SPATIAL MODELS OF POLITICAL COMPETITION
WITH ENDOGENOUS POLITICAL PARTIES

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ABSTRACT

Two of the most important action selection processes analyzed in the social sciences are the choice by citizens of parties to support in elections and the choice by party leaders of policy “packages” offered to citizens in order to attract this support. Having reviewed some of the main approaches to analyzing these choices and the reasons for doing this using the methodology of agent-based modeling, we extend a recent agent-based model of party competition to deal with the birth and death of political parties, treating the number and identity of political parties as an output of, rather than an analyst-specified input to, the process of party competition. Party birth is modeled as an endogenous change of agent type from citizen to party leader, which requires describing (cumulative) citizen dissatisfaction with the history of the system. Aggregate outputs are measured in terms the mean and standard deviation of citizens’ distances from their closest party, and the configuration of party positions is characterized in terms of mean party eccentricity. Endogenous birth and death of parties transforms into a dynamic system even an environment where all agents have otherwise non-responsive adaptive rules. A key parameter is the survival threshold, with lower thresholds leaving citizens on average less dissatisfied. Paradoxically, the adaptive rule most successful for party leaders in winning votes makes citizens on average less happy than under other policy-selection rules in birth-adapted party systems.

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1. “SPATIAL” MODELS OF POLITICAL COMPETITION

Most people who analyze politics talk sooner or later about the “positions” of political actors, whether these are citizens, voters, politicians or behind-the-scenes éminences grises. It is hard to have a serious discussion about the substance of real politics without referring to “where” key actors stand on important matters. Position implies distance (between two positions); distance implies movement; movement involves direction; I can only observe and describe your movement relative to some benchmark. Thus it is difficult to analyze real political competition without using positional language and reasoning. This is why “spatial” models of political competition are seen as being among the “workhorse” models of political science (Cox 2001).

These spatial models typically involve two types of agent: citizens and politicians. Citizens have policy preferences; politicians compete for citizens’ support by offering policy packages at elections. Citizens are typically assumed to have “single-peaked” preferences over the set of potential policy packages. This means we think of an “ideal point” for each citizen, characterizing his/her most-preferred policy package in a given choice setting and describing increasingly less-preferred policies as points in some cognitive space that are increasingly far from this ideal. Although this is not a logical necessity, spatial models of party competition almost always take the set of citizen ideal points as exogenously given and mapped into a common space. The basis vectors of real-world political spaces are typically interpreted in terms of “policy dimensions” such as: economic left-right; social liberal-conservative; foreign policy hawk-dove; and so on. Each citizen is typically assumed to support the politician offering the policy package closest to his/her ideal point. Each politician is assumed to be concerned with maximizing political support. The action selection problem for citizens is to support one of the

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1 The institutional structuring of political competition into a sequence of elections described in this way is assumed (typically implicitly) to be exogenously determined.


4 More complex models can be specified, involving “strategic” behaviors by citizens that may not imply supporting the closest policy package, but these involve specifying and solving a model of post-electoral “downstream” policy implementation and we do not consider these here. See Austen-Smith and Banks (2005) for a discussion.

5 More complex models can be specified in which politicians also have primitive policy preferences, but these involve specifying a trade-off between vote-maximization and policy implementation, in addition to specifying and solving a downstream model of policy implementation – see Austen-Smith and Banks (2005). We do not consider these here.
various policy packages on offer; the action selection problem for politicians is to offer a policy packages that maximizes citizen support, subject to certain constraints. Hinich and Munger provide an accessible introduction to spatial models of party competition (Hinich and Munger 1994; Hinich and Munger 1997). Austen-Smith and Banks provide a comprehensive technical overview (Austen-Smith and Banks 2000; Austen-Smith and Banks 2005).

Most mainstream spatial models of party competition are static: key model parameters and rules of interaction are fixed exogenously; the core intellectual mission is to specify a model and solve analytically for equilibrium. Most informed observers of politics, however, see party competition as a system in continual motion, and as a system with an endogenous dynamic: what agents do at tick \( t \) of the political process feeds back to affect the entire process at tick \( t+1 \). These observations characterize political competition as an evolving complex system. Recent work in political science has analyzed this dynamic characterization using the techniques of agent-based modeling (Kollman, Miller and Page 1992; Kollman, Miller and Page 1998; De Marchi 1999; De Marchi 2003; Kollman, Miller and Page 2003; Kollman, Miller and Page 2003; Fowler 2005; Fowler and Smirnov 2005; Laver 2005). A key feature of this work is that the number of political parties competing for votes is fixed exogenously. The innovation in this present paper is a dynamic spatial model of political competition in which the number and identity of competing parties is an endogenous \textit{output} of the model, not an exogenous \textit{input} to it. We achieve this by modeling the “birth” and “death” of political parties.

In section 2, we consider arguments for deploying agent based modeling in this setting and review current agent-based models of party competition. In section 3, we specify an agent-based model of party competition that comprehends the birth of new parties and the death of existing ones. In section 4 we report results from the systematic interrogation of this model. In section 5, we set out an agenda for future research on this problem.

2. AGENT-BASED SPATIAL MODELS OF PARTY COMPETITION

The shift from formal analytical models to agent-based computational models

Although this is still not fully articulated in the literature, there are several reasons to shift methodologies from the analytical game theoretic approach of the traditional static spatial model to the systematic interrogation of computer simulations using dynamic agent based modeling, hereafter ABM. The first reason concerns the \textit{analytical intractability} of complex dynamic spatial models. Such intractability is significant if we assume a dynamic system of several parties competing in a multi-dimensional space; it becomes catastrophic when we add endogenous
parties to the mix. Using ABMs allows us to build tractable models of multi-party competition as an evolving complex system.

