Political Institutions, Policy Choice and the Survival of Leaders

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Institutional arrangements influence the type of policies that leaders pursue. We examine two institutional variables: size of the selectorate (S) – the set of people who have an institutional say in choosing leaders – and the size of the winning coalition (W) – the minimal set of people whose support the incumbent needs in order to remain in power. The larger the winning coalition, the greater the emphasis leaders place on effective public policy. When W is small, leaders focus on providing private goods to their small group of supporters at the expense of the provision of public goods. The size of the selectorate influences how hard leaders work on behalf of their supporters. The greater the size of the selectorate, the more current supporters fear exclusion from future coalitions. This induces a norm of loyalty that enables leaders to reduce their effort and still survive. As a first step towards a theory of endogenous selection of institutions, we characterize the institutional preferences of the different segments of society based on the consequences of these institutions for individual welfare. We conclude by examining the implication of the model for the tenure of leaders, public policy, economic growth, corruption, taxation and ethnic politics.

We examine how political institutions influence the incentives of leaders to allocate resources towards the provision of public goods (such as the protection of property rights, the rule of law, transparency, protection of human rights, national security) and private goods (such as the corruption, pork, patronage, cronyism, nepotism). In particular, we identify the institutional circumstances in which the incentives facing a leader who wants to stay in office are compatible with the provision of effective public policy. We contrast these circumstances with institutional arrangements in which the incentives facing the leader are compatible with corruption, kleptocracy and other forms of inefficient governance. We demonstrate that certain institutions discourage the provision of public goods that benefit all in society; these institutions also benefit leaders' welfare in comparison with other systems that encourage the provision of public goods. We use the results of the model that we propose to characterize the institutional preferences of different segments of society. For instance, we ask how a leader, or a member of his or her coalition, would alter political institutions if there were an opportunity to do so. In this way, we move towards a theory of endogenous institutional design. We conclude with discussions of the implications of our results and directions for future research.

   A large literature exists on political institutions and governance. One strand of this literature largely focuses on institutional differences across democratic

systems.[1] Another strand focuses on non-democratic institutions, with a major emphasis devoted to leadership.[2] Less attention has been paid to the differences between these major classifications of regime types.[3] We seek to unify the study of the differences within and across the traditional regime classifications of democracy, autocracy and monarchy. To do so we focus on just two political institutions that characterize all political systems: the size of the selectorate – the set of people who have an institutional say in choosing leaders – and the size of the winning coalition that keeps the leader in office.[4]

We start by describing the problems our model addresses and the intuitions

[1] David Baron, 'Comparative Dynamics of Parliamentary Government', *American Political Science Review*, 92 (1998), 593–610; Timothy Besley and Stephen Coate, 'An Economic Model of Representative Democracy', *Quarterly Journal of Economics*, 112 (1997), 85–112; Gary W. Cox, *Making Votes Count* (New York: Cambridge University Press, 1997); Avinash Dixit and John Londregan, 'The Determinants of Success of Special Interest Groups in Redistributive Politics', *Journal of Politics*, 58 (1996), 1132–55; Anthony Downs, *An Economic Theory of Democracy* (New York: Harper and Row, 1957); John Ferejohn, 'Incumbent Performance and Electoral Control', *Public Choice*, 50 (1986), 5–26; Gene Grossman and E. Helpman, 'Electoral Competition and Special Interest Politics', *Review of Economic Studies*, 63 (1996), 265–86; Michael Laver and Norman Schofield, *Multiparty Government: The Politics and Coalitions in Europe* (Oxford: Oxford University Press, 1990); Torsten Persson, Gerard Roland and Guido Tabellini, 'Comparative Politics and Public Finance' (CEPR Discussion Paper 1737, 1997); Torsten Persson and Guido Tabellini, 'Growth, Distribution, and Politics', in Torsten Persson and Guido Tabellini, eds, *Monetary and Fiscal Policy, Vol.2: Politics* (Cambridge, Mass.: MIT Press, 1994), pp. 137–56.

[2] Gary M. Anderson and Peter J. Boettke, 'Perestroika and Public Choice: The Economics of Autocratic Succession in a Rent-Seeking Society', *Public Choice*, 75 (1993), 101–18; Herschel I. Grossman, 'Kleptocracy and Revolutions', *Oxford Economic Papers*, 51 (1999), 267–88; Samuel Huntington, *Political Order in Changing Societies* (New Haven, Conn.: Yale University Press, 1968); John Londregan and Keith Poole, 'Poverty, the Coup Trap, and the Seizure of Executive Power', *World Politics*, 49 (1990), 1–30; Martic C. McGuire and Mancur Olson, 'The Economics of Autocracy and Majority Rule: The Invisible Hand and the Use of Force', *Journal of Economic Literature*, 34 (1996), 72–96; Mancur Olson, 'Dictatorship, Democracy, and Development', *American Political Science Review*, 34 (1993), 567–76; Ronald Wintrobe, *The Political Economy of Dictatorship* (New York: Cambridge University Press, 1998).

[3] McGuire and Olson, 'The Economics of Autocracy and Majority Rule'; Olson, 'Dictatorship, Democracy, and Development'.

[4] Ours is not a theory of standard regime types dressed up in new clothes. It is a theory of political institutions that helps account for many characteristics associated with particular regimes. While most systems with large winning coalitions are democratic, and those with small coalitions and large selectorates are autocratic (monarchies and juntas usually have small coalitions and small selectorates), coalition size and selectorate size are insufficient to *define* any common regime types. Rather, selectorate and coalition size are correlated with regime classifications. For instance, democracy is often associated with a variety of characteristics, such as an independent judiciary, free press, civil liberties, norms of conduct and reliance on law, that are not part of what defines the coalition or selectorate. Rather, these features of democracy are policy consequences expected to follow from having a large winning coalition and a large selectorate. Because selectorate and coalition size do not define common regime categories, we expect the empirical implications of our theory to hold when we relate coalition size and selectorate size to an array of dependent variables, but they may not hold when we relate standard measures of regime types to the same dependent variables. Elsewhere we show that these expectations are borne out by the evidence (Bruce Bueno de Mesquita, Alastair Smith, Randolph Siverson and James Morrow, 'The Logic of Political Survival' (unpublished manuscript, 2000)).

underlying the model. Although the mathematics of the model is somewhat complex, the intuition behind it is straightforward. We then present the model and its equilibria and summarize the comparative static analysis of behaviour under equilibrium. Although space limitations preclude testing the model's claims here, we summarize empirical assessments of its validity that are presented elsewhere.[5] The proofs for our theoretical claims are displayed. We conclude with discussions of the substantive implication of our results and directions for future research.

POLITICAL INSTITUTIONS AND THE PROVISION OF GOODS

All political leaders seek to hold office, as holding a position of power is a minimal requirement to achieve any other political end. Whether a leader continues in office depends on those in society who hold the power to remove the incumbent and select her replacement. We refer to these individuals as the *selectorate*. Within the selectorate, a leader stays in power by holding the loyalty of a *winning coalition*. The composition of the selectorate and the magnitude of the winning coalition are important characteristics of any political system. In modern mass democracies, the selectorate is the electorate, and the winning coalition is determined by the specific electoral rules. In autocratic systems, the winning coalition is often a small group of powerful individuals, and the selectorate is those who have the positions (for example, military rank or party membership in a single-party system) to aspire to make and break leaders. Although the membership of each group may be detailed and complex in most political systems, we simplify our approach by assuming that all members of the selectorate have equal weight in the winning coalition. We then characterize the selectorate and winning coalition by the size of their membership and refer to those sizes as S and W respectively.

We also assume that leaders provide a mixture of public and private goods. Public goods benefit all members of the society. Private goods are excludable and awarded by the leader to specific members of society. Such private goods cover a wide range of government policies and actions that produce benefits for particular individuals, such as state-granted monopolies, access to hard currency, stores in economies with shortages, and kickbacks and bribes secured by government officials. The leader controls resources which she can use to

[5] Bruce Bueno de Mesquita, James D. Morrow, Randolph Siverson and Alastair Smith, 'Policy Failure and Political Survival: The Contribution of Political Institutions', *Journal of Conflict Resolution*, 43 (1999),147–61; Bruce Bueno de Mesquita, James D. Morrow, Randolph Siverson and Alastair Smith, 'An Institutional Explanation for the Democratic Peace', *American Political Science Review*, 93 (1999), 791–808; Bruce Bueno de Mesquita, James D. Morrow, Randolph Siverson and Alastair Smith, 'Political Institutions, Political Survival, and Policy Success', in Bruce Bueno de Mesquita and Hilton Root, eds, *Governing for Prosperity* (New Haven, Conn.: Yale University Press, 2000), pp. 59–84; Bruce Bueno de Mesquita, James D. Morrow, Randolph Siverson and Alastair Smith, 'Testing the Selectorate Explanation of the Democratic Peace' (unpublished manuscript, 2000); Bueno de Mesquita, Smith, Siverson and Morrow, 'The Logic of Political Survival'.

produce a mixture of public and private goods, retaining unallocated resources for her own benefit.[6]

The leader's coalition includes all who receive private goods from her. When a challenger emerges to the sitting leader and proposes an alternative allocation of resources, the members of the selectorate must decide whom to support. The challenger replaces the leader if he can reduce the leader's coalition to less than a winning coalition by attracting enough members of the leader's coalition to support his challenge and if he can attract enough support throughout the selectorate to create a coalition of his own that is at least a winning coalition. A leader thwarts a challenge if she either retains a winning coalition or prevents the challenger from assembling a winning coalition.

The institutions of the selectorate and winning coalition influence a contest for leadership by altering the mix of public and private goods that a leader produces in her efforts to fend off challenges. Our first central intuition is that public goods become more attractive to provide relative to private goods as the size of the winning coalition increases. A simple numerical example illustrates this insight. Imagine that the leader has a pool of $1,000 with which to provide goods and that spending the entire $1,000 would produce a public good worth $20 to everyone in society. If a winning coalition requires only ten members of the selectorate, the leader can offer each of them up to $100 of private goods which they would prefer to the public good. If a winning coalition requires a hundred members of the selectorate, the leader can only offer each of them $10 of private goods and they would prefer that she provide the public good. The pool of resources devoted to private goods is spread more thinly as the size of the winning coalition increases, and so producing more public goods becomes a more attractive way for a leader to produce value for the members of her coalition.

Of course in reality all policies contain aspects of public and private goods. Even those aspects of public policy, such as national defence, that benefit all members of society often provide private benefits for those who provide them.[7] A core feature of our study is how institutions affect the relative mix of public and private goods in public policy. While defence contractors profit from government contracts, the extent to which they receive private benefits depends upon whether contracts are put up for public bid or are awarded to cronies without regard for the quality of the product or size of the cost overrun. Such tradeoffs are a central aspect of all public policy. Robert Bates, for example, describes how many African governments use agricultural policy to reward supporters.[8] Rather than allowing the market to decide prices, market boards act

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[6] We can think of mixed goods produced by government action – those with both public and private aspects – as an allocation of some public and some private goods by the leader.

[7] Peter H. Aranson and Peter C. Ordeshook, 'Public Interest, Private Interest, and the Democratic Polity', in Roger Benjamin and Stephen Elkin, eds, *The Democratic State* (Lawrence: The University Press of Kansas, 1985), pp. 87–177.

[8] Robert H. Bates, *Markets and States in Tropical Africa: The Politics Basis of Agricultural Policies* (Berkeley: University of California Press, 1986).

as monopsonies, buying all crops from farmers at a fixed price, usually below the world market price. To maintain production at these artifically low prices governments provide a variety of subsidies such as low cost loans, subsidized fertilizer, subsidized seed and access to irrigation. While all producers benefit from a higher price, governments control access to subsidies, enabling them to privilege some farmers and not others.

Our second central intuition is that the loyalty of members of the leader's coalition increases as the size of the selectorate increases, holding the size of the winning coalition fixed. Should the challenger replace the leader, members of his coalition now cannot be certain that they will be included in his winning coalition, and so continue receiving private goods in the future. The risk of exclusion rises with the size of the selectorate because the challenger has more choices of whom to include in his coalition after he becomes the leader. Although a challenger can make promises to those who defect from the leader's coalition to support his challenge, they cannot be certain he will honor those promises should his challenge succeed. This risk of exclusion means that the challenger must offer members of the leader's coalition more than the leader has offered them to convince them to support the challenge.

