((\phi),\theta(attack,retaliate,s_C^*(\phi)))\)
\( \in \{x_B-w_B,y_B-w_B,z_B-w_B\} \geq U_B((\phi,\theta(attack,\phi, s_c^*(\phi))) = acq_B-v_B. \) Given (i), if a war occurs then \( s_c^*(BC) = attack A. \) Therefore, \( z_B-w_B-f_B \geq acq_B-v_B-f_B, \) which implies that \( s_B^*(BC) = retaliate. \)

Next show that \( s_B^*(BC) = \phi \) implies \( s_B^*(\phi) = \phi. \) We need to consider B’s decision for each possible strategy by C.

First, suppose \( s_c^*(BC) = attack A. \) then \( s_B^*(BC) = \phi \) implies that \( z_B-w_B-f_B \leq acq_B-v_B-f_B. \) \( U_B((\phi,\theta(attack,retaliate,s_c^*(\phi))) \in \{z_B-w_B,y_B-w_B,z_B-w_B\} \leq acq_B-v_B \) which implies that \( s_B^*(\phi) = \phi. \)

Second, suppose \( s_c^*(BC) = attack B. \) This implies that \( y_C-w_C-f_C-h_C \geq max[x_C-f_C-h_C,z_C-w_C-f_C]. \) Therefore, \( y_C-w_C \geq max[x_C,z_C-w_C-f_C], \) which implies that \( s_c^*(\phi) = attack B. \) Therefore, \( s_B^*(BC) = \phi \) implies that \( y_B-w_B-g_B-f_B \leq acq_B-v_B-f_B. \) If \( s_B^*(\phi) \neq \phi \) then \( U_B((\phi,\theta(attack,retaliate,s_c^*(\phi))) \geq U_B((\phi,\theta(attack,\phi,s_c^*(\phi))) \) which implies that \( y_B-w_B \geq acq_B-v_B. \) However, under these circumstances, B does not want to form an alliance. \( U_B((BC),\theta(s^*(BC))) \geq U_B((\phi,\theta(s^*(\phi))) \) implies that \( acq_B-v_B \geq f_B \geq acq_B-v_B. \) This contradicts alliances being costly \( (f_B > 0). \)

Third, suppose \( s_c^*(BC) = \phi. \) This implies that \( x_C-h_C-f_C \geq max[y_C-w_C-h_C,z_C-w_C-f_C]. \) Therefore, \( x_C \geq max[y_C-w_C,z_C-w_C-f_C] \) which implies that \( s_c^*(\phi) = \phi. \) If \( s_B^*(BC) = \phi \) then \( x_B-w_B-g_B-f_B \leq acq_B-v_B-f_B. \) If \( s_B^*(\phi) \neq \phi \) then \( x_B-w_B \geq acq_B-v_B. \) \( U_B((BC),\theta(s^*(BC))) \geq U_B((\phi,\theta(s^*(\phi))) \) implies that \( acq_B-v_B \geq f_B \geq acq_B-v_B. \) This contradicts alliances being costly \( (f_B > 0). \)

Therefore, if \( f_C > 0 \) then \( s_B^*(BC) = \phi \) implies \( s_B^*(\phi) = \phi. \)

(iii) A is weakly less likely to attack if a costly alliance forms. This means that if \( f_B > 0 \) then \( s_B^*(\phi) = \phi \) implies \( s_A^*(BC) = \phi \) and if \( f_B > 0 \) \( s_A^*(BC) = attack \) implies \( s_A^*(\phi) = attack. \)

Both these statements are proved by considering how alliance formation affects B and C’s strategies. Given (ii), the formation of a (BC) alliance makes B more likely to retaliate. The formation of a (BC) alliance also makes C more likely to attack A and less likely to attack B or remain neutral.

Therefore, \( U_A((\phi,\theta(attack,s_B^*(\phi),s_c^*(\phi))) \geq U_A((BC),\theta(attack,s_B^*(BC),s_c^*(BC))). \) Since \( U_A((\phi,\theta(s_B^*(\phi),s_c^*(\phi))) = U_A((BC),\theta(\phi,s_B^*(BC),s_c^*(BC))) = s_A^*(\phi) \) implies \( s_A^*(BC) = \phi. \) Similarly, \( s_A^*(BC) = attack \) implies \( s_A^*(\phi) = attack. \)

(iv) If alliance formation is costly then C is friends with B. If \( f_C > 0 \) then \( s_B > y_C > x_C > z_C > acq_c. \)

Suppose that C is not friends with B. Suppose that \( f_C > 0 \) and \( acq_c > y_C > x_C > z_C > sq_c. \) By (i) the outcome is either SQ, ACQ, or WarA/BC. For each outcome show that this contradicts B and C not being friends.

Suppose \( \theta(s^*(BC)) = SQ. \) \( U_B((BC),SQ) = s_B-f_C. \) If no alliance forms then \( U_C((\phi,\theta(s^*(\phi))) \in \{sq_c,acq_c,\max[x_C,y_C-w_C]\}. \) Since \( acq_c > y_C > x_C > z_C > sq_c \) this contradicts C wanting to form an alliance.

Suppose \( \theta(s^*(BC)) = ACQ. \) By (i) and (iii) \( \theta(s^*(\phi)) = ACQ. \) But then \( U_C((\phi,\theta(s^*(\phi))) > U_C((BC),\theta(s^*(BC))), \) which contradicts C wanting to form the alliance.

Suppose \( \theta(s^*(BC)) = WarA/BC. \) By (ii) and (iii) \( U_C((BC),WarA/BC) = z_C-w_C-f_C < U_C((\phi,\theta(s^*(\phi)))) \in \{acq_c,\max[x_C-w_C,x_C]\}. \) This contradicts C wanting to form an alliance. QED
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