The second reason to shift to ABMs involves a reassessment, given the complexity of the decision-making environment sketched above, of plausible behavioral assumptions about agent rationality. Classical game theory tends to assume hyper-rational agents engaged in deep strategic look-forward; equilibrium strategy sets take account of every possible future choice that might be made by every agent. When agents are faced with potentially bewildering complexity in the decision-making environment, an alternative behavioral assumption might be more appropriate, stressing adaptive learning rather than deep strategic look-forward. On this account, a shift to ABM reflects this shift of behavioral assumption and the adoption of a methodology more suitable to modeling this. This leads to a third reason for the shift to ABM, which concerns the amount and quality of information available to decision-makers. Information requirements for many analytical spatial models are extremely high; many models, for example, assume all politicians know the ideal points of all citizens. If we assume a much lower-information world, adaptive learning models, implemented using ABM, might seem to offer a more empirically plausible description of how political agents select actions.

Thus the potential shift in methodology from analytical game theory to ABM in modeling party competition involves changing several aspects of how party competition is characterized by the analyst – from a high-information static environment populated by forward-looking strategic agents, to a low-information dynamic environment populated by backward-looking adaptive agents. The shift is thus not just a change in method, but a change in the entire characterization of political competition.

Agent-based models of party competition with an exogenously fixed set of parties

The emerging literature on agent-based dynamic models of party competition can be traced to an influential early paper by Kollman, Miller and Page (1992), hereafter KMP, who developed an ABM of two-party, incumbent-challenger, competition in a multidimensional policy space that stimulated a program of subsequent work. This work typically retains a US-oriented focus on two-party incumbent-challenger competition (Kollman, Miller and Page 1998; De Marchi 1999; De Marchi 2003). The underlying spatial characterization of policy preferences is the same as in static spatial models, although computational implementations assume agents to occupy one of a small number of discrete positions on a finite set of issue dimensions. These authors follow the traditional spatial model in assuming voters to be both policy-motivated and well-informed about published party policy positions, and to support the party position that maximizes their utility.
However, they depart from traditional static models in assuming party leaders are not perfectly informed, either about the preferences of every single voter or about the uncertainties associated with these. Instead, party leaders are assumed to gather information, using private opinion poll and/or focus group feedback, on the impact of counterfactual policy moves on their electoral support levels, using this private information to select policy positions.

KMP propose and investigate three search algorithms for party leaders. A “random adaptive party” generates a random set of counterfactual policy positions in the neighborhood of its current position and uses private polling to select the alternative attracting most support. A “climbing adaptive party” envisages a systematic sequence of small counterfactual changes to its position, selecting any change that increases it support and iterating this process. A “genetic adaptive party” uses a version of the genetic algorithm. Its policy package is described by a vector of $p$ possible policy positions on $d$ issue dimensions and treated as equivalent to its “policy DNA.” A finite set of counterfactual policy vectors is generated and subjected to the genetic operators of reproduction, crossover, and mutation, with the resulting evolved positions selected if they increase party support – again measuring this using private polling. The key feature of the KMP results is that, regardless of search algorithm (KMP 1992) and the spatial distribution of voter ideal points (KMP 1998), the two party platforms systematically converge over a series of elections to positions that are centrist yet distinct. More “rugged,” less smooth, profiles of voter preferences slow down party convergence towards the center but do not change the strong tendency for this to happen.

None of the search algorithms defined by KMP is adaptive, in the sense of selecting future actions on the basis of feasible observations of past states of the system. Each KMP search algorithm involves a form of “hill-climbing”, in the sense that politicians use some device (private opinion polling and/or focus groups) to explore, within some limited search horizon, the implications of counterfactual action-selection. The action (new policy position) that is chosen optimizes across the set of possible actions (counterfactual policy positions) that are explored. At issue here is not the intrinsic efficacy of the search algorithms, but the substantive plausibility of their information requirements. This is the basis of a critique and alternative model proposed by Laver (2005), and the problem becomes particularly evident when the KMP model is extended from a two- to a multi-party environment. Thus KMP (2003) deal with up to seven parties, and this context the counterfactual question posed by each party in each round of private opinion polling must take the form: “assume all six other parties retain their current policy packages; assume we move our own policy package from $x$ to $x \pm \varepsilon$; which party would you then support?” A battery of similar questions, generating systematic sweeps of $\varepsilon$, must be asked, by each party at
each cycle of adaptation, to select the best action in a given position. Laver (2005) argued that this is a complicated and unrealistic assumption about how parties gather information.6

The model proposed by Laver (2005) defines different, inherently adaptive, algorithms party leaders may use to select future policy positions. In each of these party leaders make use, when they select future actions, of freely available information about the history of the system. These search algorithms require no more, and often much less, information than the published policy positions of all parties, plus the levels of support these parties received, during the previous two cycles of competition. As with the traditional static spatial model, Laver assumes citizen ideal points are located in a real policy space, with citizens supporting the party with the closest policy position to their ideal point. Following a random “cold start” of the system, which is of course a model artifact, party policy positions and citizens’ party support patterns evolve continuously in the forever loop described in Figure 1. Citizens choose to support the closest party position; party leaders choose a policy position according to the profile of citizen support levels, using one of a number of adaptive rules. This process iterates continuously.

The adaptive rules defined and investigated by Laver were as follows:

- **STICKER**: never change position (an “ideological” party leader);
- **AGGREGATOR**: set party policy on each dimension at the mean preference of all party supporters (a “democratic” party leader);

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6 This is in addition to violating every professional canon on the type of question that can validly be posed in an opinion survey.
• **HUNTER**: if the last policy move increased support, make the same move; else, make a random move in the opposite direction (an autocratic party leader who is a Pavlovian vote-forager)\(^7\);
• **PREDATOR**: identify largest party; if this is you, stand still; else, move towards largest party (an autocratic party leader who seeks votes by attacking larger parties).