We model this uncertainty about the future composition by assuming that all members of the selectorate have an affinity for each leader and challenger. This affinity covers any value that a member of the selectorate could derive from a leader in office, from personal friendship to ideological similarity. It gives the leader a reason to choose those selectors with high affinity for herself because they are more difficult to induce to defect from her coalition. We assume that affinities for the current leader are known, but those for the challenger are unknown, being revealed only if and after the challenger becomes the new leader.[9] This assumption leads to the risk coalition members face of exclusion from future private goods if they defect to the challenger. In equilibrium, this risk is W/S, the size of a winning coalition over the size of the selectorate. The smaller that W/S is, the greater the risk and cost associated with defecting to a challenger and, therefore, the stronger the loyalty to the incumbent.

Our model makes simplifying assumptions on the problems described above. First, we assume that all members of the selectorate are identical in their tastes for public and private goods and in their contribution to making a winning coalition. Secondly, we assume that all members of the leader's coalition receive equal amounts of private goods. Thirdly, we assume that affinities for the challenger are unknown when members of the selectorate must choose between the leader and the challenger. All of these simplifying assumptions reduce the technical complexity of the model while retaining our central intuitions. We have developed complementary models to this one that explore the significance of these issues for our argument.[10]

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[9] In Bueno de Mesquita *et al.*, 'The Logic of Political Survival', we relax these restrictions and investigate how alternative affinity assumptions influence leadership survival prospects and the other central comparative static results from our model.

[10] Bueno de Mesquita *et al.*, 'The Logic of Political Survival'.

THE SELECTORATE MODEL

The model is an infinitely repeated game for which we define a single round here. A nation is composed of N residents, divided between the selectorate and all others. The selectorate consists of S members. The incumbent L is kept in power by a winning coalition of members of the selectorate, where each member of the selectorate has equal weight in contributing to the winning coalition. For the current round, we denote the members of L's winning coalition as W_L and those in the challenger's coalition as W_c. Throughout, the subscripts L and c refer to the incumbent and challenger respectively. For convenience, we refer to L as 'she' and C as 'he'. The model has a total of $S + 2$ actors per round.

In each period there are R $(R > 0)$ resources available. A round begins with leader L specifying a coalition W_L $(W_L \subset S)$ and allocating x_L public goods and g_L private benefits to each member of her coalition. To reduce notation, we let the letter denoting a set, say X, refer both to the set X and the cardinality of the set X. Where there is a risk of confusion we denote cardinality explicitly by $|X|$. The incumbent's expenditure of public and private goods is $M_L = px_L + |W_L|g_L$. Leader L retains any remaining resources $(R - M_L)$ for her discretionary purposes.

Next, C specifies a coalition, W_c, and proposes a provision of x_c public goods and g_c private goods to each member of his coalition, in a parallel fashion to L. We denote these choices as x_c, g_c, W_c, M_c. The members of S simultaneously select L or C to be their leader, and the winner implements his or her allocation, ending the round. The rules for selecting between L and C – the deposition rules – retain L in office unless fewer than W members of W_L choose L and there are at least W members of W_c who choose C.

The incumbent L's utility for retaining office is $\Psi + R - M_L$ where $\Psi > 0$ and represents the benefits of holding office separate from discretionary resources $R - M_L$. The incumbent's utility is 0 for being removed from office. C's utility for gaining office is $\Psi + R - M_c$ and 0 for failing to replace L.

The members of the selectorate have utility function $V(x,g) + a_i$ where V is increasing, twice differentiable and concave[11] in x and g. The term a_i parameterizes i's idiosyncratic affinity for whoever holds office at the end of the round.[12] We label i's affinity for L as $a_{i,L}$ and for C as $a_{i,C}$. The full set of affinities for each candidate is a linear ordering of the members of S taking values between $-a/2$ and $a/2$. If i is the oth individual in this ordering, then

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[11] We also assume an interior solution, a sufficient condition for which is:

$$V_x(x, g)|_{x=0} > \frac{p}{W} V_g(x, g)|_{x=0} \text{ for } g > 0 \text{ and } V_x(x, g)|_{g=0} < \frac{p}{W} V_g(x, g)|_{g=0} \text{ for } x > 0.$$

[12] James M. Enelow and Melvin J. Hinich, *The Spatial Theory of Voting* (Cambridge: Cambridge University Press, 1984); Melvin J. Hinich and Michael J. Munger, *Ideology and Theory of Political Choice* (Ann Arbor: University of Michigan Press, 1994); Russel Hardin, *One for All: The Logic of Group Conflict* (Princeton, N.J.: Princeton University Press, 1997); Ethan Bueno de Mesquita, 'Coalitions, Strategic Voting, and Non-Policy Voting: The Political Causes of Electoral Reform in Israel', *Comparative Politics*, 33 (2000), 68–80.

$$a_{i,L} = a\left(\frac{o-1}{S-1} - \frac{1}{2}\right).$$

We assume this fixed scale of uniform steps in affinity for mathematical convenience. Also for convenience, we refer to the members of S by their place in L's affinity ordering, 1 being the member of S with lowest affinity for L, and S the member with highest affinity. We label S's full set of affinities for L as $a(L)$ and for C as $a(C)$ and the full set of all possible orderings as A.

Idiosyncratic factors, such as affinities, can have important political consequences for leadership selection separate from the leader's policy performance. Affinity, however, is not our main focus. Our objective is to examine how political institutions influence policy provision and leader survival. When the magnitude of affinity (a) is large enough, leaders can secure support from those predisposed towards them without reference to their policy provisions. To avoid such corner solutions we characterize equilibria when affinities are smaller than the threshold value at which affinity overwhelms all other considerations when choosing a leader. For practical purposes we consider $a \to 0$. Small affinities (small a) allow multiple coalitions to be supported in equilibrium. Fortunately, of all the coalitions that can be supported in equilibrium, the incumbent's preference for the coalition that we characterize provides a natural equilibrium refinement. Additionally, as our proof shows (see end of article), as the magnitude of affinities increases, alternative equilibria disappear.

Affinities for $L - a(L)$ – are common knowledge in each round. Values of $a(C)$ are chosen uniformly from A and revealed to all if C replaces L, becoming common knowledge for all future rounds. If C replaces L, we relabel C as L for all future rounds and choose a new C in each future round with its set of affinities drawn uniformly from A. If L retains office, a new C is the challenger in the next round. Once defeated, C or L is removed, and its place taken by a new candidate. Additionally, across rounds, all players discount the value of future payoffs with a common discount factor δ. The time line of a round of the game is given in Figure 1.

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1. L chooses W_L and divides R into g_L and x_L such that $R \geq pg_L + |W_L|x_L = M_L$.
2. C chooses W_c and divides R into g_c and x_c such that $R \geq pg_c + |W_c|x_c = M_c$.
3. Each member of S simultaneously chooses L or C. If fewer than W members of W_L choose L and at least W members of S choose C then C replaces L, $a(C)$ is revealed, and each member of W_c receives $V(x_c, g_c) + a_{iC}$, other members of S receive $V(x_c, 0) + a_{iC}$, C receives $\Psi + R - M_c$, and L receives 0. Otherwise, L retains office and each member of W_L receives $V(x_L, g_L) + a_{iL}$ and other members of S receive $V(x_L, 0) + a_{iL}$, L receives $\Psi + R - M_L$, and C receives 0.

Fig. 1. Time line of a round of the game

We present Markov perfect equilibria of the model. Repeated games typically give rise to many equilibria if the players are allowed to condition their strategies on the history of the game.[13] Markov perfect equilibria restrict strategies to depending only on the current state of the game; if the state is the same in two rounds, then the strategies played must be as well. The history of the game enters into a Markov perfect equilibrium only to the extent that it affects the current state. The state of the game in our model is $a(L)$, the set of affinities for the incumbent. Markov perfect equilibrium also requires subgame perfection within a round. The following definition specifies what strategies form a Markov perfect equilibrium of the model:

Definition. A strategy for L is a mapping, $\Lambda(a(L)) \rightarrow \{\Re^+, \Re^+, 2^S\}$, where the image gives (x_L, g_L, W_L). A strategy for C is a mapping, $\Gamma(x_L, g_L, W_L, a(L)) \rightarrow \{\Re^+, \Re^+, 2^S\}$, where the image specifies (x_C, g_C, W_c) for L's strategy and the set of affinities. A strategy for $i \in S$ is a mapping $\Sigma(x_L, g_L, W_L, x_C, g_C, W_c, a(L), i) \rightarrow \{L, C\}$ giving its choice of either L or C given their strategies.[14] Strategies $(\Lambda, \Gamma, \Sigma)$ form a Markov perfect equilibrium of the model when the following is true:

(1) Λ maximizes L's expected payoff over all rounds of the game from every choice point of L given $a(L)$, Γ and Σ,
(2) Γ maximizes C's expected payoff from every choice point of C given $a(L)$, Λ and Σ, and
(3) for each $i \in S$, Σ maximizes i's discounted expected payoff over all rounds from every choice point of S given Λ and Γ.

Members of the selectorate cannot condition their choice between L and C based on their affinities for C because they do not know those values when they must choose between the two. Further, their expected affinity for C is 0. We do not calculate C's discounted expected payoff over all future rounds in determining its strategy because it will switch to Λ in the next round if C is selected as the leader. Clearly, C's decision is a matter of gaining office in the current round because it will be removed from the game if it fails to do so. Any actions C takes in gaining power do not bind it in future rounds nor do any actions taken by L or the members of S in that round have any implications for C's payoff in future rounds.

The model has two types of equilibria depending on the size of W relative to S. Supermajoritarian systems, those with $W > (s + 1)/2$, yield a blocking equilibrium where L seeks to deny a potential winning coalition to C. All other systems, which we believe are more common, have equilibria where L seeks to

[13] Drew Fudenberg and Eric Maskin, 'The Folk Theorem in Repeated Games with Discounting and with Incomplete Information', *Econometrica*, 54 (1986), 533–45.
[14] In common with most voting models, we also insist the choices of members of the selectorate who can influence the outcome are weakly undominated (i.e., members choose as if their choice matters). This rules out equilibria in which all members of S choose the same alternative.

hold a sufficient number of members of her winning coalition.[15] We focus on this holding equilibrium because we believe such systems are more common. Later in the article, we return to the blocking equilibrium for comparison.

The Holding Equilibrium

On the equilibrium path the incumbent forms a coalition with the W individuals in S with the highest utility for her. She rewards these supporters with x^* public and g^* private goods, keeping any remaining resources for herself. The challenger attempts to depose the incumbent by forming a coalition with the lowest affinity member of L's coalition and $W - 1$ other members of the selectorate and offering to spend all available resources to reward them. The incumbent's policy provisions, x^* and g^*, match the challenger's 'best offer', ensuring loyalty from the incumbent's coalition.

Let us define some functions that are useful in stating the equilibria. Let $\hat{x}(M, w)$, $\hat{g}(M, w)$ maximize $V(x, g)$ subject to $M = px + wg$, where M is the resources spent and w is the size of the leader's coalition. The allocation $\hat{x}(M, w)$, $\hat{g}(M, w)$ provides greatest rewards to the members of a coalition of size w when their leader allocates M resources. Define $v(M, w) = V(\hat{x}(M, w), \hat{g}(M, w))$ as the indirect utility function, or welfare, that members of coalition of size w receive from the optimal allocation of M resources. Additionally, let $u(M, w)$ be the utility of receiving only the public goods portion of this optimal allocation: $u(M, w) = V(\hat{x}(M, w), 0)$. This is the reward level that citizens outside the coalition receive when leaders optimally spend M resources on a coalition of size w.

Let $x^* = \hat{x}(m^*, W)$ and $g^* = \hat{g}(m^*, W)$, where m^* is the value of m that maximizes $\Psi + R - m$ subject to the constraint

$$(1 - \delta\tfrac{W}{S})\, v(m, W) - (1 - \delta)\, v(R, W) - \delta(1 - \tfrac{W}{S})\, u(m, W) + a_{(S - W + 1), L} = 0$$

where $a_{(S - W + 1), L}$ is the affinity that the $(S - W + 1)$th individual receives from the incumbent remaining in office.

This allocation by L makes individual selector $S - W + 1$ indifferent between L and C when both include $S - W + 1$ in W_L and W_c and C optimally allocates the complete pool of resources R. We let $w_L = \{S - W + 1, \ldots, S\}$: the set of the W individuals with highest affinity for L.

