Fighting Battles, Winning Wars

ALASTAIR SMITH
Department of Political Science
Washington University

The author models warfare as a random-walk stochastic process. Rather than model war as a single-shot lottery, as is common in the literature, nations fight a series of battles. Nations do not defeat their foe in a single battle; rather, victory results from aggregate success over a series of interactions. Only by gradually reducing an opponent's capacity to resist can a nation force victory. Yet, under many circumstances, nations preempt defeat by surrendering once the tide of war moves against them. The author characterizes the distribution of resources that results in conflict. Against this background, the author examines how the preferences of leaders affect the conditions under which war occurs. Because the preferences of leaders affect the pattern of conflict, citizens' electoral choices are made contingent on the international environment. Hence, the author provides a link between conflict behavior and domestic electoral processes.

The nature of warfare is fundamental to the study of any security issue. In this article, I develop a model of warfare. This Markovian random-walk model of war decomposes wars into their constituent battles and asks how nations choose strategies under different contingent circumstances. Rather than treat war as a single instantaneous event, I model it as a dynamically evolving process. The overall process is modeled like a tug-of-war competition: nations strive to defeat their opponent. Yet, their strategy during the course of the competition depends on how well they are doing. Nations close to victory behave differently from those close to defeat.

I start by summarizing existing models of warfare. In particular, I concentrate on simple lottery-based conceptions and discuss the pros and cons of such approaches. Having outlined the basic components of the model I propose, I discuss how it overcomes the limitations of existing techniques. The model provides a flexible platform from which to address numerous security issues. Rather than depicting all these possibilities, I concentrate on examining the key concepts within the model. To this end, I detail the underlying stochastic process and how nations' decisions affect it.

Having explained the basic nature of war, I discuss the incentives that nations face and describe equilibrium behavior. Given these results, I examine what factors affect the conditions under which nations fight. In particular, I focus on how the value of the

AUTHOR'S NOTE: The ideas in this article were inspired by the “Center in Political Economy Conference on Modeling War” at Washington University in May 1996. I thank the participants for their ideas. I gratefully acknowledge the financial support of the National Science Foundation (SBR-9631990).

JOURNAL OF CONFLICT RESOLUTION, Vol. 42 No. 3, June 1998 301-320
prize for winning, the cost of fighting, and the relative power of the combatants determines when fighting occurs.

To illustrate the applicability of the model, I examine the relationship between domestic elections and conflict. In particular, I assume that an election occurs at a specific time during the conflict. How the war has progressed affects the electorate’s preferences for different candidates. Because different types of candidates are electorally advantaged under different circumstances, I examine the incentives of leaders to divert the course of the war as elections approach. I conclude with a discussion of the advantages and the general applicability of this type of model.

MODELS OF WAR

Wars are not one-shot events. They occur over a period of time, sometimes over many decades. As such, the conventional modeling technique, a simple lottery, is inappropriate. The basic idea of such models is that when war occurs, the winner is determined by a coin flip. Although one side’s probability of victory is usually described as a function of its relative strength or its choice of actions, inherently the victor is determined instantaneously.

As a modeling strategy, this eliminates any discussion of temporal issues or range of outcome: instantly, one side completely wins and the other side completely loses. Inherently, such models prevent us from examining what happens during war. Yet, we know that the progress of the war affects things such as effort levels, negotiating position, military strategy, and domestic political support. Because leaders care about these things, they affect the initial decision to engage in conflict.

Although it is generally true that most models of war use simply lotteries, there are many exceptions. For example, Powell (1990, 1996) and Fearon (1994, 1996) propose escalatory and appeasement models of warfare. However, the most common alternative is the war of attrition model (Maynard-Smith 1974). In this class of models, nations pay costs to remain in the struggle. The winner receives a prize only when its opponent quits.

The model I propose contains elements of both the standard lottery and the war of attrition models. Nations have repeated interactions, during each of which one nation could gain an advantage over the other. Over time, one nation’s advantages could accumulate until it completely overwhelms its foe; alternatively, initial successes could be offset by reverses with the war remaining in the balance. The interactions continue either until one side has accumulated so many successes that the other side no longer has the capacity to resist or until one side decides to quit. Put simply, nations fight battles until one nation decisively defeats the other or until one nation surrenders.

A useful metaphor is nations battling over forts. Initially, each nation possesses some of these forts. During the war, nations fight battles over these forts. If one nation ever captures all the forts, then it wins decisively, the other nation being unable to continue. The war is a series of attempts to capture the enemy’s forts without losing your own. A nation is only beaten decisively if it loses all its forts and is no longer able to prosecute the war. Sometimes nations are decisively defeated (e.g., Nazi Germany in 1945 or Carthage in 146 B.C.). However, a more typical situation is for a nation to
decide that the war is no longer worth pursuing. Unlike the case of Nazi Germany in World War II, during World War I, Germany surrendered before the allies had obtained a decisive victory.

Although the term battle is direct and easily understood, the interactions between states need not strictly be battles. Blood does not need to be shed for one side to gain a military advantage; simply outmaneuvering one’s opponent is often just as effective. This said, I shall use the term battle rather than the abstract term interaction. However, it should be understood that one nation might gain advantage through jockeying for position, rather than by direct bloodshed. Similarly, I shall also use the term fort. By fort, however, I mean a discrete approximation of what nations are fighting over. Although historically forts were important objectives, modern battle lines are more fluid and continuous. Yet, even these dynamic fronts are broken up by defensible geographical features, such as mountains, rivers, forests, and marshes. I assume forts have an intrinsic value to each nation, but allowing them to be intermediates that must be captured to secure the real objective does not fundamentally alter the structure of the model.  

Although winning a battle gives one side an advantage, it is not the same as winning a war. I treat each battle as a separate interaction, modeling the outcome as a lottery. It is the aggregation of these separate battles that determines the final outcome. The advantage of decomposing wars into these separates battles is that it enables decisions to be considered under the appropriate contingent circumstances. If the war is treated as a whole, then all the choices made are lumped into a single invariant decision. In the setting I propose, nations gauge their best choices as the war proceeds.

At each interaction, we can examine the decision to fight, effort levels, choice of military strategy, negotiating position, and domestic support for the war. Modeling decision making during the course of the war is intrinsically useful because it provides a handle on those questions that cannot be addressed if the war is treated as a whole. A model of decision making also edifies studies that treat wars as single events because it provides more realistic expectations about the war. For example, during prewar disputes, whether nations reach agreement depends on what they are negotiating to avoid. More realistic expectations about the war lead to more realistic expectations about bargains.