Programming this system as an ABM, Laver investigated the effects of different adaptive rules on party system evolution. The most striking findings relate to the Hunter rule, which significantly outperformed all other rules when it came to vote-maximization. Hunters typically beat Predators in the competition for votes and, less surprisingly, out-perform Stickers and Aggregators. Despite the fact that a Predator is programmed to become the largest party, it is typically the smallest party in competition with a set of Hunters.\(^8\) Hunters tend to go towards the center of the policy space but not to the dead center. The other striking finding was that all-Aggregator systems quickly reach a steady state in which all parties have stable positions and support levels. Such steady states are easy to perturb however, and are not robust to small random shocks to the ideal points of citizens.\(^9\) A final striking finding was that, in a system with both Hunters and Stickers, Hunters tend strongly to win more support than Stickers at the same position in the policy space. This is because a Sticker is occupying this position by applying an unconditional and non-responsive adaptive rule, while a Hunter finds the same position as a result of being rewarded with higher support, given the configuration of other parties in the system.

Laver (2005) set out to fit this theoretical ABM of dynamic party competition to empirical opinion poll data on changing levels of party support in a real party system. Taking a period (1986-1997) of Irish politics when the number of political parties remained constant at five, and using independent empirical estimates of voter policy positions and the “starting” policy positions of the political parties at the beginning of the period, the ABM was used to retrieve observed levels and variations in party support. The key parameter set in the model is the vector of adaptive rules used by each party. As a first pass, parties’ adaptive rules were inferred from their public pronouncements. Two parties (Fianna Fáil and Fine Gael) characterize themselves as

\(^7\) The biological analogy for the Hunter strategy seems to be klinokinetic avoidance – though typically in the political simulations the negative stimulus arises from the approach of another party towards the Hunter, and there is no clearly defined target for a Hunter, which seeks to occupy as large a policy “territory” as possible.

\(^8\) When there was more than one predator in the system, things were even worse for the Predators since they typically attacked each other whenever one became large.

\(^9\) This finding is consistent with theoretical existence results for “centroidal Voronoi tessellations”, which are closely analogous to the voter support clusters of Aggregator parties and are useful in understanding, among many other things, the territorial behavior of animals – see Du, Faber and Gunzberger (1999).
“parties of government” and were designated as Hunters; three parties (Labour, Democratic Left and the Progressive Democrats) describe themselves as “policy-driven” and were designated as ideological Stickers. A suite of simulations came close to retrieving the key observables using these behavioral assumptions about the parties. However, a sweep of all possible rule combinations revealed some substantively plausible combinations that generated voter support levels that better fit observed opinion poll series than results derived from the parties’ declared strategies, potentially revealing information about parties’ actual decision rules.

Overall, therefore, ABMs of dynamic party competition have replicated key findings of the canonical static spatial model of party competition (notably a tendency of parties to converge on the center of the policy space) and supplemented these with explanations of other systematically observed tendencies that are far more problematic for static models (notably a tendency for vote-seeking parties not to go to the dead center of the policy space). In addition, the dynamic models offer the possibility of explaining times series features of party support that static models, more or less by definition, are unable to address, but which are nonetheless important aspects of the ways in which informed observers describe the process of party competition. This is clearly a promising start.

3. MODELING THE BIRTH AND DEATH OF POLITICAL PARTIES

The models of party competition discussed above involve one important and unrealistic restriction; the set of political parties is fixed exogenously (in effect by Nature). The distinction between static and dynamic models of party competition has particular significance, however, once we treat the set of political parties as an endogenous output of political competition, not an exogenous input to this. Dynamic models are obviously suited to describing the evolution of the set of competing parties as an intrinsic feature of the process of competition itself.

“Static” models of party “entry”

The possibility of new political parties has not been ignored by scholars developing “static” spatial models using traditional analytical methods. There is growing body of literature that sets out to extend the classical spatial model by treating the identity and number of parties as endogenous, analyzing the comparative statics of actual or anticipated “entry” by new parties into an existing party configuration. Key features of the burgeoning literature on party entry and endogenous political parties are reviewed by Shepsle; more recent writing, mostly by economists, is discussed by Dhillon and by Austen-Smith and Banks (Shepsle 1991; Austen-Smith and Banks 2005; Dhillon 2005). The arguments put forward are surprisingly heterogeneous; party “entry” is
intrinsically related to party formation and many different rationales are assumed motivate the formation of political parties.

Müller and Strom provide an overview of different potential motivations for politicians, sorting these into those relating to policy, office, or votes (Müller and Strøm 1999). Some scholars analyzing party formation/entry have focused on the incentives for individual policy-motivated citizens to run as “citizen candidates” (Osborne and Slivinski 1996; Besley and Coate 1997). This leads to a consideration of incentives for citizen candidates to band together into parties, seen in effect as political clubs. These include: the value of a party “brand” in signaling policy positions to voters (Snyder and Ting 2002); the value of a mechanism for politicians to commit to positions other than their ideal points when offering policy positions to voters (Levy 2004; Morelli 2004); economies of scale in campaign costs (Osborne and Tourky 2004). Morelli extends the analysis beyond first-past-the-post to proportional representation electoral systems (Morelli 2004). Dealing explicitly with the entry into electoral competition by candidates rather than parties, Osborne modeled candidates who take up positions on a single policy dimension to deter entry of others into the competition (Osborne 1993; Osborne 2000).

This topic creates ferocious challenges for those working in the classical analytical tradition. One widely-cited piece on candidate entry, for example, assumes an exogenously determined number of potential candidates, typically three, and a single dimension of policy. At the end of a heavy-duty analytical discussion, the substantive conclusion is that one candidate enters the race at the median of the voter distribution and no other candidate enters (Osborne 1993). When there are more than three potential candidates, the results of the model are, according to its author, “limited”. In another example, the setting is a legislature with three members who are assumed to form political parties to maximize expectations combining one-dimensional policy and some distributive private payoff. Analysis of this model results in the conclusion that a majority party (i.e. the party with two legislators) will always form. Extending the approach to deal with legislatures with more than three members is acknowledged as presenting fundamental modeling issues that are not resolved (Jackson and Moselle 2002). Given the thinness and lack of generalizability of these results, it seems time to establish alternative lines of intellectual attack on this important problem.