For each individual, $i \in S$, we define $F(x_L, g_L, x_c, g_c, W_L, W_c, i) = U_i(L|x_L, g_L, W_L) - U_i(C|x_c, g_c, W_c)$, the difference between the expected rewards from the leader's offer and the challenger's offer. Where $U_i(L|x_L, g_L, W_L) =$

$$\begin{cases} V(x_L, g_L) + \frac{a_i L}{1-\delta} + \frac{\delta}{1-\delta} V(x^*, g^*) & \text{if } i \in W_L, i \in w_L \\ V(x_L, g_L) + \frac{a_i L}{1-\delta} + \frac{\delta}{1-\delta} V(x^*, 0) & \text{if } i \in W_L, i \notin w_L \\ V(x_L, 0) + \frac{a_i L}{1-\delta} + \frac{\delta}{1-\delta} V(x^*, g^*) & \text{if } i \notin W_L, i \in w_L \\ V(x_L, 0) + \frac{a_i L}{1-\delta} + \frac{\delta}{1-\delta} V(x^*, 0) & \text{if } i \notin W_L, i \notin w_L \end{cases} \quad \text{and}$$

$$U_i(C|x_c, g_c, W_c) = \begin{cases} V(x_c, g_c) + \frac{\delta}{1-\delta} (\frac{W}{S} V(x^*, g^*) + (1 - \frac{W}{S}) V(x^*, 0)) & \text{if } i \in W_c \\ V(x_c, 0) + \frac{\delta}{1-\delta} (\frac{W}{S} V(x^*, g^*) + (1 - \frac{W}{S}) V(x^*, 0)) & \text{if } i \notin W_c. \end{cases}$$

Given L's coalition, we define $\Theta(W_L)$ as the set of $(|W_L| - W + 1)$ members of W_L with the lowest $U_i(L|x_L, g_L, W_L)$. Further let $\Upsilon(X)$ be the set of $W - |X|$ selectors who are not members of W_L who have the lowest values of $U_i(L|x_L, g_L, W_L)$. As we shall see, on the equilibrium path, $\Theta(W_L) = \{S - W + 1\}$ and $\Upsilon(\Theta(W_L)) = \{1, \dots, W - 1\}$.

Given these preliminaries we describe the coalition and policies of the challenger. There are two cases.

(1) If $|W_L| \leq 2W + 1$ then we define the challenger's coalition as $\tilde{W}_c = \Theta(W_L) \cup \Upsilon(\Theta(W_L))$ and we represent the individual in this set with the highest utility for the incumbent as k:
$k = \arg \max_{i \in \tilde{W}_c} U_i(L|x_L, g_L, W_L)$. In equilibrium, this individual is $S - W + 1$ and determines the survival or ouster of the incumbent. Define $\tilde{x}(x_L, g_L, W_L)$ and $\tilde{g}(x_L, g_L, W_L)$ as the values of x_c and g_c that minimize $M_c = px_c + |\tilde{W}_c|g_c$ with respect to x_c and g_c subject to the constraint

$$V(x_c, g_c) + \frac{\delta}{1-\delta} \left(\frac{W}{S} V(x^*, g^*) + \left(1 - \frac{W}{S}\right) V(x^*, 0) \right) \geq U_k(L|x_L, g_L, W_L).$$

Define $Q(x_L, g_L, W_L) = R - p\tilde{x}(x_L, g_L, W_L) - |\tilde{W}_c|\tilde{g}(x_L, g_L, W_L)$ as the resources left after satisfying this minimization problem. If $Q(x_L, g_L, W_L) < 0$, then C has too few resources to match L's offer to k.

(2) When $|W_L| > 2W + 1$ we define $\tilde{x}(x_L, g_L, W_L)$, $\tilde{g}(x_L, g_L, W_L)$ and $\tilde{W}_c$ as the policies and coalition that minimize $M_c = px_c + |\tilde{W}_c|g_c$ subject to

$$\left| \left\{ i \in W_L \cap W_c : V(x_c, g_c) + \frac{\delta}{1-\delta} \left(\frac{W}{S} V(x^*, g^*) + \left(1 - \frac{W}{S}\right) V(x^*, 0) \right) \right. \right.$$

$$\left. \left. < U_i(L|x_L, g_L, W_L) \right\} \right|; \text{ plus } \left| \left\{ i \in W_L, i \notin W_c : V(x_c, 0) \right. \right.$$

$$\left. \left. + \frac{\delta}{1-\delta} \left(\frac{W}{S} V(x^*, g^*) + \left(1 - \frac{W}{S}\right) V(x^*, 0) \right) < U_i(L|x_L, g_{L\wedge}, W_L) \right\} \right| < W.$$

Again define $Q(x_L, g_L, W_L) = R - p\tilde{x}(x_L, g_L, W_L) - |\tilde{W}_c|\tilde{g}(x_L, g_L, W_L)$ as the resources left after satisfying this minimization problem.

The following proposition specifies a Markov perfect equilibrium of the model where L forms a minimal winning coalition:

Proposition 1. When $W \leq (S + 1)/2$ the following strategies form a Markov perfect equilibrium of the game:

(1) $\Lambda(a(L)) = (x^*, g^*, \{S - W + 1, \ldots, S\})$,
(2) $\Gamma(x_L, g_L, W_L, a(L)) = (\tilde{x}(x_L, g_L, W_L), \tilde{g}(x_L, g_L, W_L), \tilde{W}_c)$ if $Q(x_L, g_L, W_L) \geq 0$, and $(\hat{x}(R, |\tilde{W}_c|), \hat{g}(R, |\tilde{W}_c|), \tilde{W}_c)$ if $Q(x_L, g_L, W_L) < 0$,

(3) $\Sigma(x_L, g_L, x_c, g_c, W_L, W_c, i) = \begin{cases} C & \text{if } F(x_L, g_L, x_c, g_c, W_L, W_c, i) < 0 \\ C & \text{if } F(x_L, g_L, x_c, g_c, W_L, W_c, i) = 0 \\ & \quad \text{and } M_c < R \\ L & \quad \text{otherwise} \end{cases}$

Corollary 1: In the holding equilibrium, the incumbent forms a coalition of size W from the highest affinity members of S. She provides her supporters with x^* public and g^* private goods, with a total expenditure $m^* = px^* + Wg^*$, such that

$$\left(1 - \delta \frac{W}{S}\right) v(m^*, W) - (1 - \delta)v(R, W) - \delta\left(1 - \frac{W}{S}\right) u(m^*, W) + a_{(S - W + 1), L} = 0 \quad (1)$$

and

$$\frac{V_x(x^*, g^*)}{p} = \frac{V_g(x^*, g^*)}{W}. \quad (2)$$

The challenger forms a coalition of size W, from the lowest affinity member of L's coalition (individual $S - W + 1$), together with $W - 1$ individuals with lowest affinity for L from outside L's coalition. He proposes expending all R resources to reward his coalition with optimal provision of public and private goods: $x_c = \hat{x}(R, W)$ and $g_c = \hat{g}(R, W)$.

The incumbent survives as all members of L's coalition remain loyal, with $\{S - W + 2, \ldots, S\}$ strictly preferring L's offer to that of the challenger and individual $k = S - W + 1$ being indifferent between L's and C's offers. Individuals outside L's coalition choose the challenger.

Corollary 2. When affinities are small $(a \to 0)$, $S!/(W!(S - W)!)$ coalitions – corresponding to all possible combinations of picking W individuals from S – can be supported in analogous equilibria to that defined in proposition 1. However, from the perspective of the incumbent leader, the coalition $\{S - W + 1, \ldots, S\}$ (Proposition 1) yields a higher payoff than the payoff associated with any other coalition supported in equilibrium. Further, once $a > \delta(S - 1)(v(m^{\ddagger}, W) - u(m^{\ddagger}, W))$, where $m^{\ddagger}$ solves $(1 - \delta W/S) v(m^{\ddagger}, W) - (1 - \delta)v(R, W) - \delta(1 - W/S) u(m^{\ddagger}, W) + a_{(S - W), L} = 0$ these alternative equilibria no longer exist.

In equilibrium, L forms a minimal winning coalition of the W members of the selectorate with the highest affinities for her.[16] She allocates the mix of public and private goods that counters C's best offer and saves the most resources for L's own use. C can offer to spend the entire budget to attract a winning coalition

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[16] William H. Riker, *The Theory of Political Coalitions* (New Haven, Conn.: Yale University Press, 1962).

that includes the one member of *L*'s winning coalition who least likes *L*'s allocation and $W - 1$ others. *L* must match the value of *C*'s best offer to the member of her own winning coalition targeted by *C*. *L* can do so because that member faces the risk of exclusion from *C*'s winning coalition in following rounds if *C* replaces *L*. Because that member does not know her affinity for *C* when she must choose between *L* and *C*, she believes her chance of being included in *C*s winning coalition in following rounds is W/S. This risk of exclusion allows *L* to offer the member in question less in this round than *C* can and still hold her loyalty. In equilibrium then, *L* holds all members of her winning coalition, and so builds a winning coalition of exactly *W* members.

The holding equilibrium requires the constructive vote of no confidence in our selection rules. We are exploring the consequences of not requiring *C* to assemble a coalition of size *W* in continuing research and give some conjectures here. If *C* does not need to form a coalition of size *W* or greater, then he only needs to induce enough members of *L*'s winning coalition to defect to reduce that coalition below size *W*. In equilibrium then, we believe that *L* will 'oversize' her coalition (include more than *W* members) to force *C* to induce defections by multiple members of *L*'s winning coalition in order to replace *L* in office.[17] In the current setting, the incumbent has no incentive to oversize her coalition since, unless she increases her coalition above $2W - 1$, increases in her coalition size do not force the challenger to attract additional supporters.

If the members of *S* are indifferent between *L* and *C*, any selection between *L* and *C* is a best reply. To avoid the need for an ε-equilibrium concept, we break ties in favour of the challenger if the challenger does not spend all available resources. The intuition here is that *C* could spend ε more to secure support. However, when *C*'s offer exhausts the available resources, $M_c = R$, we assume indifferent selectors choose *L*, since she could better *C*'s offer by spending ε more. While these variations produce alternative equilibria, they do not alter the policy provisions of the incumbent and therefore are of no substantive importance.

How Policy Changes with Institutions

We first discuss the comparative statics associated with changes in the size of *W*, *S*, *R* and δ. After discussing the implications of these comparative statics, we examine comparative statics regarding preferences over institutions.

If $W \leq (S + 1)/2$, then an increase in *W* increases the proportion of public goods relative to private goods: $d(x^*/g^*)/dW > 0$. The incumbent's expenditure, m^*, is increasing in *W* and *R*, and decreasing in *S* and δ: $dm^*/dW > 0$, $dm^*/dR > 0$, $dm^*/dS < 0$, and $dm^*/d\delta < 0$. These comparative statics follow directly from Equations 1 and 2.

[17] Timothy Groseclose and James M. Snyder Jr, 'Buying Supermajorities', *American Journal of Political Science*, 90 (1996), 303–15.

The comparative static for the holding equilibrium shows that as W increases, public goods make up a larger proportion of expenditure. The reason is that as the coalition gets larger, private goods must be distributed to more people, increasingly making the provision of public goods – enjoyed by all – more cost-effective. Additionally, as the size of the winning coalition increases, incumbents must spend a higher proportion of available resources to defeat political rivals. These findings are similar to McGuire and Olson's in that as the inclusiveness of society increases, more public goods are provided and the ruling coalition expropriates fewer resources.[18] The expropriations by the coalition in their model are akin to the private good transfers in our model. Rather than explicitly modelling the survival of leaders, the McGuire and Olson model asks what policies and tax rate maximize the welfare of the coalition in the absence of a challenger.[19] In contrast, we show that a leader need not always spend all available resources in order to match the best possible challenge a rival can offer. Additionally, because we model political competition, we can make predictions regarding leadership survival prospects and preferences over institutions selection that are outside the Olson or McGuire and Olson framework.[20] To understand how institutions shape the ability of leaders to withstand the challenge of rivals and still retain resources for their discretionary use requires the introduction of the *loyalty norm*.

In equilibrium, the challenger cannot credibly commit to keep an individual in his long-term coalition, just as individuals in the transition coalition cannot commit to continue supporting the new incumbent once they learn their affinity for him. Because the challenger will build his coalition only of those with the highest affinity for him if he takes office, members of the current coalition are reluctant to support the challenger. Although the challenger might offer to spend all available resources to provide high levels of rewards for those who sweep him to power, he can only promise longer-term access to private goods probabilistically. Since the new leaders form coalitions of size W in the future, then members of the current coalition have only a W/S probability of being retained in the new leader's long-term coalition. Even though their support might be crucial in the incumbent's ouster, with probability $1 - (W/S)$ potential defectors are not retained once the challenger learns their affinities. Those who are dropped lose access to the future flow of private goods, having gained a one-time large windfall payment from the new incumbent.

In contrast to the challenger, the incumbent guarantees her supporters access to future private goods since her coalition is already composed of her natural supporters, the high affinity types. The smaller the coalition and the larger the selectorate, the greater the risk of exclusion. This risk $(1 - W/S)$, coupled with the cost of exclusion (lost future private goods), drives the loyalty norm that

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[18] McGuire and Olson, 'The Economics of Autocracy and Majority Rule'.

[19] Bueno de Mesquita *et al.*, 'The Logic of Political Survival', introduce an endogenously determined tax rate within the context of the current model.