In this article, I detail the core components and properties of the model. Hence, initially I examine only the decision to keep fighting. However, the flexible framework I propose can be readily adapted to address many questions, as I shall later illustrate by interacting it with a model of domestic political competition. Indeed, this and related frameworks are already being used in various contexts. For example, Morrow’s (1997) closely related model examines the choice of military strategy. Within the context of this model, the optimal level of armaments and effort for nations engaged in war is being addressed by Judd and Smith (see Judd 1985; Budd, Harris, and Vickers 1993). Smith (1997b) uses this framework to tackle the questions of stability and the distribution of power, bargaining (see also Wagner 1994, 1997), the effect of regime type on conflict, and weapons development.

1. Smith (1997b) examines how the occurrence of war varies under these different assumptions.
A MARKOVIAN RANDOM-WALK MODEL OF WAR

In this section, I detail the components of the model. The underlying structure is similar to the random-walk model common in the stochastic processes literature (Goodman 1988; Grimmett and Stirzaker 1992; Taylor and Karlin 1994). The key element is a random variable, known as the state variable. In the context of this model, the state variable is the distribution of forts. Assuming there are \( N \) identical forts, the state variable at time \( t \), \( X^t \), describes how many forts nation A controls. Thus, there are \( N + 1 \) possible states: 0, 1, 2, \ldots, \( N - 1, N \). To avoid possible confusion, the term state always refers to the number of forts. Nations fight over this state space. Nation A wants to increase the state variable; nation B wants to reduce it. Should nation A capture all the forts (\( X^t = N \)), then B is decisively beaten and A is the victor. At this point, nation B no longer has the capacity to fight, and the war is permanently ended in favor of nation A. State \( X^t = 0 \) represents total victory for nation B. In this state, nation A no longer has the ability to prosecute the war.

During the course of the game, the state changes. The future state, \( X^{t+1} \), depends on the current state, \( X^t \), and the action of the nations. If nation A surrenders, then the next state is \( X^{t+1} = 0 \); the state becomes \( N \) if B surrenders. If the nations choose to fight, then either nation A could gain the advantage, capturing a fort from B, nation B could gain a fort, or the interaction could result in no advantage for either. Hence, each battle ends in either a win, a loss, or a draw for nation A, and the probability of each event is \( p \), \( q \), and \((1 - p - q)\), respectively.

This is conveniently represented in matrix form. The matrix \( P_{\text{fight}} \) describes the probability of moving between states. Each row corresponds to the current state, \( X^t \), and each column corresponds to the next state, \( X^{t+1} \). Each row describes the probability distribution over states after a single period of fighting. For instance, the first row, which corresponds to state \( X^t = 0 \), tells us that with probability 1 the next state is also \( X^{t+1} = 0 \); once A is defeated, it remains defeated. The second row corresponds to state \( X^t = 1 \) and shows that the next state could be either 0, 1, or 2, with probabilities \( q \), \( 1 - p - q \), and \( p \), respectively.

\[
P_{\text{fight}} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
q & 1 - p - q & p & : & : & : \\
0 & q & 1 - p - q & : & : & : \\
0 & 0 & q & : & : & : \\
0 & 0 & 0 & 1 - p - q & p & 0 \\
: & : & : & q & 1 - p - q & p \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}.
\]

For tractability, I assume that the transitions probabilities, \( p \) and \( q \), are constant across states. However, relaxing this assumption is straightforward (Smith 1997b).²

². See also Wagner (1994, 1997).
ENDING THE WAR

The matrix above, \( p_{\text{fight}} \), assumes that nations fight until the bitter end. Sometimes nations fight until their capability to resist is completely destroyed. For example, the Carthaginian resisted until Scipio led the Romans in the sack of Carthage (146 B.C.). Similarly, Nazi Germany continued to fight until Germany was overrun by the allies and the central authority could no longer resist. Yet, the majority of wars end before one side is totally destroyed. Typically, once a nation realizes that it is close to defeat, it surrenders.

Nations decide on the conditions under which they will fight. At some point, if a nation continues losing battles, then it might decide to quit. Technically, a nation’s strategy is a decision whether to fight in each state. Such strategies are called Markov strategies because the action choice is made conditional on the state but is independent of all other aspects of the history of play (i.e., nations choose based on where they are and not how they got there).

WHAT ARE NATIONS FIGHTING OVER: SPOILS AND COSTS

The distribution of forts (the state variable) and the actions taken determine the rewards that nations receive each period. Nations value the resources that they capture from their foe. Ideally, a nation wants to capture all the forts because then it enjoys the resources and its foe can no longer prosecute the war. For each fort that nation A owns at the end of the period, it receives a payoff of \( a \). However, if conflict occurs during any period, then both nations pay a cost of \( k \) in that period. Hence, A’s per period payoff is \( aX^{t+1} - k \) if war occurs and \( aX^{t+1} \) otherwise. B’s corresponding payoffs are \( b(N - X^{t+1}) - k \) and \( b(N - X^{t+1}) \).

Given that nations receive payoffs in every period, what is the average value of being in a particular state? This value, which I call the continuation value, reflects not just today’s payoff but also what nations expect to receive in the future. As is standard, to reflect the declining value of future rewards, future payoffs are discounted at the rate \( \delta \). This means that a payoff of 1 in the next period has a present value of \( \delta < 1 \). Thus, A’s continuation value in state \( i \) reflects what A expects to receive today, what A expects to receive in the next period, what A expects to receive in the period following that, and so on.

EQUILIBRIUM BEHAVIOR

In this section, I describe equilibrium behavior. The appendix contains the formal characterization of these properties. The analysis here concentrates on intuition rather than mathematical rigor. The higher the state (i.e., the more forts A has), the greater A’s incentives to continue fighting. When A has few forts and B is close to decisively defeating it, then A often quits. Formally, it is convenient to define the first state in which each nation fights. Thus, in states \( f_A \) and above, nation A fights. When the state,
Figure 1: A Pictorial Representation of Monotone Strategies

\( X' \) is below \( f_A \), then nation A surrenders. Similarly, nation B continues to fight when it has sufficient forts, \( X' \leq f_B \), but quits when its supply of forts dwindles: \( X' > f_B \). Thus, in those states between \( f_B \) and \( f_B \), war occurs. Above state \( f_B \), B surrenders and A prevails. In low states (those below \( f_A \)), A surrenders and B prevails. Figure 1 contains a pictorial representation of this equilibrium behavior.