Here, we extend Laver’s (2005) ABM of dynamic party competition to a setting in which the identity and number of parties is endogenous to the process of political competition. New parties are “born” as a result of this process; existing parties “die”. We model party birth as an endogenous change of agent type, from citizen to party leader, arising from dissatisfaction of an individual citizen with the existing offer of party policy positions. In this sense our model can be
seen as a dynamic ABM implementation of the intuitions of the “citizen candidate” approach (Osborne and Slivinski 1996; Besley and Coate 1997).

**Citizen dissatisfaction**

To allow us to compare our work with static spatial models of party competition, we begin by modeling citizen dissatisfaction using the classic assumption about party choice used by these models. This is based on the Euclidean distance\(^{10}\) between the citizen ideal point and some characterization of the configuration of policy positions offered by party leaders. The first big question is thus to characterize the particular configuration of policy positions on offer that citizens are assumed to evaluate. It is this that distinguishes different spatial models of politics.

The original spatial model (Downs 1957) and its direct intellectual descendents, involve a description of “proximity” voting by citizens. Each citizen is assumed to support the closest party, and to be more dissatisfied, the more distant this closest party. This assumes non-strategic behavior by citizens, who make no attempt to select an action in anticipation of the eventual consequences of this for downstream policy implementation. To do otherwise, citizens need a model of post-electoral policy implementation and must reason backwards from this model’s predictions when they select which way to vote. Models assuming citizens to do all of this have certainly been proposed (Austen-Smith and Banks 1988; Kedar 2005), but are analytically complicated, and posit extraordinarily high levels of citizen rationality and instrumental behavior. This problem is less serious in an assumed two-party system – where the implicit assumption about policy implementation is that one party wins outright implements its policy position – but grows to epic proportions in multiparty systems in which post-electoral negotiations on government formation must be anticipated. Seeking to avoid these complications, we assume citizens vote for the closest party, without getting into heavy-duty strategic calculations about the downstream implications of this.\(^{11}\)

In what follows we thus describe the level of a citizen C’s dissatisfaction, \(D_c\), during the current tick of the political process in terms of the distance between C’s ideal point and the closest party policy position on offer. If the distance between C’s ideal point and the position of

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\(^{10}\) We use Euclidean distances so the distance metric is not a source of difference between our model and others. We have the great luxury in a computational model of being able to plug in any other type of distance metric or loss function we feel is more appropriate and will make this a significant focus of future work.

\(^{11}\) Analysts with a taste for more a strategic spatial model of voter choice in large electorates are encouraged to specify such a model, analyze how voters solve it, program it, and see for themselves how all of this impacts upon endogenous party birth.
Party P is $\delta_{cp}$, then $D_c = \min_d(\delta_{cp})$.\footnote{It would be a computationally trivial matter to implement other non-linear loss functions (such as quadratic or exponential) of this distance.} The most basic measure of C’s dissatisfaction with the party system at tick $t$ of the political process $D_{ct}$, is simply the distance of the closest party from C’s ideal point at tick $t$.

**Updating citizen dissatisfaction**

It seems substantively implausible in a dynamic setting, however, to assume that action selection by citizens (in this case party support) responds only to the current instantaneous state of the system. While this is never made explicit in existing accounts of party competition, it seems more plausible to treat citizens as using their observations of the system at tick $t$ to *update* rather than completely *determine* their evaluations of the party system. In what follows, therefore, we propose a very simple updating model. We describe C’s updated dissatisfaction with the party system at tick $t$, $D^*_{ct}$, in the following recursive fashion:

$$D^*_{ct} = D_{ct} + \alpha \cdot D^*_{c(t-1)} : 0 \leq \alpha < 1$$

This type of update is analogous in some ways to updates modeled in the extensive literature on reinforcement learning, to which Sutton and Barto provide an excellent introduction (Sutton and Barto 1998). If $\alpha = 0$, then updated citizen dissatisfaction reduces to $D_{ct}$, instantaneous dissatisfaction at tick $t$. As $\alpha$ approaches unity, then instantaneous dissatisfaction at tick $t$ adds little to overall dissatisfaction, which is mainly affected by the past history of the system. If $\alpha = 0.5$, then only one percent of updated dissatisfaction contains information about states of the world earlier than $t-6$, and only 0.1 percent contains information about states earlier that $t-9$, although a small amount of information about all previous states of the world remains impounded in current dissatisfaction.

**Updated citizen dissatisfaction and party switching**

Existing dynamic models of party competition assume citizen C switches party support the instant some other party moves closer to his/her ideal point than the party currently supported. Once we move away from an “instantaneous” to an “updated” account of citizens’ evaluations of the party system, we must reconsider the assumption of instantaneously responsive party-switching. Here we assume C’s dissatisfaction with Party P at time $t$, $\delta_{cpr}$, to be a (linear) function of the Euclidean
distance between C’s ideal point and P’s policy position. Applying the updating model set out above, C’s cumulative dissatisfaction with Party P at time \( t \) would be:

\[
\delta^*_{c pt} = \delta_{c pt} + \alpha \cdot \delta^*_{c pt(t-1)} : 0 \leq \alpha < 1
\]

A “retrospective” voter in this setting would always select the party minimizing \( \delta^*_{c pt} \), updated dissatisfaction with each party. If \( \alpha = 0 \), then we model hyper-reactive citizens who always instantly switch to the closest party. This in effect is the answer adopted by existing dynamic models – the answer to a question that static models cannot comprehend. The larger is \( \alpha \), (the less the citizen discounts the past history of the system), the more ticks of the system for which a “new” closest party must remain closest to C, other things being equal, before C switches parties.