[20] Bueno de Mesquita *et al.*, 'The Logic of Political Survival'.

makes it hard for the challenger to pay enough to attract defectors. The risk of exclusion becomes larger as the pool of available supporters (the selectorate, S) rises and as the number of supporters required (the minimum winning coalition, W) falls.

The prospective loss of private goods following defection is exacerbated when the coalition is small because the smaller the coalition, the larger the welfare advantages of membership over exclusion. When the coalition is small, the incumbent's supporters are highly privileged relative to the population at large because the majority of the benefits are derived from private goods. This welfare difference diminishes as the coalition size grows because the incumbent increasingly relies on public goods which benefit supporters and opponents alike. Specifically, our model indicates that for any fixed level of expenditure, the welfare of the coalition declines as the size of the coalition increases and, although members of the coalition always receive higher levels of rewards than those outside the coalition, the relative difference between these rewards decreases as coalition size increases (i.e., $v(m^*, W)/u(m^*, W)$ decreases as W increases). When W is small, many more rewards are private in nature and the relative difference in welfare between those inside and those outside the coalition is large. In this case, exclusion from future coalitions is extremely costly and, therefore, increasing S sharply increases loyalty, allowing leaders more latitude for kleptocracy.[21]

The size of available resources and the relative patience of citizens also influences the incumbent's effort level. The more resources that are available, the greater the challenger's best credible offer and, therefore, the more the leader must spend to stay in office. Consequently, when the resource pool is larger, so is the total expenditure on the optimal mix of private and public goods. This finding, that rich societies enjoy higher levels of policy provision than poor societies, is unsurprising. More surprising is the result that the more patient citizens are, the lower the level of policy provisions they receive. This result is best explained by reference to the incumbency criterion (Equation 1).

In equilibrium, the incumbent spends m^* resources on the coalition in each period. In making his best credible bid for power, the challenger offers to spend everything in the first period. Of course, in subsequent periods, he behaves as the current incumbent does, spending m^* resources on a coalition of size W.[22]

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[21] Yoram Barzel, *The Economic Analysis of Property Rights* (New York: Cambridge University Press,1989); Robert H. Frank and Philip J. Cook, *The Winner-Take-All Society* (New York: Free Press,1995); Paul Milgrom and John Roberts, *Economics, Organization, and Management* (New York: Prentice Hall, 1990); Adam Przeworski, *The State and the Economy under Capitalism* (Chur; Harwood Academic Publishers,1990); Adam Przeworski, *Democracy and the Market* (New York: Cambridge University Press, 1991); Robert Tollison, 'Rent-Seeking', *Kyklos*, 35 (1982), 575–602; Gordon Tullock, 'The Transitional Gains Trap', *Bell Journal of Economics*, 6 (1975), 671–8; Bueno de Mesquita *et al.*, 'Testing the Selectorate Explanation for Democratic Peace'.

[22] Our formal analysis exploits a property of Markov perfect equilibria. An implication of such equilibria is that different leaders, finding themselves in structurally identical situations, behave identically. For instance, if the optimal coalition size for the previous incumbent was w, then when confronted by identical incentives, the new leader also forms a coalition of size w.

When individuals choose between leaders, they are uncertain as to whether they will be included in the challenger's long-term coalition. Effectively this means the challenger can only promise private goods probabilisitically while the incumbent promises then with certainty to her current (post-transition) coalition. With regard to future payoffs, the incumbent can promise more than the challenger. The discount factor weights the importance of these future payoffs relative to payoffs today. When citizens are patient (high δ), the incumbent's inherent advantage in providing future private goods weighs heavily in a current supporter's calculations. This means that in the current period the incumbent can offer fewer rewards than the challenger and still look like the more attractive leader. When citizens are impatient (low δ), heavily discounting future rewards compared to current ones, incumbents must spend more to survive in office. In this circumstance, the incumbent's inherent advantage in providing private goods in the future is worth less relative to rewards in the current period. As a result, the incumbent must spend more resources in order to match the challenger's offer. This deduction stands in contrast to much of the literature on co-operation and regimes.[23] If our theory is correct, patience is not a virtue.

We have seen now that the loyalty norm indicates an incumbency advantage and that the extent or value of the incumbency advantage depends upon political institutions. As W increases, the incumbency advantage is diminished through two mechanisms. First, the probability of receiving private goods after defecting, W/S, increases. Secondly, the cost of being excluded from the coalition decreases since an increasingly large proportion of the rewards are provided via public rather than private goods as the winning coalition gets larger. The size of the selectorate, S, also influences the incumbency advantage. The larger the selectorate, the greater the risk of exclusion from future private goods.

Although incumbents are always retained in our model, we can speculate about how the sizes of the selectorate and winning coalition affect leader tenure. Let us hypothesize that surplus resources held by the leader make it easier for her to hold on to office in a crisis because she can use those surplus resources to reward loyal supporters. The smaller the coalition and the larger the selectorate, the easier it is for incumbents to keep the loyalty of their backers and, therefore, the easier it is for them to survive in office. Given their incumbency advantage, leaders in small W, large S systems (for example, autocracy) spend the least on their backers and find it easiest to skim off resources for their own personal goals. In contrast, leaders in systems in which

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[23] Robert Axelrod, *The Evolution of Cooperation* (New York: Harper Collins, 1984); Robert Axelrod and Robert O. Keohane, 'Achieving Cooperation Under Anarchy: Strategies and Institutions', in Kenneth A. Oye, ed., *Cooperation Under Anarchy* (Princeton, N.J.: Princeton University Press, 1984). For a result related to ours, though in a different context and from a different model, see Robert Powell, *In the Shadow of Power* (Princeton, N.J.: Princeton University Press,1999).

W is large and W/S ($S > W$) is large (for example, democracy) find it hardest to survive in office, have to spend the most, and can skim off the fewest resources. The difference in expenditures provides more opportunities for small W leaders to, among other things, maintain a reserve fund to save their incumbency in case of a rainy day. This reserve fund ($R - M_L$) makes it easier to compensate cronies for the unanticipated exigencies of politics, thereby preserving political tenure. This benefit is on top of the greater incumbency advantage ((i.e., $v(m^*, W)/u(m^*, W)$ increases as W decreases) small W leaders attain in general through their disproportionate provision of private goods.

We have utilized the size of the incumbent's coalition, W, and the size of the selectorate, S, to show how loyalty is induced and to show what this implies about leadership tenure. Further, we have shown that the relative allocation of private and public benefits is directly dependent on the size of the winning coalition. Now we show how these factors shapes the institutional preferences of different sections of a polity.

Preferences Over Institutions

The results summarized thus far show how changes in institutions change the mix and quantity of goods allocated by leaders. They also show how institutional changes influence the pool of discretionary resources at the incumbent's disposal and the implications for tenure in office. Now we pull these strands together to discuss what the model suggests about preferences over institutional arrangements. We assess the preferences over institutions implied by the model for four groups: members of the winning coalition; members of the selectorate outside the winning coalition; disenfranchised residents in the polity, that is, those who are not members of the selectorate; and the incumbent leader. We characterize the welfare, or level of benefits, that members of each of these groups receive under different institutional arrangements. Implicitly, we are asking how each group would modify institutions if it had unilateral freedom to change the existing institutions. This characterization of institutional preference is a necessary first step in a theory of endogenous institutional change.

The welfare of those outside the winning coalition, $V(x^*, 0) = u(m^*, W)$, is increasing in W, R, and decreasing in S and δ: $dV(x^*, 0)/dW > 0$, $dV(x^*, 0)/dR > 0$, $dV(x^*, 0)/dS < 0$, $dV(x^*, 0)/d\delta < 0$. The welfare of the winning coalition, $V(x^*, g^*) = v(m^*, W)$, is increasing in R, and decreasing in S and δ. With respect to the size of the winning coalition, $dV(x^*, g^*)/dW = dv(m^*, W)/dW = v_m(m^*, W)\partial m^*/\partial W + v_w(m^*, W)$, the effect of increasing W depends upon the size of W.

The leader receives a payoff of $\Psi + R - M_L$ for each period she survives in office and 0 if she is deposed. Ψ reflects the inherent value of office holding and $R - M_L$ represents the amount of resources she can retain for her discretionary use. In equilibrium, the incumbent can always meet the incumbency criterion, defeating the challenger. To do so, she must take into account how institutions

influence the equilibrium level of expenditure, m^*, required for her political survival. In that regard, m^* provides a measure of the difficulty of surviving in office. The higher m^* must be to stay in office, the lower the leader's welfare.

Since we have already characterized the comparative statics for m we can directly state a leader's institutional preference: $\partial m^*/\partial W > 0$, and $\partial m^*/\partial S < 0$. Leaders prefer small winning coalitions and large selectorates. The importance that such institutional arrangements place on private goods induces a strong loyalty norm as we have noted. In common parlance, leaders most prefer autocratic regimes with universal suffrage (implying rigged elections). Universal suffrage is a way of signalling that almost anyone could, with a very low probability, make it into a winning coalition and small coalitions (with rigged elections) ensure that a small, elite group gets to share the valuable private goods that the leader dispenses. These two conditions increase the strength of the loyalty norm that keeps autocrats in office for long periods.

The winning coalition's welfare, exclusive of affinities, is $V(g^*, x^*) = v(m^*, w)$. How do changes in W and S affect the winning coalition's level of benefits? We start by considering W. Of course, no members of the winning coalition want institutional changes that remove them personally from the coalition. So, conditional on remaining a member of the winning coalition, it is possible for the membership to prefer to expand or to contract the size of the coalition. Whether altering the size of the winning coalition through institutional change increases or decreases the payoffs to the members depends upon the initial conditions.

An increase in W has two competing effects on the welfare of the winning coalition.[24] First, the increase in W means that each member's share of rewards is diluted since the overall number of people who receive rewards has increased. This effect reduces welfare. Secondly, the increase in W reduces the loyalty norm, thereby forcing leaders to spend more resources on keeping their coalition loyal. An increase in expenditure improves the welfare of members of the winning coalition. Which of these two effects dominates depends upon the specific conditions. Figure 2 shows a plot of the winning coalition's welfare, $v(m^*, W)$, as a function of W. The asymmetric, nonmonotonic pattern it illustrates is indicative of the incentives held by members of the winning coalition. When W is small, increases in coalition size diminish their rewards. Beyond a turning point, further increases in W improve the coalition members' welfare, although at a diminishing marginal rate. This happens because the increases in W from this point forward improves the odds of being in a successor coalition faster than it decreases the value of private goods. As the probability of being in a successor coalition improves, the loyalty norm is weakened and so the incumbent must try harder, spending more to satisfy her supporters.

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[24] These may be expressed as: $\frac{dv(m^*, W)}{dW} = \left(\frac{\partial v(m^*, W)}{\partial m}\right)\left(\frac{\partial m^*}{\partial W}\right) + \left(\frac{\partial v(m^*, W)}{\partial W}\right)$, where $\frac{\partial v(m^*, W)}{\partial m} > 0$, $\frac{\partial m^*}{\partial W} > 0$, and $\frac{\partial v(m^*, W)}{\partial W} < 0$; and as $\frac{dv(m^*, W)}{dS} = \left(\frac{\partial v(m^*, W)}{\partial m^*}\right)\left(\frac{\partial m^*}{\partial S}\right) < 0$.
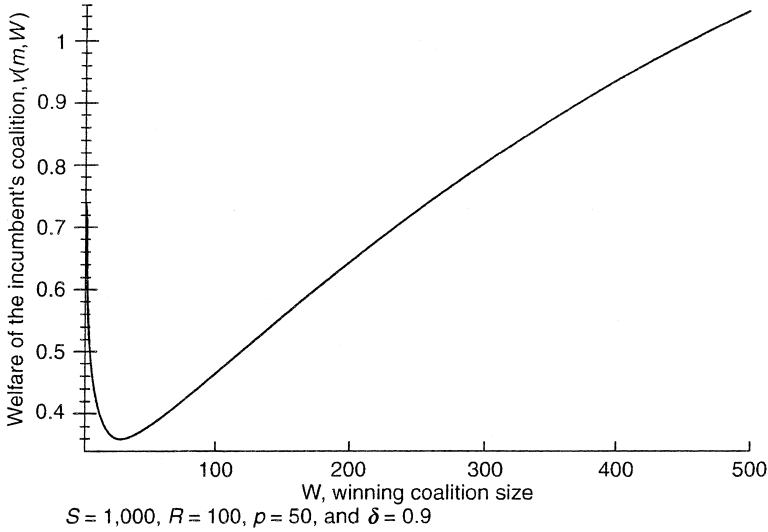
Fig. 2. *Welfare of the incumbent's coalition as a function of winning coalition size*

Increasing the selectorate size strictly diminishes the welfare of the incumbent coalition.[25] Increasing S reduces the effort the incumbent needs to make to maintain the support of her coalition. This is true because an increase in the selectorate size diminishes the chances that current members of the winning coalition would be in a successor coalition. This strengthens their loyalty to the incumbent which reduces the amount the incumbent must spend to keep them loyal.