To understand this behavior, first suppose that nation A has captured nearly all the forts (\( X' \) is high). A has no incentive to quit. Its current rewards are large because it already has a lot of forts that it does not want to give up. In addition, requiring only a few more victories to decisively defeat nation B, it is most likely to be the eventual winner. Thus, under these circumstances, A wants to fight. However, as the state becomes lower, then A's probability of eventual victory declines and the average time that it takes for A to win increases. This makes fighting less attractive and eventually, as its stock of forts becomes low, fighting is no longer worthwhile and A surrenders.

Being more precise, in states above \( f_B \), nation B always surrenders if nation A attacks. Therefore, if A fights in these states, it will capture all the forts and receive its maximum payoff of \( aN \) in all future periods. Thus, to nation A the value of states in which B surrenders is \( v_A = aN + \delta aN + \delta^2 aN + \cdots = \frac{aN}{1 - \delta} \) for all \( i > f_B \). In state \( f_B \), nation B resists. However, nation A only needs a single victory before nation B will surrender. Hence, in state \( f_B \), nation A is likely to be the eventual winner and, on average, A should win quickly. As B gains more forts, then the value of fighting declines. For example, in state \( f_B - 1 \), nation A must win two battles to defeat B. Hence, the probability of eventual victory declines, and the average time to secure this victory increases. As B
gets more forts, the probability that A ever gets to state $f_B + 1$, where B surrenders, becomes increasingly unlikely. Hence, the value for fighting declines as A’s supply of forts dwindles.

If A ever quits the fight, then its payoff in all future periods is 0. The value of any state in which A quits is therefore 0. Because A’s value from fighting declines as the state variable declines, if A ever quits, it quits when the state is low. Indeed, $f_A$ is defined as the lowest state that has a nonnegative value for fighting. State $f_A - 1$, the highest state in which A does not fight, has a value of 0, but A’s expected payoff if it decided to fight in this state would be negative because the eventual probability of victory is so low and A would, on average, have to bear the costs of fighting for so long even if it did eventually win.

Given B’s decision to fight in states $f_B$ and below, it is straightforward to calculate the states in which A fights, $f_A$ and above. Similarly, given A’s strategy, it is straightforward to calculate the optimal $f_B$. In the appendix, I prove that there are always pairs of $f_A$ and $f_B$ such that both nations are both simultaneously behaving optimally. It is how $f_A$ and $f_B$ vary in response to different circumstances that I explore next.

WHEN DO NATIONS FIGHT?

Nations fight in those states between $f_A$ and $f_B$. In lower states, A surrenders; in higher states, B surrenders. However, substantively what we want to know is what affects the value of $f_A$ and $f_B$. In particular, I explore how the value of holding forts ($a$ and $b$), the cost of fighting ($k$), and probability of winning battles ($p$ and $q$) determine the conditions under which fighting occurs.

The effect of changing parameter values on A’s equilibrium behavior occur through two mechanisms. First, changes directly affect nation A’s choices. Second, if A alters its strategy, then B might also alter its behavior to compensate. Hence, changes in A’s behavior reflect not just a direct change but also an anticipation of B’s response. To illustrate, I examine the effect of changing the value of forts on the occurrence of war.

Suppose the value of holding forts varies. Some nations might regard holding forts as highly desirable relative to the cost of fighting; others might not. I use the stylization of hawks and doves to clarify this distinction. If nation A is a hawk, finding forts highly desirable, it receives a payoff of $a_H$ per fort held. Doves receive a payoff of $a_D$ for each fort held (where $a_H > a_D$).

The question I address is, How does the nation’s type affect the occurrence of war? Changing A’s type alters the behavior of both A and B. First, if A is hawkish, then it fights in lower states than if it were a dove ($f_A(a_H) \leq f_B(a_D)$). Second, when facing a hawk, B surrenders sooner than it would against a dove ($f_B(a_H) \leq f_B(a_D)$). Hence, the set of states in which fighting occurs shifts to lower states as nation A becomes more hawkish. Figure 2 shows a numerical example of this. In this case, if A is a hawk, then fighting occurs in states 3, 4, and 5. If A is a dove, then fighting occurs in states 5 and 6.3

3. For example, $p$ and $q$ might depend on the state variable.
Before explaining how changes in other parameters affect behavior, I explain why both sides change their behavior in response to changes in the parameter values associated with one nation. As holding forts becomes more valuable, A’s value associated with fighting increases. Hence, A is prepared to continue the fight under less favorable conditions because the stakes are higher. This has consequences not just for nation A but also for nation B. Because hawks fight harder (which in the current context means fighting in lower states), they are harder to defeat. Nation B requires more victories against a hawk than against a dove. Hence, the incentives to continue fighting against a hawk are smaller: the probability of eventually defeating a hawk is lower and, on average, securing this victory takes longer. Therefore, B surrenders sooner against a hawk than against a dove.

Increases in A’s evaluation of forts make it more aggressive and make B more compliant. This dependence between A’s decision to fight and B’s decision to fight holds for other parameters as well. For example, if A’s prospects for victory improve (either p increases or q decreases), then A fights harder. These changes in the probability of winning battles have a dual effect on nation B. First, A fights harder, and B needs more victories against A to win. Second, each victory is harder to obtain because the probability of winning a battle declines. These effects discourage B from resisting. Hence, as A becomes more powerful, the zone of conflict shifts toward lower states.
Cost also affects when fighting occurs. As A’s cost of fighting increases, then it becomes more reluctant to resist and surrenders in higher states. If only A’s costs are affected, then increasing costs means that B fights sooner and surrenders later. However, if increasing costs affect both sides, then B has mixed incentives. Because A surrenders sooner, it has an incentive to attack, but its own increasing costs also encourage it to surrender. In general, as costs get high, there are multiple equilibria: fighting only occurs in a few states, but which few states can vary. If the cost of fighting is extremely high, then war never occurs, but there are numerous equilibria that differ depending on which nation surrenders in each state.

Changes in parameter values typically make one nation fight longer and make the other nation quit sooner. The Markovian random-walk model of war offers a flexible modeling platform to examine decision making within wars. Because the purpose of this article is to introduce the modeling strategy, I restrict the choices at each stage to the decision to continue fighting. However, the analysis can be readily expanded to consider a wider set of choices (Smith 1997b). The model can also be used to examine how decision making within a war interacts with other processes. The next section demonstrates this by introducing domestic elections into the model.