**Mean citizen dissatisfaction as a system output**

Having defined citizen dissatisfaction at tick \( t \) in both an instantaneous (\( D_{c t} \)) and an updated (\( D^*_{c t} \)) form, both the mean and the variance of dissatisfaction across all citizens are of considerable substantive interest. Mean dissatisfaction measures the extent to which party positions reflect the ideal points of all citizens. This can be interpreted as a measure of party system efficiency in adapting to the preferences of citizens, as well as a social welfare indicator for the system as a whole. The standard deviation in dissatisfaction across all citizens is a measure of the level of inequality with which the party system reflects the ideal points of citizens. Neither of these substantively interesting measures is generated by those analyzing traditional spatial models of party competition. Both are intractable to calculate analytically for an arbitrary configuration of citizen ideal points and party policy positions; both are trivial to calculate within a computational model and provide striking measures that characterize of the output of any party system.

**De facto survival thresholds for political parties**

If we treat the number and identity of political parties in the system as endogenous, we are forced to think about “threshold” support levels – below which parties cannot survive or be born. While plausible in common sense terms and by analogy with biological systems, party survival thresholds raise surprisingly complicated issues.

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13 Once more we note that other functions can very easily be programmed but we currently wish to remain as close as possible to the classical Downsian formulation.
14 Once more we note that different analysts may have different tastes about the loss function deployed. Other social welfare indices might be devised, but these all depend in some way or another on having a well-specified model of post-electoral policy implementation, which we do not consider here.
Explicit and implicit *representational* thresholds are embedded in all electoral systems, arising from the interaction of electoral formulae, constituency size and constituency-level concentrations of party support (Cox 1997), as well as matters such as campaign finance laws, the public funding of political parties, and so on. Many political parties continue to survive, however, even when they fall far below representational thresholds. An intriguing but little-studied British example is the Monster Raving Looney Party (MLRP), founded by the late Screaming Lord Sutch who, as the party’s website notes, “fought and lost more elections than anyone else in the UK’s political history”\(^{15}\) Should we include the MLRP in a model of party competition in Britain? Most sane political scientists would say no, but this involves an implicit argument that there is a set of parties “below the radar” of mainstream party competition – in the sense that whatever they do has no impact whatsoever on other, more consequential parties.

One answer to this puzzle can be found in spatial models of party competition that include a “valence” term (Ansolabehere and Snyder 2000; Adams 2001; Groseclose 2001; Aragones and Palfrey 2002; Schofield 2003; Aragones and Palfrey 2004; Schofield 2004; Schofield 2005). Valence captures “candidate quality”, viewed in a very general sense – bundling together all non-policy candidate characteristics that enhance appeal to voters. The crucial theoretical result in the present context, supported by some experimental evidence (Aragonès and Palfrey 2004) is that, if two candidates take identical or very similar policy positions, the higher valence candidate wins all the votes that are available. In these terms, we do not need to consider the MLRP because it can plausibly be assumed to have zero valence.\(^{16}\) Here, therefore, we propose a rule of thumb: parties that fall below a certain size threshold, \(T_s\), for a certain number of system ticks, \(T_t\), have zero valence and are unable to win votes in policy competition with parties having non-zero valence.\(^{17}\) Such parties are “below the radar” of mainstream party competition. They exist in some senses of the word but in another sense they are the walking dead, of insufficient consequence to affect action-selection by other parties. Parties that sink below this threshold in effect die as far as mainstream party competition is concerned. This gives us a rationale for a death threshold. What about party birth? Assuming that citizens who consider forming new parties do not want to form parties that operate below the radar of mainstream party competition, we apply the same threshold logic to party birth. A citizen contemplating forming a

\(^{15}\) www.omrlp.com.

\(^{16}\) Note that valence models do not in themselves *explain* valence, but rather *assume* valence as one of the premises of an explanation of particular party policy positions.

\(^{17}\) Note that \(T_t\) and \(q\), while dealing with different things, are intimately related to each other. In the interest of reducing the number of free parameters in the model, we will in future work make \(T_t\) a function of \(q\), so that the survival threshold number of cycles is the number of cycles it takes for the impact of past memories to decline to insignificance. (On this account, note that \(q = 0.5 \Rightarrow T_t = 9\)).
new party will not do so unless it would have achieved the threshold size, $T_s$, for the threshold number of ticks, $T_t$. As we shall see the de facto survival threshold, $T_s$ is one of the most important parameters in the model, and the remainder of this paper is to a large extent concerned with analyzing the impact of survival thresholds on party systems configurations.

### The birth of political parties

So far, we have been regarding cumulative citizen dissatisfaction as an output from a dynamic model of party competition; we now treat it as feedback into the model. Our assumption about party birth is that, the more dissatisfied a citizen, the more likely s/he is to create a new party by changing type to party leader. Specifically, we assume every citizen always has the implicit option of forming a new party by changing type from citizen to party leader. For citizen $C$, the probability of doing this is $p_c$, where $p_c$ is some function of updated dissatisfaction, $D^*_ct$, whereby relatively more dissatisfied citizens are more likely to become citizen candidates. Here, we assume that the probability $C$ will form a new party increases in direct proportion to $C$’s dissatisfaction relative to other citizens. Thus:

\[ p_c = \beta \cdot \frac{D^*_ct}{\text{mean}(D^*_ct)} \]

Substantively $\beta$ describes how sensitive to $C$’s relative dissatisfaction with the system is the probability that s/he will form a new party.\(^{19}\)

### 4. ANALYZING THE BIRTH AND DEATH OF POLITICAL PARTIES

The model set of above was programmed as an ABM and interrogated using suites of systematic simulation experiments.\(^{20}\) Before activating the birth and death of political parties in these experiments, we used Monte Carlo simulations to benchmark the impact of model parameters on our measures of system output. This benchmarking exercise is described and reported in the electronic Appendices to this paper. We have already noted that one crucial system output is mean citizen dissatisfaction with the configuration of party positions. We characterize this

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\(^{18}\) Note that, if birth and death thresholds are not set at the same level, the system will experience a secular growth, or decline, in the number of parties.

\(^{19}\) When there many citizens, $\beta$ will be very small if there are not going to be a huge number of new parties.