The welfare of the winning coalition is also enhanced by an increase in the resources available, R, and by having a relatively impatient citizenry (i.e., small δ). Both these results arise directly from increased expenditure on the part of the leader.

The coalition's interests can be broadly interpreted in terms of common regime types. Our results suggest that members of the coalition prefer monarchy or democracy to autocracy. In a monarchy, both the coalition and the selectorate are small. When the winning coalition is sufficiently small, it is contrary to the interests of its members to see it expand. Such expansion dilutes each member's rewards. Hence in monarchies, we should expect, at least initially, that the court and the king share a common goal of restricting the size of the court. However,

[25] Robert H. Bates, Avner Greif, Margaret Levi, Jean-Laurent Rosenthal and Barry Weingast, *Analytic Narratives* (Princeton, N.J.: Princeton University Press,1998); Barrington Moore Jr, *Social Origins of Dictatorship and Democracy* (Boston, Mass.: Beacon, 1966); Hilton Root, *The Foundation of Privilege* (Berkeley: University of California Press, 1994); Charles Tilly, *Coercion, Capital, and European States* (Cambridge, Mass.: Basil Blackwell, 1990); Douglass North and Barry Weingast, 'Constitutions and Commitment: The Institutions Governing Public Choice in Seventeenth Century England', *Journal of Economic History*, 44 (1989), 803–32.

if the winning coalition expands beyond a certain size, its members prefer further expansion, while the king does not. As a monarchy becomes more inclusive, a wedge is driven between the monarch and his court. The king continues to prefer a small winning coalition while the court, with its members' share of private goods already diluted, prefer to push for additional expansion in the winning coalition to force the monarch to work harder on their behalf. Numerous studies of economic history support the effects on transitions from monarchy to more democratic forms suggested here.[26]

Members of the selectorate outside the winning coalition and those outside the selectorate receive only public goods. As such, their welfare, exclusive of affinity, is equal to the value of the public goods provided, $V(x^*, 0) = u(m^*, W)$. Therefore, these two groups always prefer to increase the size of the winning coalition and to decrease the size of the selectorate. An increase in W induces leaders to increase the provision of public goods, the only form of rewards that these groups benefit from. Further, increases in W and decreases in S undermine the loyalty norm making the leader work harder and allocate more resources to the provision of policy. So, those outside the winning coalition prefer polities, such as democracy, with a large winning coalition and relatively small selectorates, to other styles of government.

The Blocking Equilibrium

The holding equilibrium requires $W \leq (S + 1)/2$, which we believe characterizes most political systems. When $W > (S + 1)/2$, L shifts her efforts from keeping her winning coalition to trying to prevent C from assembling a winning coalition. The selection rules require both that L lacks the support of a winning coalition of size W and that C assembles a winning coalition of at least size W for C to supplant L in office. When $W > (S + 1)/2$, blocking the latter is easier than preventing the former.

Again, some definitions assist in the statement of the blocking equilibrium. These functions parallel those in the holding equilibrium with W_L reduced to $S - W + 1$ members, so we denote these functions and terms with a bar over them. Let $\bar{x}^* = \hat{x}(\bar{m}^*, S - W + 1)$ and $\bar{g}^* = \hat{g}(\bar{m}^*, S - W + 1)$, where $\bar{m}^*$ is that value of m that maximizes $\Psi + R - m$ subject to the constraint

$$\left(1 - \delta \frac{(S - W + 1)}{S}\right) v(m, S - W + 1) - (1 - \delta)v(R, W)$$

$$- \delta\left(\frac{W - 1}{S}\right) u(m, S - W + 1) + a_{W, L} = 0.$$

This allocation by L makes individual selector W indifferent between L and

[26] Bueno de Mesquita *et al*., 'Political Institutions, Political Survival and Policy Success'; Bueno de Mesquita *et al*., 'The Logic of Political Survival'; Ronald Wintrobe, *The Political Economy of Dictatorship* (New York: Cambridge University Press, 1998).

C when both include selector W in W_L and W_c and C optimally allocates all of the resources R.

We let $\bar{w}_L = \{W, \ldots, S\}$: the set of the $S - W + 1$ individuals with highest affinity for the incumbent. For each individual, $i \in S$, we define $\bar{F}(x_L, g_L, x_c, g_c, W_L, W_c, i) = \bar{U}_i(L|x_L, g_L, W_L) - \bar{U}_i(C'|x_c, g_c, W_c)$, the difference between the expected rewards from the leader's offer and the challenger's offer, where $\bar{U}_i(L|x_L, g_L, W_L) =$

$$
\begin{cases}
V(x_L, g_L) + \frac{a_{iL}}{1-\delta} + \frac{\delta}{1-\delta} V(\bar{x}^*, \bar{g}^*) & \text{if } i \in W_L, i \in w_L \\
V(x_L, g_L) + \frac{a_{iL}}{1-\delta} + \frac{\delta}{1-\delta} V(\bar{x}^*, 0) & \text{if } i \in W_L, i \notin w_L \\
V(x_L, 0) + \frac{a_{iL}}{1-\delta} + \frac{\delta}{1-\delta} V(\bar{x}^*, \bar{g}^*) & \text{if } i \notin W_L, i \in w_L \\
V(x_L, 0) + \frac{a_{iL}}{1-\delta} + \frac{\delta}{1-\delta} V(\bar{x}^*, 0) & \text{if } i \notin W_L, i \notin w_L
\end{cases}
\quad \text{and}
$$

$\bar{U}_i(C|x_c, g_c, W_c) =$

$$
\begin{cases}
V(x_c, g_c) + \frac{\delta}{1-\delta} \left((\frac{S-W+1}{S}) V(\bar{x}^*, \bar{g}^*) + (1 - \frac{S-W+1}{S}) V(\bar{x}^*, 0) \right) & \text{if } i \in W_c \\
V(x_c, 0) + \frac{\delta}{1-\delta} \left((\frac{S-W+1}{S}) V(\bar{x}^*, \bar{g}^*) + (1 - \frac{S-W+1}{S}) V(\bar{x}^*, 0) \right) & \text{if } i \notin W_c.
\end{cases}
$$

Given the incumbent's coalition, we define $\bar{\bar{W}}_c$ to be the W members of S with the lowest value for $\bar{U}_i(L|x_L, g_L, W_L)$. We index the member in this set with the highest value of $\bar{U}_i(L|x_L, g_L, W_L)$ as $\bar{k}$: i.e. $\bar{k} = \arg\max_{i \in \bar{W}_c} \bar{U}_i(L|x_L, g_L, W_L)$. In equilibrium we shall see that $\bar{\bar{W}}_c = \{1, \ldots, W\}$; and individual $\bar{k}$ is W, who determines whether the incumbent survives.

Let $\bar{\bar{x}}(x_L, g_L, W_L)$ and $\bar{\bar{g}}(x_L, g_L, W_L)$ be the values of x_c and g_c that maximize $R - px_c - Wg_c$ subject to

$$
V(x_c, g_c) + \frac{\delta}{1-\delta} \left(\left(\frac{S-W+1}{S} \right) V(\bar{x}^*, \bar{g}^*) \right.
$$

$$
\left. + \left(1 - \frac{S-W+1}{S} \right) V(\bar{x}^*, 0) \right) \geq \bar{U}_{\bar{k}}(L|x_L, g_L, W_L).
$$

Define $\bar{Q}(x_L, g_L, W_L) = R - p\bar{\bar{x}}(x_L, g_L, W_L) - W\bar{\bar{g}}(x_L, g_L, W_L)$ as the resources left after satisfying this maximization problem. If $\bar{Q}(x_L, g_L, W_L) < 0$, then C has too few resources to match L's offer.

The following proposition specifies a Markov perfect equilibrium of the model where L forms a blocking coalition of size $S - W + 1$:

Proposition 2. When $W > (S + 1)/2$, the following strategies form a Markov perfect equilibrium of the game:

(1) $\Lambda(a(L)) = (\bar{x}^*, \bar{g}^*, \{W, \ldots, S\})$,
(2) $\Gamma(x_L, g_L, W_L, a(L)) = (\bar{\bar{x}}(x_L, g_L, W_L), \bar{\bar{g}}(x_L, g_L, W_L), \bar{\bar{W}}_c)$ if $\bar{Q}(x_L, g_L, W_L) \geq 0$, and $(\hat{x}(R, W), \hat{g}(R, W), \bar{\bar{W}}_c)$ if $\bar{Q}(x_L, g_L, W_L) < 0$,

$$
(3) \ \Sigma(x_L, g_L, x_c, g_c, W_L, W_c, i) =
\begin{cases}
C & \text{if } \bar{F}(x_L, g_L, x_c, g_c, W_L, W_c, i) < 0 \\
C & \text{if } \bar{F}(x_L, g_L, x_c, g_c, W_L, W_c, i) = 0 \\
& \text{and } M_c < R \\
L & \text{otherwise.}
\end{cases}
$$

Corollary 3. In the holding equilibrium, the incumbent forms a coalition of size S − W + 1 from the highest affinity members of S. She provides her supporters with $\bar{x}^*$ public and g^* private goods, with a total expenditure $\bar{m}^* = p\bar{x}^* + (S − W + 1)\bar{g}^*$, such that

$$\left(1 − \delta\frac{S − W + 1}{S}\right) v(\bar{m}^*, S − W + 1) − (1 − \delta)v(R, W)$$

$$− \delta\left(1 − \frac{S − W + 1}{S}\right) u(\bar{m}^*, S − W + 1) + a_{W, L} = 0 \tag{3}$$

and

$$\frac{V_x(\bar{x}^*, \bar{g}^*)}{p} = \frac{V_g(\bar{x}^*, \bar{g}^*)}{S − W + 1}. \tag{4}$$

The challenger forms a coalition of size W from those selectors with the lowest affinity for the incumbent. C proposes expending all available resources to reward his coalition with optimal provision of public and private goods: $x_c = \hat{x}(R, W)$ and $g_c = \hat{g}(R, W)$.

Since the incumbent spends just enough resources to match the challenger's best possible offer to selector W, the incumbent survives in office. Selector W is indifferent between L's and C's policy provisions. The remaining members of L's coalition, $\{W + 1, \dots, S\}$, are not members of C's coalition and prefer the incumbent. All selectors outside of the incumbent's coalition are included in W_c and they choose the challenger over the incumbent.

When the winning coalition requires a supermajority, *L* seeks to deny a winning coalition to *C*. The pivotal member of the selectorate is the *W*th member of the affinity ordering. *L* offers sufficient benefits to this member and all those with higher affinities, leading to a coalition smaller than winning size. *C* seeks its winning coalition by offering to reward the *W* members of the selectorate with the lowest affinity for *L*. However, *C* cannot offer enough benefits in equilibrium to overcome the *W*th member's risk of omission from *C*'s winning coalition in future rounds.

Comparative Statics on Policy in the Blocking Equilibrium: If $W > (S + 1)/2$, then the relative proportion of public goods to private goods is increasing in *S* and decreasing in *W*:

$$\frac{d\frac{\bar{x}^*}{\bar{g}^*}}{dS} > 0 \text{ and } \frac{d\frac{\bar{x}^*}{\bar{g}^*}}{dW} < 0.$$

The incumbent's expenditure, $\bar{m}^*$, is increasing in *S* and *R*, and decreasing in *W* and δ:

$$\frac{\partial \bar{m}^*}{\partial S} > 0, \frac{\partial \bar{m}^*}{\partial R} > 0, \frac{\partial \bar{m}^*}{\partial W} < 0, \text{ and } \frac{\partial \bar{m}^*}{\partial \delta} < 0.$$

These comparative statics follow directly from Equations 3 and 4.

Welfare Analysis in Blocking Equilibrium: The welfare of those outside the winning coalition, $V(\bar{x}^*, 0) = u(\bar{m}^*, W)$, is increasing in S, R, and decreasing in W and δ:

$$\frac{dV(\bar{x}^*, 0)}{dW} < 0, \frac{dV(\bar{x}^*, 0)}{dR} > 0, \frac{dV(\bar{x}^*, 0)}{dS} > 0, \text{ and } \frac{dV(\bar{x}^*, 0)}{d\delta} < 0.$$

The welfare of the winning coalition, $V(\bar{x}^*, \bar{g}^*) = v(\bar{m}^*, S - W + 1)$, is increasing in S and R, and decreasing in δ. With respect to the size of the winning coalition,

$$\frac{dV(\bar{x}^*, \bar{g}^*)}{dW} = \frac{dv(\bar{m}^*, S - W + 1)}{dW} = v_m(\bar{m}^*, S - W + 1)\frac{\partial \bar{m}^*}{\partial W} - v_w(\bar{m}^*, S - W + 1),$$

the effect of increasing W depends upon the size of W.