DOMESTIC POLITICS

So far I have assumed that nations are unitary actors. Although personification of nations is a useful simplification, we know that decisions are made by the interaction of individuals within the political system. Given this, I start by outlining a simple model of domestic politics. Because I want to explore the effect of elections on foreign policy, I assume that individuals differ in how much they value holding forts. For simplicity, all individuals are either hawks or doves, receiving a payoff of $a_H$ or $a_D$, respectively, for each fort held.

I assume that the portion of the population—the electorate—is responsible for selecting a leader. In turn, this leader determines foreign policy: whether to continue the war. Ignoring all other issues, first I ask, On the basis of foreign policy, does the electorate prefer a hawk or a dove for leader? Second, I examine how the prospect of domestic elections affects a leader’s foreign policy choices.

WHICH LEADER DOES THE ELECTORATE HIRE?

To start the analysis, I examine which type of leader the electorate wants to hire to make foreign policy decisions on its behalf. Once elected, hawks and doves fight the war differently. As an illustration, consider the numerical example above and suppose the electorate in nation A must decide whether to elect a hawk or a dove. Figure 2 shows in which states hawks fight and in which states doves fight.

Suppose that the war is in state 0, 1, or 2. In these states, both hawks and doves surrender. Thus, on the basis of foreign policy, the electorate is indifferent between a hawk and a dove. Similarly, in states 7, 8, and 9, nation B always surrenders regardless of the type of leader in nation A. In these states, the electorate gets the same payoff
whether they elect a hawk or a dove. However, what happens in between—states 3, 4, 5, and 6—depends on whether a hawk is elected. If the electorate chooses a hawk, then fighting occurs between states 3 and 5. However, if a dove is elected, then fighting occurs in states 5 and 6.

Suppose that the current state is 3 or 4. If a hawk is elected, then fighting occurs. However, a dove would not contest these states. Therefore, in these states, the electorate’s choice between hawk and dove leaders is effectively a choice between continuing the war or not. Whether the electorate considers the war worthwhile depends on whether it is hawkish or dovish. Hawk voters, like hawk leaders, receive a high benefit from holding forts. Thus, a hawk electorate, like a hawk leader, wants to fight in states 3 and 4. However, in these states, a dove electorate does not want to fight and hence prefers a dove leader. This information is shown diagrammatically in Figure 3. Figure 3a shows the payoff that a dove electorate expects to receive from both hawk and dove leaders. Figure 3b shows the corresponding payoffs if the electorate is a hawk.

Next consider state 6. In this state, a hawk leader deters nation B from resisting. Yet, nation B would continue to fight if faced with a dovish opponent. This paradoxical result—that the doves continue the war—profoundly influences voting at the domestic level. A vote for a dove means that the war continues; a vote for a hawk means that nation B surrenders, and nation A achieves an immediate victory. Both hawk and dove electorates prefer complete victory without having to fight. Figure 3a shows that doves receive a payoff of 45 if a hawk leader is elected but receive only 13 if a dove is elected. The corresponding numbers for a hawk electorate are 90 and 44.2, respectively (Figure 3b). Therefore, on the basis of foreign policy, both types prefer a hawk rather than a dove. In state 6, a hawk is always elected regardless of the nature of the electorate.

In state 5, both hawk and dove leaders will fight with nation B. However, the difference is that a hawk is much closer to victory than a dove. If a hawk wins a single battle, then nation B surrenders. Yet, against a dove, B fights longer. The advantage of electing a hawk is that victory is more likely. This produces an electoral bias in favor of hawkish leaders. In state 5, doves receive a payoff of 0.2 if they elect a dove but receive 9.4 from a hawk leader (Figure 3a). Similarly, hawks prefer hawks to doves (40 vs. 18.5; see Figure 3b).

Wars influence domestic politics. Because hawks continue to fight longer than doves, they are harder to defeat. This means that opponents surrender sooner against hawks than against doves. Because even doves want to win wars, although not to the same extent as hawks, they are often inclined to pick a hawkish leader who discourages the enemy from continuing to resist. This biases electoral decisions toward hawkish candidates. We can imagine some of the unfortunate consequences that this can produce. The electorates in both nations have an incentive to pick the most hawkish candidates even if they themselves are doves. Although these leaders continue to be supported electorally, both sets of electorates would prefer surrender rather than continue the war. The electoral advantage of hawks might also encourage leaders to appear as hawkish as possible. Leaders who care about reelection should mimic hawks even if they are doves. Indeed, a stylized fact of the U.S. political system is that first-term presidents adopt more hawkish postures than those in their second term
Figure 3: Payoffs for Doves and Hawks from Different Leaders
(Smith 1996a). Although in the current complete information setting such mimicking is not possible, the presence of elections still affects the foreign policy choices that leaders make.

THE PROSECUTION OF WARS IN THE FACE OF ELECTIONS

War influences the outcome of elections. As the previous section shows, it creates a bias in favor of hawk candidates. In this section, I informally expand the analysis to show how the prospect of future elections also influences war. To proceed, I assume that leaders care about office holding. I then suppose the incumbent leader is facing an election. How does the leader modify his or her behavior prior to the election?

As noted above, there is a bias toward hawk candidates in elections that occur during wars. This bias goes away without a war. This provides the starting point for the analysis. If the war continues through the election, then this tends to advantage a hawk incumbent and disadvantage a dove incumbent. Without a war, in terms of foreign policy, the differences between hawks and doves disappear. Therefore, hawks want the war to continue through the election, but doves want the war concluded before an election. Thus, the following scenarios can occur: doves surrender prior to an election in states where they would otherwise fight. Hawks continue to fight in states where they would normally surrender.

I consider each scenario in turn. Hawk electorates always vote against dove leaders and, in some cases, even dove electorates prefer hawk leaders. This disadvantages doves electorally. On the basis of policy, doves might want to continue the struggle. However, if they do, then they increase the risk of being removed from office. If leaders care sufficiently about office holding, then they surrender prior to the election. This removes the war from being a political issue at the election. There exist states in which dove leaders surrender when both hawks and doves prefer that the war continues.

On the other extreme, hawks want the war to continue. Hawk voters always prefer hawk leaders, and in some states, even the doves prefer hawk leaders. This places hawks in a privileged situation at elections. Hawks do not want the war to end. This can have several consequences. First, prior to the election, hawks might continue fighting in those states where they would normally surrender. Should the hawk get lucky and win a string of victories, then it might reach a state where it would fight after the war. Once in one of these states, hawk voters and possibly dove voters as well are biased toward the hawk. Thus, hawks might fight unjustified wars prior to elections in the hope that a few victories will help their electoral chances. Second, although beyond the scope of the model, if the war is going well, then hawks have no incentives to fight hard or to negotiate in good faith prior to an election. In both scenarios, leaders act against the interests of the electorate. The electorate has the power to remove these leaders but, paradoxically, this ability causes leaders to act against the interests of the voters in the first place.