\(^{20}\) The original agent-based model on which the present model is based (Laver 2005) was programmed in NetLogo. Since our model of party endogenous party birth adds significant complexity to this, we ported the model to the Repast environment and programmed all extensions in the Python implementation of Repast. All programs are available from the authors.
configuration in terms of the “policy eccentricity” of each party, defined as its distance in standard deviation units from the mean of the distribution of citizen ideal points. A headline result from these benchmarking runs that will aid the interpretation of subsequent findings is that mean citizen dissatisfaction with the party system tends to be lowest when parties are on average located 1.5 standard deviation units from the center of the voter distribution – in other words when party policy positions are quite decentralized.

The impact of survival thresholds on birth-adapted party systems: citizen dissatisfaction
Our simulations of “birth-adapted” party systems were designed to explore the impact of party survival thresholds $T_s$ on: the number of parties in the system; mean citizen dissatisfaction; the spatial locations where new parties are born; and the mean policy eccentricity of all parties. Each experiment involved ten 2000-tick runs, for each parameter setting for each rule. Each run was initialized with a random-normal configuration of citizen ideal points and five parties with random policy positions, and each suite of experiments stepped through values of $T_s$, setting $\alpha$ at 0.5, $\beta$ at 0.0001, and $T_t$ at 10 ticks.

Figure 2 shows the impact of survival thresholds on the number of parties in the system, for each adaptive rule, for “ticks” 1001-2000 of the 10 simulations for each rule – that is, after any impact of the random cold-start had burnt off and the simulated party system had reached a “mature” state. The 20% survival threshold tends to result in 3- or 4-party systems, the 15% threshold in a 4- or 5-party system, depending on adaptive rule, the 10% threshold in 6- or 7-party systems, the 7.5% threshold in 8 to 10-party systems, and the 5% threshold in systems with 12-15 parties. This gives us a sense, for example, that real party systems with six or seven parties must have a de facto birth threshold, however this might in practice operate, of about 10 percent. The same basic pattern can be observed for all adaptive rules, though we expect birth-adapted systems with parties using decision rules that themselves adapt to the system state to have systematically fewer parties, since they are less likely than parties with random locations to leave pools of dissatisfied citizens. This is what we observe. Especially at low levels of the de facto survival threshold, systems with Hunters and Aggregators tend to have fewer parties for any given survival threshold. In general, all-Hunter systems tend to have the fewest parties for any given survival threshold.

21 Viewed in these terms, the benchmarking runs were a suite of experiments for which $\beta = T_s = 0$. Recall that $T_t = 10$ is consistent with $\alpha = 0.5$. The original model outputs reported in Laver (2005) are thus outputs of a special case of the current system, for which $\alpha =\beta = T_s = T_t = 0$ (there is no party birth, death, survival threshold, or updating of citizen dissatisfaction).
Figure 2 shows the impact of survival thresholds on mean citizen dissatisfaction. Citizens in birth-adapted systems tend to be almost equally dissatisfied with Stickers and Aggregators, with a tendency to be more satisfied with Aggregators when survival thresholds are high. Mean citizen dissatisfaction is much higher, and very much more variable, in all-Hunter systems, for all survival thresholds. For all decision rules, the strong pattern is for citizen dissatisfaction to increase as the survival threshold increases – higher survival thresholds imply fewer parties and a smaller chance that a citizen finds a party close to his/her ideal point.

The impact of party decision rules on citizen dissatisfaction can be explained by the spatial locations typically occupied by birth-adapted parties, summarized in Figure 4. Hunters tend to locate much closer to the center of the policy space, replicating in a more general context the pattern found by Laver (2005). Stickers tend to be found in more eccentric locations, and Aggregators in more eccentric locations still. Thus, at all thresholds, birth-adapted Hunters are tending to search for votes much “too close” to the center to minimize the mean dissatisfaction of all citizens. Although birth-adapted Aggregators and Sticker systems have similar effects on mean citizen dissatisfaction, Stickers locate a little too close to the center, and Aggregators a little too far from the center, to minimize mean citizen distances from the closest party.
Figure 3: Birth-adapted citizen dissatisfaction, by survival threshold and adaptive rule

Figure 4: Birth-adapted party eccentricity, by survival threshold and adaptive rule
Table 1 shows that enabling party birth and death can in some, but not all, circumstances make citizens more satisfied than they are in systems in which the number and identity of parties is fixed exogenously. To see this, since there is no survival threshold for party systems with no birth and death, we must compare results for the birth-adapted systems with those for benchmark systems with the same number of parties.

Table 1: Birth-adapted means of mean citizen distances from closest party, by number of parties, by adaptive rule (tick>1000)

<table>
<thead>
<tr>
<th>No. of parties</th>
<th>Sticker</th>
<th>Aggregator</th>
<th>Hunter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Benchmark</td>
<td>Birth-adapted</td>
<td>Benchmark</td>
</tr>
<tr>
<td>3</td>
<td>1.70</td>
<td>1.33</td>
<td>1.28</td>
</tr>
<tr>
<td>4</td>
<td>1.49</td>
<td>1.21</td>
<td>1.14</td>
</tr>
<tr>
<td>5</td>
<td>1.36</td>
<td>1.09</td>
<td>1.04</td>
</tr>
<tr>
<td>6</td>
<td>1.25</td>
<td>1.02</td>
<td>0.96</td>
</tr>
<tr>
<td>7</td>
<td>1.17</td>
<td>0.94</td>
<td>0.90</td>
</tr>
<tr>
<td>8</td>
<td>1.10</td>
<td>0.89</td>
<td>0.85</td>
</tr>
<tr>
<td>9</td>
<td>1.05</td>
<td>0.84</td>
<td>0.80</td>
</tr>
<tr>
<td>10</td>
<td>1.00</td>
<td>0.82</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Parameter settings: \( \alpha = 0.5 \), \( T_t = 10 \), \( \text{tick} > 1000 \).