The comparative statics of policy with respect to the sizes of the selectorate and winning coalition are reversed under the blocking equilibrium. Raising W/S reduces the size of the leader's coalition, leading her to rely more on private goods than public goods. The effects of increasing resources and the discount factor are the same. Similarly, the welfare of individuals and hence their preferences over institutions are also reversed in the blocking equilibrium. We speculate that these results explain why such supermajority systems are rare. Democracies commonly have large winning coalitions around about half of the size of their selectorates. Increasing the winning coalition further reduces the benefits of democracy – greater provision of public goods and lesser diversion of resources to the leader's benefit. The members of the selectorate have leverage over their leader in such systems and are likely to act to block changes to expand the size of the winning coalition further.

IMPLICATIONS

We touched on several implications of the selectorate model as we developed our argument. Here we pull some of the more interesting implications together. In particular we address questions of leadership tenure, economic growth, the provision of public policy and corruption. We also discuss how the model might be extended to address the questions of taxation and ethnic politics.

The model suggests that it is easier for autocrats (leaders with small coalitions) to survive in office than democrats (i.e., leaders with large coalitions). There is considerable empirical evidence for this claim.[27] Further, in the context of the model we can account for finer differences between the survival of leaders. The difference in survivability between autocracy and democracy stems from differences in the relative importance of private and public goods. In large winning coalition systems leaders must compete over the provision of public goods. Although the incumbent is privileged in the supply of private goods, her advantage is small since political competition centres around the ability to produce public goods.

In contrast, in autocratic systems the small winning coalition size means

[27] Bueno de Mesquita *et al.*, 'The Logic of Political Survival'.

political competition is focused on the provision of private goods. Once the incumbent has identified those members of the selectorate with the highest affinity for her, she finds satisfying the incumbency criterion relatively easy. She can credibly commit to include these individuals in every future coalition, while the challenger can offer them access to future private goods only probabilistically. For the purposes of parsimonious modelling we assume affinities are fully revealed when a new leader arises. Yet realistically the learning process takes years and until it is over, members of the winning coalition cannot be certain of inclusion in future coalitions. This implies that initially an autocrat's coalition is relatively unstable, since members fear exclusion. However, as the learning process continues, it becomes increasingly unlikely that supporters will be replaced and so their fear of exclusion diminishes and the loyalty norm strengthens. In contrast to incumbents who depend on a large coalition, like democrats, for whom the hazard rate always remains high, the risk of removal from office diminishes over time for those who depend on a small coalition, like autocrats or leaders of military juntas. While the average tenure in office of all autocrats is about twice as long as that of all democrats, those who survive the first few years typically survive a long time, often only succumbing to old age and ill health.

Our main deductive predictions relate to the quantity and quality of public policy provision. In particular, because democrats rely on large winning coalitions, they must provide more public goods than those who depend on small winning coalitions. Indeed, David Lake and Mathew Baum extensively survey the literature and examine a wide variety of public policy issues to conclude that democrats provide significantly more public goods than autocrats.[28] These policy differences between regime types carry implications about economic growth.[29]

[28] David Lake and Mathew Baum, 'The Invisible Hand of Democracy' (unpublished manuscript, Department of Political Science, University of California, San Diego, 2000). Gary Cox, *The Efficient Secret* (Cambridge: Cambridge University Press, 1987) provides illustration in his study of nineteenth-century British electoral reform. At the begining of the century many electoral districts were small, some having only a handful of voters. The major business of the House of Commons at that time was bills proposed by private members on behalf of specific constituents. Much of the population remained unrepresented. By the end of the century, various reform acts, through redistricting of rotten boroughs and the enlargement of the franchise, produced large, roughly even, electoral districts. The effects on British politics were profound. With increased district size, members of parliament could no longer reward their constituents with private goods. Both bribery of the electorate and private members bills declined to be replaced by government initiated public policy.

[29] Sampling of influential works on the controversy over the relationship between democracy and economic performance includes Robert Barro, *Determinants of Economic Growth* (Cambridge, Mass.: Harvard Institute of International Development, 1997); Stanley L. Engerman and Kenneth L. Sokoloff, 'Factor Endowments, Institutions, and Differential Paths to Growth', in Stephen Haber, ed., *How Latin America Fell Behind* (Stanford: Stanford University Press, 1997), 269–304; John F. Helliwell, 'Empirical Linkages between Democracy and Economic Growth', *British Journal of Political Science*, 24 (1994), 225–48; Stephen Knack and Philip Keefer, 'Institutions and Economic Performance: Cross-Country Tests Using Alternative Institutional Measures', *Economics and Politics*, 7 (1995), 207–27; Seymour Martin Lipset, 'Some Social Requisites of Democracy',

To the extent that the provision of public goods such as protecting property rights, rule of law, transparency, national security, etc. promote economic growth and prosperity, we expect large coalition systems to be richer and to experience higher average growth. Elsewhere we discuss the literature on the controversial topic of the relationship between democracy and growth and test the empirical relationship between growth (and about thirty other indicators of policy performance) and empirically derived measures of S and W as well as measures of democracy.[30] We show stronger, more consistent results when the focus is on S and W than when the focus is on democracy–autocracy, as argued in fn. 4. Furthermore, we demonstrate that increased growth follows increases in the size of W but increases in growth do not lead to change in coalition or selectorate size, thereby establishing the empirical direction of causality in a manner consistent with the logic of the theory.

Large coalition systems are also expected to grow faster because they experience less corruption than their small coalition counterparts.[31] To some extent corruption persists in all polities. Yet, the extent to which leaders attempt to detect and eradicate corruption depends upon institutional arrangements.[32] Our model suggests three motives for corruption, all of which are encouraged by small winning coalition systems. To the extent that eliminating corruption and encouraging the development of institutions that promote the rule of law are public goods, leaders with small winning coalitions have few incentives to find and eliminate corruption. Hence small W encourages complacency. In addition to failing to root out corruption, leaders with small coalitions might endorse corruption as a way of rewarding supporters. Particularly in nations with underdeveloped infrastructures, collecting taxes with which to reward supporters is inefficient relative to granting supporters the right to expropriate resources for themselves. Hence, some authoritarian leaders encourage corrupt practices as a reward mechanism. A final form of corruption predicted by our model is kleptocracy, as discussed above.[33]

The model assumes leaders have a fixed quantity of available resources. Elsewhere, we extend the analysis to consider the origins of these resources by modelling how institutions shape tax rates, t, and how hard citizens choose to work.[34] In particular, we model extensions in which leaders pick a tax rate, and

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(F'note continued)

American Political Science Review, 53 (1959), 69–105; Douglass C. North, *Structure and Change in Economic History* (New York: W. W. Norton, 1988); Adam Przeworski and Fernando Limongi, 'Modernization: Theory and Facts', *World Politics*, 49 (1997), 155–83; John Roemer, 'On the Relationship between Economic Development and Political Democracy', *Economic Design*, 1 (1994), 15–40.

[30] Bueno de Mesquita *et al.*, 'The Logic of Political Survival'.

[31] Bueno de Mesquita *et al.*, 'Testing the Selectorate Explanation for Democratic Peace'.

[32] José Eduardo Campos and Hilton Root, *The Key to the Asian Miracle: Making Shared Growth Credible* (Washington, D.C.: The Brookings Institution, 1996).

[33] Grossman, *Kleptocracy and Revolutions*.

[34] José Antonio Cheibub, 'Political Regimes and the Extractive Capacity of Government', *World Politics*, 50 (1998), 349–76; Ronald Findlay, 'The New Polical Economy: Its Explanatory Power for

the citizens decide how hard to work. To the extent that taxes inhibit the incentives to work and invest, setting a tax rate presents a mixture of incentives.[35] When the leader has only to reward a small number of people (small *W*) the best way to do so is to maximize government revenue. This large pool of resources is then distributed to the leader's supporters. The high tax rate required to maximize revenue harms the members of the winning coalition. Yet, the leader has so many resources that she can easily compensate her supporters for these losses or create exemptions from taxation for the members of the winning coalition. When the winning coalition is small, leaders act as 'reverse Robin Hoods'; they use taxes to redistribute as many benefits as possible from the citizenry, who benefit little from the system, to herself and her coalition who benefit greatly. As such, taxes in systems with small winning coalitions have large redistributive consequences.

In contrast, when the winning coalition is large, maximizing revenue harms the interest of the winning coalition. When the winning coalition includes most citizens then there is little redistribution associated with taxing and spending. High taxes might increase government revenues, enabling leaders to supply more benefits to the citizens, but the citizens must pay for these benefits. At the extreme, with everyone in the winning coalition, there are no redistributive consequences. Unfortunately, high taxes reduce the incentive to work, so the overall size of the benefits that citizens share is reduced in size. To maximize the welfare of the winning coalition, the leader keeps taxes low, providing only those public goods that have a higher marginal value than their marginal cost in terms of the private goods citizens give up through taxation. This argument parallels the logic of McGuire and Olson.[36]

When the winning coalition is large leaders want to maximize the size of the pie to be divided since everyone receives the same sized share. However, when the winning coalition is small, leaders sacrifice the overall size of the pie to ensure that the privileged few get as much as possible.

Though we have not yet explored the full implications, the model also offers insight into ethnic conflict. The affinity variable can be thought of as having a lumpy distribution that depends on ethnic, social, religious or other group-oriented characteristics. Since affinities can offset a leader's failure to provide public and private benefits while still leaving the incumbent in office, the model provides an avenue for exploring bloc-voting and ethnic or other group identities. We leave these issues for future research.

In conclusion, the selectorate model yields a broad array of empirically testable and falsifiable implications. Elsewhere using Polity II data we measure

(F'note continued)

LDCs', *Economics and Politics*, 2 (1990), 191–222; McGuire and Olson, 'The Economics of Autocracy and Majority Rule'; Mancur Olson, 'Autocracy, Democracy, and Prosperity', in Richard E. Zeckhauser, ed., *Strategy and Choice* (Cambridge: Cambridge University Press, 1991), pp. 131–57; Przeworski, *The State and Economy under Capitalism.*

[35] Barro, *Determinants of Economic Growth.*

[36] McGuire and Olson, 'The Economics of Autocracy and Majority Rule'.

W and S.[37] In particular, we generate a five-point scale to represent coalition size using the variables REGTYPE, XRCOMP, XROPEN and PARCOMP. The polity variable, Legislative Selection (LEGSELEC), is used as an index of selectorate size. We test the model's predictions using controls (where appropriate) for gross domestic product per capita, region–year fixed effects and the level of democracy against a wide variety of dependent variables. We find that income, growth, investment, civil liberties, property rights, peace (meaning the absence of civil or international war), transparency (measured by whether or not polities report economic and tax data), and a host of public health, education and social security measures all increase as our measure of coalition size increases and as our measure of selectorate size decreases. In contrast, corruption (as measured by black market exchange rate premiums, construction as a proportion of the economy, and Transparency International's index) and the extent to which government expenditures and revenues do not match are higher in small W, large S systems. Coalition and selectorate size also significantly influence the foreign policies of states[38] and the risk of coups, revolution, emigration, immigration and many other factors.[39] While the proposed model is skeletal, it explains many empirically observable phenomena.

PROOFS

Our proof of proposition 1 has three steps. First we examine the selectorates' choice. Secondly, we examine the challenger's coalition and allocation choices. Thirdly, we examine the incumbent's strategy. We then conclude with a discussion of corollary 1. Since the proof of proposition 2 is largely analogous we dispense with a full proof and discuss only aspects that differ.

The principle of dynamic optimality, sometime called Bellman's principle of optimality, ensures that only one period defections from equilibrium behaviour need to be considered. Given strategy in future periods, we calculate optimal strategies for all players in the current period. If the strategy profile s^* in the current period is a subgame perfect equilibrium given players use strategy profile s^* in all future periods, then s^* is a subgame perfect equilibrium in the whole game.[40]

[37] We use Polity II because subsequent Polity releases do not update one of the variables (REGTYPE) we use to construct our indicator of coalition size. We are developing alternative indicators that take advantage of updated data.

[38] Bueno de Mesquita et al., 'An Institutional Explanation for Democratic Peace'; Bueno de Mesquita et al., 'Testing the Selectorate Explanation for Democratic Peace'.

[39] Bueno de Mesquita et al., 'The Logic of Political Survival'. For details of the variable construction and analyses, see Bueno de Mesquita et al., 'Policy Failure and Political Survival'; 'Political Institutions, Political Survival, and Policy Success'; 'Testing the Selectorate Explanation for Democratic Peace'; and 'The Logic of Political Survival'.