So far, the discussion has focused on pre-election behavior in nation A. However, the presence of an election also creates incentives for nation B. Nation B's incentives depend on the state and who its opponent is. Suppose the leader in nation A is a hawk. Although the presence of a war privileges a hawk, reelection is never certain. Should
a dove get elected, then it becomes easier for nation B to win the war. This means that nation B wants to wait until after the election before deciding whether to surrender. Likewise, against a dove, nation B has incentives to wait until the election before surrendering. As explained above, elections encourage doves to surrender. B does not want to surrender if nation A will quit anyway.

What happens internationally affects domestic elections. This, in turns, influences the choices of political leaders. Leaders pursue those strategies that create the international circumstances that are likely to get them reelected. This suggests that the initiation and the termination of conflict are affected by the electoral cycle (Gaubatz 1991; Smith 1996b; Stoll 1984).

CONCLUSION

The Markovian random-walk model of war provides a flexible modeling platform. Like Morrow (1997), it combines aspects of lottery-based and war-of-attrition-based models. As the conflict develops over time, nations are faced with different contingent circumstances and, as a result, choose different strategies. Although here the set of possible actions that nations can take is restricted to the decision to fight, extensions to more complex stage games are straightforward. For example, extensions address the questions of negotiated settlements, effort levels, weapons development, choices of military strategy, and how the distribution of resources affect the stability of the international system (Smith 1997b). The model has three principal advantages over simple lottery-based models of war: first, it allows decisions made during the course of a war to be studied. Second, it provides more detailed expectations about the likely outcome of conflict, which enhances our ability to explain pre-war behavior. Third, the modeling framework allows conflict to be combined with models of other political processes, such as domestic elections.

As the other articles in this volume stress, leaders’ decisions change as wars progress (Gartner 1998 [this issue]). For example, the choice of military strategy depends on the military balance (Goemans 1997; Morrow 1997). Domestic political support and the capacity of different regimes to continue to fight vary during the course of the war (Gartner and Segura 1998 [this issue]; Reiter and Stam 1998 [this issue]; Bennett and Stam 1998 [this issue]). Whether nations can reach negotiated settlements also depends on the progression of the war (Smith 1997b; Wagner 1997; Werner 1998 [this issue]). These factors are clearly of interest to scholars, yet they cannot be addressed by a modeling strategy that assumes conflict is instantaneously resolved. Rather than taking the war as a single event, I break the conflict down into a series of interactions (battles). The analysis focuses on strategy choice during each battle. By doing so, the model examines decision making under the contingent circumstance that leaders actually face. It does not assume that choices are fixed for the entire duration of the war.

Treating conflict as a series of interactions rather than as an instantaneous coin flip provides a more accurate picture of the likely consequences and the likely outcomes of war. Some wars are long, drawn-out affairs, but others are rapidly resolved through
negotiated settlements. By examining decisions made during fighting, the model provides a powerful tool for examining pre-war behavior. The more realistic our expectations about the nature of war, the better our understanding of pre-war decision making. As an example, consider the effect of these expectations on crisis bargaining. If a war is likely to be rapidly settled by negotiation anyway, then nations might risk becoming involved in a war. However, the prospects of a long bloody conflict might encourage nations to adopt more conciliatory postures. Hence, improving our model of warfare enhances our ability to explain pre-war behavior.

Decisions made during conflict also interact with other political processes. I illustrated this using the case of domestic elections. By treating the war as a string of battles rather than as a snapshot event, the model explains how conflict affects domestic political events. Because the progress of the war affects whether leaders retain office, leaders pick foreign policies in anticipation of impending elections. There are causal pathways in both directions: the war affects elections, and elections affect the war. Without a temporal domain, a model of war cannot separate these effects. Treating both as an instantaneous event, we are left knowing that conflict and elections are correlated but not knowing the nature of the relationship. Such theorizing is essential in understanding the causal relationships between political processes.

In conclusion, I model war as a stochastic process in which nations fight a series of battles. The fighting continues until one nation is decisively defeated or surrenders. I examine how decision making varies in each battle. Nations that are close to winning behave differently from nations facing defeat. The Markovian random-walk model of war is a first step in systematically analyzing the internal dynamic of decision making during war.

APPENDIX

FORMAL STRUCTURE OF THE GAME

In each state, nation A decides whether to fight. Let \( \sigma_A^i \) be the probability with which nation A fights in state \( i \), and let \( \sigma_A = [0 \ \sigma_A^1 \ \sigma_A^2 \ \ldots \ \sigma_A^{N-1} \ 1]^T \) be a vector representing A's decision in every state.

The transition matrix depends on the strategy profile: \( P = \sigma_A \ast (\sigma_B \ast P_{\text{fight}} + (1 - \sigma_B) \ast P_{\text{B_surrender}}) + (1 - \sigma_A) \ast P_{\text{A_surrender}} \), where \( P_{\text{B_surrender}} \) is an \((N+1) \times (N+1)\) transition matrix where the next state is always \( N \), \( P_{\text{A_surrender}} \) is the appropriate transition matrix when nation A surrenders, and \( \ast \) represents the dot product matrix operator.

In each period, nations' payoffs depend on the state and the actions taken. Specifically, nation A receives a payoff of \( a \) for each fort that it holds at the end of the period, less the cost \( k \) if any fighting occurs. Let \( c_A^i \) represent nation A's payoff associated with state \( i \), and let the vector \( C_A \) represent A's payoff in each state. Given the initial state \( i \), A's current payoff is \( c_A^i = aX^{i+1} - k \) if fighting occurs and \( c_A^i = aX^{i+1} \) if no fighting occurs. B's corresponding payoffs are \( c_B^i = b(N - X^{i+1}) - k \) and \( c_B^i = b(N - X^{i+1}) \).