The results are striking, and give us a clear sense of the systematic effect of enabling party birth and death. Look first at all-Sticker systems, and compare birth-enabled systems with the “static” benchmark for the same number of parties. Enabling Sticker birth and death reduces mean citizen dissatisfaction in a system for any given number of parties. This is because, while Stickers never change policy position, the citizen candidate dynamic means that parties tend to die in areas where there are fewer citizen ideal points and to be born in areas where there are more. The pattern is very different in all-Aggregator and all-Hunter systems, where we see no effective difference between benchmark and birth-adapted systems for any given number of parties. The reason is straightforward – while new Hunter or Aggregator births may occur at particular points in the policy space, the set of party positions as a whole quickly adapts away from this, so the impact of adding a new party at a particular policy location is quickly dissipated.

Enabling party birth and death under different survival thresholds thus has two quite distinct effects, each with a strong impact on citizen dissatisfaction. The most striking effect concerns the impact of the survival threshold on the number of parties in the system. Endogenizing party birth and death with lower survival thresholds makes the average citizen far more satisfied, in the sense of having an ideal point close to some party position on offer, than doing the same thing with higher survival thresholds. This holds true for all adaptive rules under
investigation and is the main systematic effect of endogenizing party birth and death. In addition, enabling party birth and death also introduces a form of evolutionary adaptation for Sticker parties, which tend to die in less popular locations in the policy space and to be born in more popular ones. This effect does not extend to Aggregators and Hunters, which tend to adapt away from their initial birth locations.

The impact of survival thresholds: locations and rates of party births and deaths
Precise coordinates of party births and deaths are of course arbitrary. More interesting is the distance of party births from the center of the distribution of citizens’ ideal points. This is the policy eccentricity of births and deaths which, under the Euclidean metric, is invariant to any rotation or substantive interpretation of the policy space. Figure 5 describes the policy eccentricity of births and deaths for various survival thresholds, for a “mature” party system (tick>1000) in which all start-up effects have been burnt off. In all-Aggregator systems, for example, party births tend strongly to occur towards the center of the space, with deaths happening towards the periphery. Looking back at Figure 4, we see deaths tending to occur precisely where Aggregators tend to be found, while births occur in a much more central location. Aggregators tend to arrange themselves in a dispersed pattern that most displeases more centrist citizens. Figures 4 and 5 thus show a continuous flow of Aggregators through a “mature” party system, with new Aggregators tending to be born towards the center then evolving towards the periphery of the space, leaving centrist citizens once again dissatisfied and the center once again ripe for party births.

The opposite pattern can be seen in all-Hunter party systems. Party births tend to occur further from the center of the space than deaths (Figure 5). This is because, from the perspective of the average citizen, Hunters are looking for votes “too close” to the center (Figure 4), leaving a larger number of dissatisfied citizens, and thus a larger number of party births, near the periphery of the space. All-Hunter systems have the most “churning” of parties – the highest mean rates of party births and deaths – with birth and death rates strongly related to survival thresholds. Hunters thus tend to be born in the periphery of the space and to adapt towards the center, leaving the periphery once more ripe for party births.

In a “mature” all-Sticker system, there seems to be no systematic pattern in the locations of births and deaths. It is also the case that births and deaths are relatively rare in a mature all-Sticker system. In this setting, most of the action arises in the immature system, immediately following a random cold start. Sticker births then systematically tend to occur closer to the center
than deaths, which is how a birth-enabled all-Sticker system adapts to citizen preferences, and tends to reach something resembling an ergodic state.

![Graphs by rule]

Figure 6: Eccentricity of births and deaths, by threshold and rule (tick > 1000)

5. DISCUSSION AND FURTHER WORK

The results reported above can be summarized quite crisply. Enabling endogenous party birth and death involves implementing a *de facto* survival threshold. Different survival thresholds imply different numbers of political parties for a given adaptive rule, with higher survival thresholds implying fewer parties. For all birth-enabled systems, the higher the survival threshold, the less eccentric the mean party location and the less satisfied are citizens as a whole. Parties using decision rules that adapt party positions tend to adapt away from their birth locations, causing a higher “churning” of party systems as the result of a continuous series of party births and deaths. We know from Laver (2005) that vote-seeking Hunters (which are in effect run by autocratic voter-seeking leaders) tend very systematically to beat Aggregators (which are in effect run according to a rule of intra-party democracy) and Stickers (which are in effect run by unresponsive ideologues) in the competition for citizens’ votes. The paradox that emerges from
these results is that, in birth-adapted party systems, it is the parties most successful in winning votes, the Hunters, who tend strongly to make citizens least happy. This is because Hunter positions tend to be closer to the center of the space, and thus less close to the ideal point of the average citizen, than the more dispersed positions of either Stickers or Aggregators.

Future work will investigate the impact of the updating parameter, stepping through values of $\alpha$ to explore the extent to which different party adaptive rules are affected by changing from an assumption that citizens switch to the party closest at any given instant, to an assumption that citizens switch only very slowly when the party they currently support is no longer closest to them. Low values of $\alpha$ correspond most closely to the implicit assumption in the current spatial model; higher values correspond to an assumption that citizens develop more long-standing party affiliations that respond only slowly to a changing configuration of party positions. Future work will also systematically interrogate the model to explore the implications with “mixed” sets of party adaptive rules, exploring the performance of rules in competition with each other in birth enabled party systems, moving on to look at the evolution of sets of rules in settings where the evolutionary fitness of a decision rule is conditioned on its past success in winning votes, before modeling the endogenous evolution of the adaptive rules themselves.