[40] See Drew Fudenberg and Jean Tirole, Game Theory (Cambridge, Mass.: MIT Press, 1991), chap. 5; or David Kreps, A Course in Microeconomic Theory (Princeton, N.J.: Princeton University Press, 1990), App. 2.

The Selectorate's Choice

Suppose that in every future period the incumbent survives in office, forms coalition w_L, contingent upon the state $a(L)$, and rewards its members with x^* and g^*.

We now calculate i's payoff dependent upon i's coalition membership. Suppose i is in the incumbent's current and future coalitions and is also a member of the challenger's coalition. If the incumbent survives then i's expected payoff is a combination of current and future payoffs. In the current period, i receives the current rewards, $V(x_L, g_L)$, plus the value of his affinity for the incumbent. In addition, i expects to receive $V(x^*, g^*) + a_{i, L}$ in every future period.

If $i \in W_L$, $i \in w_L$, then $U_i(L|x_L, g_L, W_L) = V(x_L, g_L) + a_{i, L} + \dfrac{\delta}{1 - \delta} (V(x^*, g^*) + a_{i, L})$.

If C suceeds in ousting the incumbent then i's expected payoff (if $i \in W_c$) is

$$U_i(C|x_c, g_c, W_c) = V(x_c, g_c) + \frac{\delta}{1 - \delta} \left[\frac{|w_L|}{S} V(x^*, g^*) + \left(1 - \frac{|w_L|}{S} \right) V(x^*, 0) \right] + \frac{1}{1 - \delta} E[a_{ic}].$$

Should the challenger attain office, i receives the rewards he is provided with as a result of his membership in the transition coalition. However, since $a(C)$ is unknown at the time of C's accession, i is only included in future coalitions probabilistically. Given the uniform distribution of affinity orderings, the probability that i will be included in the challenger's long-term coalition is $|w_L|/S$ ($= W/S$). The differences between $U_i(L|.)$ and $U_i(C|.)$ define $F(x_L, g_L, x_c, g_c, W_L, W_c, i)$. The definitions of $F(\ldots)$ for the other seven combinations of coalition membership follow directly from parallel comparisons of $U_i(L|.)$ and $U_i(C|.)$.

When $i \in S$ is indifferent between L and C, we assume that i selects C if $M_c < R$ and L otherwise. The argument here is that when $M_c < R$, C could provide ε more resources to make i prefer C to L. When $M_c = R$, L typically could provide ε more resources, leading i to prefer L to C. This selection rule for indifference among members of S avoids the need for an ε-equilibrium concept, but is of no substantive importance. Hence, the selectors' strategies are best responses.

C's Selection of Coalition and Policy Provision

Given Σ, we specify the challenger's utility function. Given Σ and the nature of deposition, ouster occurs iff $|\{i \in W_L | F(x_L, g_L, x_c, g_c, W_L, W_c, i) < 0$ or $(F(x_L, g_L, x_c, g_c, W_L, W_c, i) = 0$ and $M_L < R)\}| < W$ and $|\{i \in W_c | F(x_L, g_L, x_c, g_c, W_L, W_c, i) < 0$ or $(F(x_L, g_L, x_c, g_c, W_L, W_c, i) = 0$ and $M_L < R)\}| \geq W$. We say $OUST = 1$ if these conditions hold; $OUST = 0$ otherwise. The payoffs associated with these eventualities are $\Psi + R - px_c - |W_c|g_c$ and 0, respectively.

The criteria for ouster imply two conditions on the challenger's coalition. First, $|W_c| \geq W$, and secondly, C does not need to attract the $|W_L| - W + 1$ members of W_L with the highest values of $U_i(L|.)$. Hence if $W_L \leq 2W - 1$ then C forms a coalition the $|W_L| - W + 1$ individuals who are easiest to attract and supplements this coalition with individuals with lowest $U_i(L|.)$ outside of this set to produce a coalition of size W. Define k to be the individual in W_c with the highest value $U_i(L|.)$. Note that in expectation, all $i \in S$ receive the same future payoff from the accession of the challenger. Given $F(.)$, if this individual, k, chooses C, then so do all other members of W_c. Hence gaining the support of all the members of his coalition reduces to the following programming problem in which C has to maximize his payoff conditional upon gaining the support of k.

$\max_{x_c g_c} \Psi + R - px_c - |W_c|g_c$ subject to $F(x_L, g_L, x_c, g_c, W_L, W_c, k) \leq 0$.

Since $F(., k) = U_k(L|.) - U_k(C|.)$, the constraint is equivalent to $V(x_c, g_c) - X \leq 0$, where

$$X = V(x_L, g_L) + \frac{a_{k, L}}{1 - \delta} + \frac{\delta}{1 - \delta} \left(\left(1 - \frac{W}{S} \right) V(x^*, g^*) + \left(1 - \frac{W}{S} \right) V(x^*, 0) \right).$$

This is a standard Hicksian demand maximization. The challenger's choice of public and private goods, $\tilde{x}(x_L, g_L, W_L)$ and $\tilde{g}(x_L, g_L, W_L)$, corresponds to these optimal allocations, minimizing C's expenditure subject to the constraint that $F(.,k) = 0$ given coalition size. If C enlarges his coalition this increases his expenditure. Similarly, picking alternative individuals within W_L increases the reward level he must provide. Hence when $|W_L| \leq 2W - 1$ the challenger forms a minimal coalition including all members of W_L except the $(W - 2)$ hardest to attract and he expends the minimum resources to ensure all members of his coalition are at least indifferent between him and the incumbent.

When $|W_L| > 2W + 1$ the challenger's programming problem is to minimize $M_c = px_c + |\tilde{W}_c| g_c$ with respect to x_c, g_c and W_c subject to the constraints that

$$\left| \left\{ i \in W_L \cap W_c : V(x_c, g_c) + \frac{\delta}{1 - \delta} \left(\frac{W}{S} V(x^*, g^*) + \left(1 - \frac{W}{S}\right) V(x^*, 0) \right) < U_i(L | x_L, g_L, W_L) \right\} \right| \text{ plus}$$

$$\left| \left\{ i \in W_L, i \notin W_c : V(x_c, 0) + \frac{\delta}{1 - \delta} \left(\frac{W}{S} V(x^*, g^*) + \left(1 - \frac{W}{S}\right) V(x^*, 0) \right) < U_i(L | x_L, g_L, W_L) \right\} \right| < W$$

and $|W_c| \geq W$.

We define $\tilde{x}(x_L, g_L, W_L)$, $\tilde{g}(x_L, g_L, W_L)$ and $\tilde{W}_c$ as the solutions to this problem.

Hence, C's strategy is a best response.

Since oversized coalitions ($|W_L| \geq 2W$) are off the equilibrium path we do not explicitly characterize optimal strategies under these conditions. Rather here we consider only a single strategy, which we shall refer to as C's constrained oversized strategy that is sufficient to satisfy these constraints. Characterizing this strategy places an upper bound on how much C needs to spend to oust the incumbent. While there might be a better strategy for C under some circumstances, C can never do worse than playing this strategy. That is to say, an unconstrained maximum is never less than a constrained maximum.

Specifically the constraints mean the challenger need not attract the $(W - 2)$ most loyal supporters in W_L. Define k as the individual with the $(W - 1)$th highest value of $U_i(L | i \in W_L)$. This individual supports the challenger providing

$$V(x_L, g_L) + \frac{\delta}{1 - \delta} v(m^*, W) + \frac{1}{1 - \delta} a_{S - W + 1} \leq V(x_c, g_c) + \frac{\delta}{1 - \delta} \left(\frac{W}{S} v(m^*, W) \right.$$
$$+ \left. \left(1 - \frac{W}{S}\right) u(m^*, W) \right).$$

If the challenger maintains a coalition of size W, then

$$V(x_L, g_L) + \frac{\delta}{1 - \delta} u(m^*, W) + \frac{1}{1 - \delta} a_{S - 2W} \leq V(x_c, 0) + \frac{\delta}{1 - \delta} \left(\frac{W}{S} v(m^*, W) \right.$$
$$+ \left. \left(1 - \frac{W}{S}\right) u(m^*, W) \right)$$

ensures all $i \leq S - 2W + 1$ prefer C to L. The challenger minimizes his expenditure, $M_C = px_c + Wg_c$ subject to these two constraints.

L's Selection of Coalition and Levels of Goods

Thus far we have shown the strategies described in Proposition 1 are best replies for $i \in S$ and C given the equilibrium strategies of the other players. The selectors choose the leader who offers them the highest level of rewards, with ties being decided on the basis of whether C is spending all his available resources. The greatest possible offer the challenger can make is to spend all available resources optimally on the smallest possible coalition. Again we assume all players play their equilibrium strategies in all future rounds and examine one-period deviations for L.

There are three possible types of deviation for L; she could oversize her coalition by adding members to w_L ($|W_L| > W$), she could drop a member of w_L either including someone with lesser affinity for her in place of a member of w_L ($i \in W_L | i < S - W + 1$) or not ($|W_L| < W$), she could allocate R differently ($x_L \neq x^*$ or $g_L \neq g^*$), or any combination of these. Start by examining

adding members to W_L. If $2W > |W_L| > W$, L still needs to hold individual $S - W + 1$ to retain office and C allocates R to W members of W_c, so L must allocate rewards worth at least $V(x^*, g^*) = v(m^*, W)$. However, since $v(m^*, |W_L|) < v(m^*, W)$ for $|W_L| > W$, L must spend additional resources if she enlarges her coalition. Hence adding a member to w_L is not a best reply for L.

Now consider $|W_L| \geq 2W$. Suppose in response to this, the challenger plays C's constrained oversized strategy. Remember that C could never do worse than play this strategy but could potentially do better making it even harder for L to match C's offer. Let M be the minimum resource expenditure that just exhausts the challenger's resources under the constrained oversized strategy given coalition W_L. This implies that of the following identities either $J1$, $J2$ and $J3$ hold or $J1$, $J3$ and Equation 2 holds.

$$J1 = v(M, |W_L|) + \tfrac{1}{1-\delta} a_{S-W+1} - V(x_c, g_c) + \tfrac{\delta}{1-\delta} \left((1 - \tfrac{W}{S}) \left(v(m^*, W) - u(m^*, W)\right)\right) = 0,$$

$$J2 = v(M, |W_L|) + \tfrac{1}{1-\delta} a_{S-2W} - V(x_c, 0) - \tfrac{\delta}{1-\delta} \tfrac{W}{S} \left(v(m^*, W) - u(m^*, W)\right) = 0 \text{ and}$$

$$J3 = R - px_c - g_c W = 0.$$

The latter case reduces to $J1' = v(M, |W_L|) + \tfrac{1}{1-\delta} a_{S-W+1} - v(R, W) + \tfrac{\delta}{1-\delta} ((1 - W/S)(v(m^*, W) - u(m^*, W))) = 0$, which by the implicit function rule yields

$$\tfrac{dM}{d|W_L|} = - \tfrac{\partial J1'}{\partial |W_L|} / \tfrac{\partial J1'}{\partial M} = - v_w(M, |W_L|) / v_m(M, |W_L|) > 0.$$

In the former case, the implicit function and Cramer's rules yields

$$\tfrac{dM}{d|W_L|} = - \tfrac{v_w(M, |W_L|)W(V_x(x_c, 0) - V_x(x_c, g_c)) + pV_g(x_c, g_c))}{v_m(M, |W_L|)(W(V_x(x_c, 0) - V_x(x_c, g_c)) + V_g(x_c, g_c)p)} = \tfrac{-v_w(M, |W_L|)}{v_m(M, |W_L|)} > 0.$$

Hence a single period deviation to increase coalition size harms L.

Next consider dropping a supporter, $|W_L| < W$. Then $W_c = \{1, \dots, W\}$ because C no longer has to convince a member of W_L to choose C to deny a winning coalition to L. Clearly, losing office reduces W's payoff and so dropping a member of W_L cannot be a best reply.

Replacing $j \in w_L$ with $i < S - W + 1$ increases the amount L must allocate to hold the member of W_L with the lowest value of $U_i(L|.)$ from m^* to $m' = px' + Wg'$ such that

$$V(x', g') = V(x^*, g^*) + \frac{a_{S-W+1} - a_i}{1 - \delta} + \frac{\delta}{1 - \delta} (V(x^*, g^*) - V(x^*, 0)).$$

As the last two terms are positive, $V(x', g') > V(x^*, g^*)$, and $m' > m^*$.