Given these current payoffs, the continuation value associated with each state, \( v_A \), is the discounted infinite sum of the current payoffs that nations expect to receive in future periods. The continuation value is straightforward to calculate for the absorbing states. If the state is \( X^i = 0 \)
(A is decisively defeated), then all future states are also 0. Because the current payoff in every period is 0, the continuation value must also be 0. If A achieves a decisive victory, \( X' = N \), then all future states are also \( N \). A receives a current payoff of \( aN \) in every period. Thus, the continuation value, \( v_A' = aN + \delta aN + \delta^2 aN + \cdots = aN(1 + \delta + \delta^2 + \cdots) = aN \frac{1}{1-\delta} \). Calculating the continuation values for other states is slightly trickier because it involves calculating the discounted current payoff for all future periods. The continuation values, \( v_B \), are given by the following infinite sum: \( v_A = C_A + \delta PC_A + \delta^2 C_A + \cdots \). Calculating the continuation values is simplified by defining \( V_A = C_A + \delta PV_A \). Hence, \( V_A = (I_{N+1} - \delta P)^{-1} C_A \) and \( V_B = (I_{N+1} - \delta P)^{-1} C_B \), where \( I_{N+1} \) represents an identity matrix of dimension \( (N + 1) \).

The solution concept is Markov perfect equilibrium (MPE). In addition, I restrict attention to a subset of strategy profiles: monotone strategies. Markov perfect equilibria in which nations play monotone strategies always exist. Indeed, when the cost of fighting \( k \) is low, they are the only MPE. For a discussion of MPE in nonmonotone strategies, see Smith (1997a).

**Monotone Strategies**

Intuitively, a monotone strategy is one in which once nation A surrenders in a particular state, then nation A will not fight in a lower state, where the military balance is even worse.

Let \( f_A \) represent the first (lowest) state in which A fights: \( \sigma_A^i = (0,1) \). Thus, in all states above \( f_A \), A fights, \( \sigma_A^i = 1 \) if \( i > f_A \), and in all states below \( f_A \), A surrenders \( \sigma_A^i = 0 \) if \( i < f_A \). Similarly, when nation B is close to defeat, it does not resist: \( \sigma_B^i = 0 \) if \( i > f_B \). At state \( f_B \), B sometimes fights, \( \sigma_B^i = (0,1) \). As B gets closer to defeating A, it never stops fighting, \( \sigma_B^i = 1 \) if \( i < f_B \). Thus, in states less than \( f_A \), nation A surrenders; in states above \( f_B \), nation B surrenders. Between these two states, both nations prosecute the war. Let \( F \) represent the set of states in which fighting occurs: \( F = \{ X' : f_A < X' < f_B \} \). Formally, if A's strategy, \( \sigma_A \), takes this form, then \( \sigma_A \in \Sigma_A \). Similarly, let \( \Sigma_B \) represent the set of monotone strategies set for nation B.

**RESULTS AND DISCUSSION**

**Lemma 1.** \( v_A' \geq 0 \) for all \( i \). \( \sigma_A^i < 1 \) implies that \( v_A^i = 0 \). \( v_A^i > 0 \) implies that \( \sigma_A^i = 1 \).

Proof: Consider any state \( i \in \{1, \ldots, N - 1 \} \), \( U_A(\text{no fight}X' = i, \sigma_B) = 0 \). In equilibrium, nation A is utility maximizing in all states. Because A can always get a payoff of 0 by not fighting, this is the minimum continuation value. The second claim is true because if not fighting is the best response, then it is utility maximizing. Therefore, there is no other action that generates a bigger expected payoff. If \( v_A^i > 0 \), then not fighting is not a best response. Therefore, nation A must always fight. QED.

**Lemma 2.** Let \( BR_A(\sigma_B) \) represent A's set of best responses to B's strategy, \( \sigma_B \). If nation B plays a monotone strategy, \( \sigma_B \in \Sigma_B \), then A's best response are all monotone strategies, \( BR_A(\sigma_B) \subset \Sigma_A \). Furthermore, all strategies that are elements of \( BR_A(\sigma_B) \) have the same unique \( f_A \).

Proof:

1. Consider those states above \( f_B \): \( X' > f_B \). Because B stops fighting in these states, A's payoff is \( aN \) in each period, which corresponds to a continuation value of \( v_A' = aN \frac{1}{1-\delta} \)

(continued)
2. States where B fights: $X^i \leq f_B$. First, I demonstrate that in regions where fighting occurs, A’s payoffs are strictly increasing in state. Suppose there exists some region where nation A fights B. Let $j$ represent the highest state in which A never fights: $\sigma_A^j = 0$. This implies that $c_A^j = 0$ and $v_A^j = 0$. In all states above $j$, A fights; $\sigma_A^j = 1$ for $j < i \leq f_B$. Because fighting is better than surrendering, $v_A^i \geq 0$ for all states $i < j$. For these fighting states, the current payoffs are $a(i + p(i + 1) + q(i - 1)) - k = a(i + p - q) - k$. Let $Z^0$ be the column vector of current payoffs restricted to states $j$ to $f_B + 1$: $Z^0 = [0 \ z \ z + a \ 2a \ldots z + (f_B - j)a + aN]'$, where $z = a(j + p - q) - k$. The transition matrix over these states is given by $P$.

$$P = \begin{bmatrix}
1 & 0 & \ldots & 0 & 0 \\
q & (1 - p - q) & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & (1 - p - q) & p \\
0 & 0 & \ldots & 0 & 1
\end{bmatrix}$$

Strictly speaking, the states $j$ and $f_B + 1$ are transient states that always result in states 0 and $N$, respectively. For ease, I treat these states as directly absorbing.

The continuation values associated with state $j$ to $f_B + 1$ are given by the following infinite sum:

$$\bar{V} = Z^0 + \delta P Z^0 + \delta^2 P^2 Z^0 + \ldots = Z^0 + \sum_{i=1}^{\infty} \delta^i P^i Z^0.$$ 

Let $Z^i = P Z^0$. Given that $Z^0$ is strictly increasing, $Z^i$ is also strictly increasing. For any interior state $i$, $z^{i,i-1} = q z^{i-1,i-1} + (1 - p - q) z^{i-1,i-1} + p z^{i-1,i} = z^{i-1,i} - pa - qa$. Thus,

$$Z^i = [0 \ z(1 - p) + pa \ z + a + a(p - q) \ z + 2a + a(p - q) \ldots$$

$$\ldots + (f_B - j - q)a + paN a N]'$$

Thus, if $Z^0$ is strictly increasing across states, then $Z^i$ is strictly increasing across states. Inductively, if $Z^i$ is strictly increasing across states, then $Z^{i+1}$ is strictly increasing across states. The column vector of continuation values is the infinite sum of terms that are all strictly increasing across states. Therefore, the continuation values are also strictly increasing across states. These infinite sums are bounded between 0 and $aN(1 - \delta)$.