REFERENCES


ELECTRONIC APPENDIX TO PAPER

Benchmarking party system outputs

Before activating the birth and death of political parties, we use Monte Carlo simulations to benchmark the impact of model parameters on our measures of system output. These simulations involve repeated random-normal scatters of 1000 citizens and an exogenously fixed number, $p$, of party leaders in a two-dimensional policy space, stepping though values of $p$. Experimental design depended on adaptive rule. For Stickers (who never change position) an experiment comprised, for each value of $p$, 1000 independent random-normal scatters of 1000 citizen ideal points and party policy positions. For sets of Aggregators, whose positions invariably reach steady state in less than 100 ticks (Laver 2005), we ran 500 runs of 200-ticks each, for each value of $p$. The standard deviations of the estimated quantities (Table A1) imply strongly that adding more runs of the same experiment would yield substantially unchanged results. For sets of Hunters, who generate a system that never reaches steady state, we ran 10 runs of 2000 ticks each for each value of $p$. The first 1000 ticks of each run were discarded as a burn-in, and data were used from the latter 1000 ticks. Table A1 shows benchmark relationships between the number of parties, and the mean and variation in citizen dissatisfaction.

Unsurprisingly and regardless of the adaptive rule used by parties, the more parties there are, the lower are both mean citizen dissatisfaction and the variation in citizen dissatisfaction. Table A1 also shows clear differences between adaptive rules. Both rules selecting party policy in response to the location of citizen ideal points leave citizens on average more satisfied than they are in an all-Sticker system. Aggregator, which adapts party strategy to the policies of current party supporters, makes citizens on average happier than Hunter, in which party leaders search for policy positions that increase party vote share. But Hunter makes citizens happier on average than they are in a system with only policy Stickers at random locations. Aggregator is by far and away the most “egalitarian” strategy; there is substantially less variation in citizen dissatisfaction than for either of the other rules. Indeed Hunter is even less egalitarian, in this sense, than a system with randomly located Stickers.

22 The 200-tick limit for each run was very conservative because all-Aggregator systems typically reach steady state from a random start after 50-60 ticks. The 200-tick limit was set to catch any unusual system configurations for which this was not the case; none were found.

23 The 10 independent 2000-tick runs are used to generate different random scatters of the 1000 citizen ideal points, though these were not found to have a significant impact on results.
One reason why some party rules make the average citizen more satisfied than others has to do with where in the policy space each rule tends to locate a typical party. Since our distance metric is Euclidean and the basis vectors of our policy space have no intrinsic meaning, we can simply use the policy eccentricity of each party – its distance from the origin of the distribution of citizen ideal points – to characterize its spatial location.24

Table A2 benchmarks mean party policy eccentricity for each of the adaptive rules. We would not expect either the mean eccentricity of a randomly located Sticker to vary with the number of parties and indeed it does not. Table A2 shows very systematic effects for the other party decision rules. Hunters tend strongly to search for votes closer to, but not at, the center of the policy space. The mean eccentricity of Hunters increases as the number of parties increases – forcing some Hunters to hunt further from the center. Thus a three-Hunter system will find parties off-center by, on average, 0.8 SD units. In a ten-Hunter system, mean party eccentricity is nearly 1.5 SD units. Aggregators tend to be found substantially further from the center than Hunters, for any size of party system. Indeed, for systems with seven or more parties, Aggregators are typically even more eccentric than randomly located Stickers.

Figure A1 shows, for randomly located Stickers, the relationship between mean citizen dissatisfaction and mean party eccentricity, for 3-, 5-, 7- and 9-party systems, plotting each individual case for the 1000-run Sticker benchmarking for each party system size, together with the quadratic trend line for these observations. These relationships are clearly non-linear; aggregate citizen dissatisfaction tends to be minimized at intermediate levels of mean party centrality – party configurations where parties are located on average between one and two standard deviation units away from the center of the distribution of citizen ideal points.

---

24 Thus a party with an eccentricity of zero is at the mean of the distribution of citizen ideal points. The units for measuring this distance are standard deviations (SD units) of the distribution of supporter ideals, so a party with an eccentricity of 3 is close to the de facto edge of this distribution.
Table A1: Mean and standard deviation in citizen dissatisfaction (measured in standard deviation units), by number of parties

<table>
<thead>
<tr>
<th>No. of parties</th>
<th>Mean across trials of mean citizen dissatisfaction levels</th>
<th>Mean across trials of std. dev. citizen dissatisfaction levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sticker</td>
<td>Aggregator</td>
</tr>
<tr>
<td>3</td>
<td>1.70</td>
<td>1.28</td>
</tr>
<tr>
<td>4</td>
<td>1.49</td>
<td>1.14</td>
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<tr>
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<td>1.25</td>
<td>0.96</td>
</tr>
<tr>
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<td>0.90</td>
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<td>0.85</td>
</tr>
<tr>
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<td>0.80</td>
</tr>
<tr>
<td>10</td>
<td>1.00</td>
<td>0.76</td>
</tr>
</tbody>
</table>

N ticks/cell | 1000 | 100 | 10,000 | 1000 | 100 | 10,000 |

Std dev range | .271-.086 | .013-.022 | .107-.030 |

Sticker cell means calculated over 1000 independent n-party draws
Aggregator cell means calculated for tick 200 of 100*200-tick runs for each n-parties (α=0.5)
Hunter cell means calculated for tick>1000 of 10*2000-tick runs for each n-parties (α=0.5)

Table A2: Mean party eccentricity (SD units), by number of parties, by adaptive rule

<table>
<thead>
<tr>
<th>No. of parties</th>
<th>Stickers</th>
<th>Aggregators</th>
<th>Hunters</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.93</td>
<td>1.59</td>
<td>0.79</td>
</tr>
<tr>
<td>4</td>
<td>1.92</td>
<td>1.75</td>
<td>1.03</td>
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<td>1.80</td>
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<td>6</td>
<td>1.92</td>
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</tr>
<tr>
<td>7</td>
<td>1.91</td>
<td>1.93</td>
<td>1.34</td>
</tr>
<tr>
<td>8</td>
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<td>1.38</td>
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<td>1.92</td>
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</tr>
<tr>
<td>10</td>
<td>1.92</td>
<td>2.09</td>
<td>1.46</td>
</tr>
</tbody>
</table>

Range of SDs | .32-.58 | .04-.08 | .13-.22 |

Simulation design as in Table 2
Figure A1: Mean citizen dissatisfaction by mean party eccentricity, by number of parties