Different allocations of R cannot be best replies either. By definition, x^* and g^* maximize $V(m, w_L)$, so a different allocation of m^* means that $S - W + 1$ will prefer C to L when C plays its equilibrium strategy and L will be removed from office. Allocating $m > m^*$ reduces L's payoff. Allocating $m < m^*$ means that $S - W + 1$ will prefer C to L when C plays its equilibrium strategy and L will be removed from office.

Finally, combinations of these deviations from the equilibrium strategy cannot be best replies because each single deviation either reduces L's payoff or leads to L's removal. Combining one deviation with another only compounds these losses.

Refinements and Corollaries

In Proposition 1, we consider a specific coalition, $W_L = \{S - W + 1, \dots, S\}$. There are a variety of equilibria in which L forms a coalition with the same set of W individuals in each period. The key aspect of the equilibrium is that the same individuals appeared in W_L in every period. Therefore, there are $S!/(W!(S - W)!)$ coalitions that L could form, corresponding to all possible combinations of picking W individuals from S. L's policy allocations in each case solve Equation 2 and the generalized version of Equation 1 below where j is the Wth highest affinity member in W_L:

$$\left(1 - \delta \frac{W}{S}\right) V(x_L, g_L) - (1 - \delta)V(\hat{x}(R, W), \hat{g}(R, W)) - \delta\left(1 - \frac{W}{S}\right) V(x_L^*, 0) + a_{j, L} = 0.$$

Of course $j = S - W + 1$ minimizes L's expenditure and so is preferred by L to other equilibria. As the following lemma shows, when affinities are large, these alternative equilibria disappear.

Lemma 4. When $a > \delta(S - 1)(v(m^{\ddagger}, W) - u(m^{\ddagger}, W))$, where $m^{\ddagger}$ solves $(1 - \delta \frac{W}{S}) v(m^{\ddagger}, W) - (1 - \delta)v(R, W) - \delta(1 - \frac{W}{S}) u(m^{\ddagger}, W) + a_{(S-W),L} = 0$ there are no equilibria where $|W_L| = W$ and $W_L \neq \{S - W + 1, \ldots, S\}$.

Proof: Suppose L picks a coalition of size W which excludes individual $S - W + 1$: specifically consider $W_L = \{S - W, S - W + 2, \ldots, S - 1\}$. Now we show when affinity is high L prefers the coalition $W_L = \{S - W + 1, S - W + 2, \ldots, S - 1\}$ in the current period even though individual $S - W + 1$ is not a member of her long-term coalition. We present the proof only for this hardest case.[41]

If $W_L = \{S - W, S - W + 2, \ldots, S - 1\}$ is part of an equilibrium then the incumbency criterion implies L must spend $m^{\ddagger}$ to survive in office, where

$$V(m^{\ddagger}, W) + \frac{1}{1-\delta} a_{S-W, L} + \delta \frac{1}{1-\delta} (v(m^{\ddagger}, W)) = v(R, W) + \delta \frac{1}{1-\delta} (\frac{W}{S} V(m^{\ddagger}, W)$$

$$+ \frac{S-W}{S} u(m^{\ddagger}, W)).$$

Now consider a single period defection of including $S - W + 1$ instead of $S - W$ in the coalition: $W'_L = \{S - W + 1, \ldots, S\}$. To survive

$$v(m', W) + a_{k', L} + \frac{\delta}{1-\delta} a_{k', L} + \frac{\delta}{1-\delta} (u(m^{\ddagger}, W)) \geq v(R, W) + \delta \frac{1}{1-\delta} (\frac{W}{S} V(m^{\ddagger}, W)$$

$$+ \frac{S-W}{S} u(m^{\ddagger}, W)),$$

where $k' = S - W + 1$.

If

$$v(m^{\ddagger}, W) + a_{k', L} + \frac{\delta}{1-\delta} a_{k', L} + \frac{\delta}{1-\delta} (u(m^{\ddagger}, W)) > v(m^{\ddagger}, W) + a_{k, L} + \frac{\delta}{1-\delta} a_{k, L}$$

$$+ \frac{\delta}{1-\delta} (v(m^{\ddagger}, W))$$

then $m' < m^{\ddagger}$ and so L strictly prefers shifting coalition and $W_L = \{S - W, S - W + 2, \ldots, S - 1\}$ cannot be part of an equilibrium.

$$v(m^{\ddagger}, W) + \frac{1}{1-\delta} a_{k', L} + \frac{\delta}{1-\delta} u(m^{\ddagger}, W) > v(m^{\ddagger}, W) + \frac{1}{1-\delta} a_{k, L} + \frac{\delta}{1-\delta} v(m^{\ddagger}, W)$$

implies

$$\frac{1}{1-\delta} (a_{k'} - a_k) = a \frac{1}{S-1} \frac{1}{1-\delta} > \frac{\delta}{1-\delta} [v(m^{\ddagger}, W) - u(m^{\ddagger}, W)].$$

Therefore if $a > \delta(S - 1)(v(m^{\ddagger}, W) - u(m^{\ddagger}, W))$ then $W_L = \{S - W, S - W + 2, \ldots, S - 1\}$ cannot be an equilibrium. $\square$

If affinities are very large,

$$a \geq 2(1 - \delta) (\frac{S-1}{S-2W+1}) v(R, \max (W, \frac{S-2W+1}{2})),$$

there is an equilibrium where L allocates $x = g = 0$ because affinities alone hold the

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[41] This is the hardest case since rearranging the coalition only improves the affinity of the least loyal member by one position in the affinity order and $m^{\ddagger}$ is already the minimum possible expenditure for any coalition $W_L \neq \{S - W + 1, \ldots, S\}$.

loyalty of $\{S - W + 1, \dots, S\}$. This equilibrium can only occur in the case with $W < (S + 1)/2$, the holding equilibrium case.

Finally, if we drop the constructive aspect of the deposition rule and assume instead that L is deposed if she is left with fewer than W of her supporters (without restriction on the size of W_c), there are multiple equilibria in which L forms oversized coalitions ($|W_L| \geq W$) and spends $\underline{m}$ resources such that Equations 2 and 5 hold.

$$v(\underline{m}, |W_L|) - v(R, |W_L| - W + 1) + \frac{1}{1 - \delta} a_{S - W + 1} + \frac{\delta}{1 - \delta}(1 - \frac{|W_L|}{S})(v(\underline{m}, |W_L|)$$
$$- u(\underline{m}, |W_L|)) = 0, \tag{5}$$

the comparative statics of which are very similar to those of the holding coalition.[42] Since the challenger is not restricted to forming a minimal winning coalition, equilibria resembling the blocking equilibrium do not exist with the constructive aspect of the deposition rule removed.

Supermajoritarian Systems: Proposition 2

Since the logic behind the proof is analogous to Proposition 1, we only highlight the differences. First, the strategy of the selectors ensures they pick the candidate offering them the highest expected rewards. Secondly, the challenger's choice of coalition is somewhat simpler than in the initial case. The constructive nature of deposition requires that C have a coalition of at least size W. Since $W > (S + 1)/2$, picking a minimal coalition ($|W_c| = W$) also addresses the problem of ensuring the incumbent has less than W supporters. C picks the W easiest supporters to attract.

Thirdly, the incumbent's coalition is of size $S - W + 1$. This coalition size ensures the challenger must pick off at least one of her supporters. Increasing W_L beyond this size does not affect the size of the challenger's coalition and so only increases the incumbents expenditure. Given this choice of coalition, L spends just enough to maximize the best possible offer the challenger could make to a coalition of size W (Equation 4).

Comparative Statics

Case 1: $W \leq (S + 1)/2$. In equilibrium Equation 1 holds with equality:

$$I = I(m^*, W, S, R, \delta) = \frac{1}{1 - \delta}[(1 - \delta \frac{W}{S})v(m^*, W) - (1 - \delta)v(R, W) - \delta(1 - \frac{W}{S})u(m^*,$$
$$W) + a_{(S - W + 1), L}] = 0.$$

Given this incumbency criterion, the implicit function rule guarantees the following comparative statics:

$\frac{\partial m^*}{\partial W} = -I_W/I_m{}^* > 0$, $\frac{\partial m^*}{\partial S} = -I_S/I_m{}^* < 0$, $\frac{\partial m^*}{\partial R} = -I_R/I_m{}^* > 0$, and $\frac{\partial m^*}{\partial \delta} = I_\delta/I_m{}^* < 0$ since

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[42] In these multiple equilibria, the incumbent's payoff depends upon coalition size $|W_L|$. A natural refinement is to examine the incumbent's most preferred coalition size; a maximization which yields the following FOC:

$$v_w(\underline{m}, |W_L|) - v_w(R, |W_L| - W + 1) + \frac{\delta}{1 - \delta}(1 - \frac{|W_L|}{S})(v_w(\underline{m}, |W_L|) - u_w(\underline{m}, |W_L|))$$
$$- \frac{1}{S}\frac{\delta}{1 - \delta}(v(\underline{m}, |W_L|) - u(\underline{m}, |W_L|)) = 0.$$

$I_{m^*} = \frac{1}{1-\delta} [v_M(m^*, W)(1 - \delta \frac{W}{S}) - \delta(1 - \frac{W}{S})u_M(m^*, W)] > 0,$

$I_W = \frac{1}{1-\delta} [v_w(m^*, W)(1 - \delta \frac{W}{S}) - \frac{\delta}{S} (v(m^*, W) - u(m^*, W)) - $
$\qquad \delta(1 - \frac{W}{S})u_W(m^*, W) - \frac{a}{S-1}] < 0,$

$I_S = \frac{1}{1-\delta} [\delta \frac{W}{S^2}(v(m^*, W) - u(m^*, W)) + a\frac{W-1}{(S-1)^2}] > 0,$

$I_R = \frac{1}{1-\delta} [-v_M(R, W)(1 - \delta)] < 0$

$I_\delta = \frac{1}{1-\delta} [-\frac{W}{S} v(m^*, W) - (1 - \frac{W}{S})u(m^*, W) + v(R, W)] > 0.$

The comparative statics of welfare follow readily.

Case 2: $W > (S + 1)/2$. In equilibrium Equation 4 holds with equality:

$$\bar{I} = \bar{I}(\bar{m}^*, W, S, R, \delta) = \frac{1}{1-\delta} [(1 - \delta \frac{(S-W+1)}{S})v(\bar{m}^*, (S - W + 1))$$
$$- (1 - \delta)v(R, W) - \delta(1 - \frac{(S-W+1)}{S})u(\bar{m}^*, (S - W + 1)) + a_{W, L}] = 0.$$

Given this incumbency criterion, the implicit function rule guarantees the following comparative statics:

$\frac{\partial \bar{m}^*}{\partial W} = -\bar{I}_W/\bar{I}_{\bar{m}^*} < 0,$ $\frac{\partial \bar{m}^*}{\partial S} = -\bar{I}_S/\bar{I}_{\bar{m}^*} > 0,$ $\frac{\partial \bar{m}^*}{\partial R} = -\bar{I}_R/\bar{I}_{\bar{m}^*} > 0,$ and $\frac{\partial \bar{m}^*}{\partial \delta} = -\bar{I}_\delta/\bar{I}_{\bar{m}^*} < 0$

since

$\bar{I}_{\bar{m}^*} = \frac{1}{1-\delta} [(1 - \delta \frac{(S-W+1)}{S})v_M(\bar{m}^*, (S - W + 1)) - \delta(1 - \frac{(S-W+1)}{S})u_M(\bar{m}^*, (S - W + 1))] > 0,$

$\bar{I}_W = \frac{1}{1-\delta} [(1 - \delta)(-v_w(\bar{m}, (S - W + 1)) - v_w(R, W)) - \delta(1 - \frac{(S-W+1)}{S})[v_w(\bar{m}^*, (S - W + 1)) - u_w(\bar{m}^*, (S - W + 1))] + \frac{\delta}{S} [v(\bar{m}^*, (S - W + 1)) - u(\bar{m}^*, (S - W + 1))] + \frac{a}{S-1}] > 0,$

$\bar{I}_S = v_w(\bar{m}^*, (S - W + 1)) + \frac{\delta}{1-\delta} (1 - \frac{(S-W+1)}{S})[v_w(\bar{m}^*, (S - W + 1)) - u_w(\bar{m}^*, (S - W + 1))] - \delta \frac{W-1}{(1-\delta)S^2} [v(\bar{m}^*, (S - W + 1)) - u(\bar{m}^*, (S - W + 1))] - a\frac{W-1}{(S-1)^2} < 0,$

$\bar{I}_R = -v_M(R, W) < 0,$

$\bar{I}_\delta = \frac{1}{(1-\delta)^2} (1 - \frac{(S-W+1)}{S})[v(\bar{m}^*, (S - W + 1)) - u(\bar{m}^*, (S - W + 1))] + \frac{a_{W, L}}{(1-\delta)^2}] > 0.$