So far, I have assumed that B fights with probability 1 in state $f_B$. If B randomizes its choice, then obviously the result still holds. The final situation to consider is that A randomizes at state $j + 1$. This implies that $v_A^{j+1} = 0$, and the result still holds, although continuation values are only strictly increasing for states $i$ such that $j < i \leq f_B$. Thus, if fighting occurs across a series of states, then $v_A^i$ is strictly increasing in $i$ in which fighting occurs.

Next, I show that once a nation has stopped fighting in some state, it never starts fighting again in a lower state. If $\sigma_A^i < 1$, then $\sigma_A^i = 0$ for all $i < i' \leq f_B$. $\sigma_A^i < 1$ implies that $U_A$(fight $X^i = i'$, $\sigma_B) \geq v_A^i = 0$. Suppose that nation A fights in state $i' - 1$. This implies that $U_A$(fight $X^i = i' + 1$, $\sigma_B) = v_A^{i+1} \geq U_A$(no fight $X^i = i' + 1$, $\sigma_B) = 0$. But if A fights, then payoffs are strictly increasing across types. Therefore, $U_A$(fight $X^i = i'$, $\sigma_B) > U_A$(fight $X^i = i' + 1$, $\sigma_B) = v_A^{i+1}$. But this contradicts $\sigma_A^i < 1$.

Thus, best responses to monotone strategies are themselves monotone strategies: if $\sigma_B \in \Sigma_B$, then $BR_B(\sigma_B) \subset \Sigma_A$. Because payoffs are strictly increasing across states in regions where fighting occurs, then the lowest state in which A fights, $f_A$, is generically unique. If $U_A$(fight $X^i = f_A$, $\sigma_B) = v_A^{f_A} = 0$, then $\sigma_A^f \in [0, 1]$. QED.
APPENDIX Continued

EXISTENCE

MPE exist in stochastic games with a finite number of states and actions (Fudenberg and Tirole 1991, 504). This guarantees that equilibria exist in the random-walk model of war. Unfortunately, it does not guarantee that equilibria exist in which nations play monotone strategies. In this section, I show that such equilibria do exist. I start by outlining the proof to the general claim because I shall use a similar approach.

The proof that MPE exist proceeds as follows. Consider a strategic form equivalent of the game. Rather than think of a single player making choices for every state, consider every combination of player and state as an agent. Because there is a finite number of players and a finite number of states, there is a finite number of agents. Every agent’s action set is finite because there are finite choices in the corresponding Markov game. Therefore, we have a strategic game with a finite number of agents, each of whom can take a finite number of actions. There always exist Nash equilibria in these strategic games. By the definition of a Nash equilibrium, every agent is utility maximizing given the strategy of every other agent. In the Markov game, this implies that every player is utility maximizing for every state. Hence, MPE exist in the corresponding Markov game.

To show existence of MPE in monotone strategies, I follow a similar approach. In monotone strategies, nations only stop fighting in single states. Thus, I define the following strategic form game. Each player, A and B, chooses a state in which to first fight: $s_A$ and $s_B$. (In the notation above, this is equivalent to announcing $f_A$ and $f_B$.) The structure of the game means A always fights in states above $s_A$, and B always fights below $s_B$. In the strategic game, there is no state-by-state decision to fight only a decision of the first fighting state. This strategic game has a finite number of players, A and B, and a finite action set, $\{0, 1, 2, \ldots, N - 1, N\}$. Therefore, Nash equilibria exist for the strategic form game. Now consider the corresponding Markov game. The existence of Nash equilibria only ensure that nations are utility maximizing in those states where they first fight, $s_A$ and $s_B$ or $f_A$ and $f_B$ in the Markov game. The Nash equilibrium does not ensure that nations are optimizing in other states because behavior in these states was fixed by the structure of the strategic form game. However, lemma 2 ensures that if it is optimal to fight at $s_A$ ($f_A$), then it is optimal to fight in all higher states and stop in all lower states. Therefore, all players behave optimally in every state. Therefore, MPE exist in which nations play monotone strategies.

Finally, it is worth noting that finiteness only ensures the existence of Nash equilibria in mixed strategies. It is worth considering what this implies for the structure of the corresponding MPE. Here I reverse the situation and use properties of the best response correspondence for monotone strategies to characterize the properties of Nash equilibria. Suppose there is a Nash equilibrium in which nation A randomizes over starting fighting in two nonadjacent states. For example, A fights in $s_A$ with prob. $w$ and fights in $s_A'$ with prob. $(1-w)$ and $s_A > s_A' + 1$. Lemma 2 shows that this cannot be utility-maximizing behavior. Consider state $s_A$, such that $s_A > s_A' > s_A'$. Because A fights in $s_A$, its expected payoff is nonnegative in this state. But lemma 2 implies that the payoff from fighting in state $s_A$ is strictly greater. Therefore, only starting to fight in state $s_A'$ cannot be optimal. Although Nash equilibria in the strategic form game might be mixed-strategy Nash equilibria, lemma 2 ensures that the randomization only occurs between adjacent states. Given this, I state proposition 1.

**Proposition 1.** MPE exist in which both nations play monotone strategies.
Appendix Continued

Comparative Statics

As discussed in the main text, changes in parameters affect the properties of equilibria. Here I present the formal analysis to support those results. I consider only changes in the value of forts. The arguments for changes in other parameters are analogous. If A’s payoff varies, then different types adopt different strategies. To express this, I allow all the notation to be contingent on A’s type. For example, σₐ(a) corresponds to the strategy of type a, and νₐ(a’) is type a’ continuation value in state i.

Lemma 4. Consider two types a and a’, such that a > a’. If nation B plays strategy σₜ ∈ Σₜ then if σₐ ∈ BRₐ(σₜ), then σₐ(a’) > 0 ⇒ σᵢₜ(a) = 1. Thus, fₛ(ₐ) ≤ fₛ(a ’).

Proof: Given lemma 2, best responses to monotone strategies are themselves monotone strategies: σₐ(a) ∈ Σₜ and σₐ(a’) ∈ Σₜ. Therefore, we can restrict our attention to restricted strategies. If σₐ(a’) ∈ BRₐ(σₜ), then σₐ(ₐₜ)(a’) > 0, which implies that νₐ(ₐₜ)(a’) ≥ 0, where fₛ(ₐ) is the first fighting state if A plays the strategy σₐ(a’). Suppose type a mimics type a’. Because Vₐ = Cₐ + δPₐCₐ + δ²PₐCₐ + δ³PₐCₐ + ..., then νₐₜ(a) > νₐₜ(a’), and νₐₜ(a) > νₐₜ(a’) if σₐₜ(a) > 0. Therefore, if type a’ (weakly) wants to fight, then type a’ strictly wants to fight. Thus, type a fights in at least as many states as type a’. QED.

Lemma 5. If σₐ(a) and σₐ(a’) are monotone strategies and σₐ(a) > σₐ(a’) for some i, then nation B first surrenders in a (weakly) higher state against type a than against type a’; σₐₜ(a) > σₐₜ(a’) for some i implies that fₛ(σₐ(a)) ≤ fₛ(σₐ(a’)).

Proof: Calculate B’s best response against the strategy σₐ(a). Let fₛ(σₐ(a)) represent the first state in which nation B fights if B plays a best response to nation A’s strategy σₐ(a). Thus, B’s continuation value for playing a best response against strategy σₐ(a) in state fₛ(σₐ(a)) is vₛ(σₐ(a))(fₛ(σₐ(a))) ≥ 0. But the payoff from fighting in state fₛ(σₐ(a)) + 1 is negative.

If B plays the same strategy against σₐ(a’), then because A fights less νₛ(σₐ(a’)) ≥ νₛ(σₐ(a)) for all i and νₛ(σₐ(a’)) > νₛ(σₐ(a)) if σₐₜ(a) > 0 and σₜ > 0. Thus, if B weakly prefers to fight against the strategy σₐ(a), then B strictly prefers to fight against the strategy σₐ(a’). Hence, fₛ(σₐ(a)) ≤ fₛ(σₐ(a’)). QED.

The substance of lemmas 4 and 5 is straightforward. Lemma 4 states that as the rewards become more valuable, then given any fixed strategy by nation B, nation A is more likely to fight in any particular state. Lemma 5 states that if nation A fights in more states, then nation B stops fighting sooner. The initiation is that because A fights longer, nation B must fight longer to reach an absorbing state and achieve a decisive victory. This reduces the incentives to fight. Hence, nation B gives up earlier. I now combine lemmas 4 and 5 with the existence result to show that, as A’s type increases, nation A first fights in a lower state and the highest state in which nation B fights also increases. In some sense, the equilibria move to the left: A fights longer, and B surrenders sooner.

Proposition 6. If the strategy profile (σₐ(a’), σₜ(a’)) is an MPE in monotone strategies where A’s type is a’, then there exists a monotone strategy profile (σₐ(a), σₜ(a)), which is an MPE where A’s type is a and σₐₜ(a’ ≤ σₐₜ(a) and σₜₜ(a) ≥ σₜₜ(a) for all i.
APPENDIX Continued

Proof: Given the strategy profile \((\sigma_A(a')^*, \sigma_B(a')^*)\), let \(f_B(\sigma_A(a')^*, \sigma_B(a')^*)\) represent the highest state in which nation B fights. Let \(BR_A(\sigma_B, a)\) represent the set of best responses for nation A, of type a, to play against the strategy \(\sigma_B\). First note that if \(\sigma_A(a')^* \in BR_A(\sigma_B(a')^*, a)\), then we are done. The equilibrium is the same regardless of nation A's type. So suppose this is not the case.

I use a similar trick to that used in the earlier proof of existence. Consider a strategic form game in which each nation's action set is a choice of state in which to start fighting. Thus, nation A chooses \(s_A\) in the strategic game, which in the corresponding Markov game means that A fights in all states \(s_A\) and above. Similarly, if B announces \(s_B\) in the strategic form game, then B fights in all states \(s_B\) and less in the corresponding Markov game.

Because the strategic game has finite actions and finite players, Nash equilibria exist. Furthermore, lemma 2 ensures that in these equilibria, any mixed strategies only involve randomizations in adjacent states. Thus, there is a direct one-to-one correspondence between monotone Markov strategies and any Nash equilibrium strategies in strategic form game.

Now consider a monotone strategic form game in which the action set for both players is limited. Nation A can only choose states \(f_A(\sigma_A(a')^*, \sigma_B(a')^*)\) or below: \(s_A \in S_A = \{0, 1, 2, \ldots, f_A(\sigma_A(a')^*, \sigma_B(a')^*)\}\). Nation B may only choose states below \(f_B(\sigma_A(a')^*, \sigma_B(a')^*)\): \(s_B \in S_B = \{0, 1, 2, \ldots, f_B(\sigma_A(a')^*, \sigma_B(a')^*)\}\).

Lemma 5 tells us about the comparative statics of best responses. Specifically, as A gets more aggressive, B stops sooner. Similarly, lemma 5 implies that as B stops sooner, then A gets more aggressive. By supposition, \(\sigma_A(a')^* \notin BR_A(\sigma_B(a')^*, a)\). In the strategic form game, this implies that A's best responses to any of player B's possible strategies involves announcing some \(s_A < f_A(\sigma_A(a')^*, \sigma_B(a')^*)\).

Now consider nation B's best response to any possible strategy by nation A in this monotone strategic game. If nation A plays the strategic game strategy that corresponds to the Markov strategy \(\sigma_A(a')^*\), then a best response for B is to announce \(s_B = f_B(\sigma_A(a')^*, \sigma_B(a')^*)\). For all more aggressive strategies by A, B's best response is to announce \(s_B\) (weakly) less than \(f_B(\sigma_A(a')^*, \sigma_B(a')^*)\).

The restricted strategic game has finite actions and finite players. Therefore, Nash equilibria exist in this restricted game. However, lemma 5 allows us to show that any equilibrium in this restricted game is also an equilibrium in the unrestricted strategic form game. Finally, lemma 2 shows that if nations play the Markov strategies that correspond to the Nash equilibrium, then players behave optimally in all states. QED

**Corollary 7.** There exist parameters such that in the equilibria \((*\sigma_A(a'), *\sigma_B(a'))\) and \((*\sigma_A(a), *\sigma_B(a))\), \(f_A(a) < f_A(a')\) and \(f_B(a) > f_B(a')\).

Proof by construction: The main text contains an example where this is true.

\[\text{\footnote{\(f_A(\sigma_A(a')^*, \sigma_B(a')^*)\) represents the lowest state in which A fights in the strategy profile \((\sigma_A(a')^*, \sigma_B(a')^*)\).}}\]
REFERENCES


———. 1996. Bargaining over objects that influence future bargaining power. Typescript, Department of Political Science, University of Chicago.


