Contracting with the Powerful: the Politico-Economic Foundations of Feudalism

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Abstract

I model decentralized contract enforcement and exchange in an environment that captures the key elements of the steady-state equilibrium in Hafer (2001) in which property is ultimately secured through force (rather than through appeals to third-party enforcement), but in which no conflict occurs. I characterize the equilibrium of the exchange game and examine the extent to which potential gains from trade are realized in equilibrium. I show that one of the equilibrium institutions of enforcement has the structure of familiar feudal relations between successive generations of resource-owners and hired labor.

1 Introduction

1.1 The State of Nature

The present model builds directly on the model of conflict and production in the state of nature studied in Hafer (2003). Prior to the establishment of effective property rights, broadly defined, agents secure their possessions individually by force. In a decentralized setting, i.e. a setting characterized by the absence of exceptionally powerful actors, agents are roughly similar in their ability to use force in the sense that each agent is capable of depriving any other agent of her possessions, although doing so is costly. Thus, in the absence of prohibitive punishments from third-party enforcers, engaging in conflict to obtain or retain the use of goods is a viable and economically rewarding alternative to production.

Thus, in the state of nature, agents allocate their time between productive and appropriative activities. Appropriation is modeled as an incomplete information war of attrition between two randomly matched players who desire the same prize, in this case a scarce input necessary for one of the production technologies. (I refer to this input as "land."[1]) The game is essentially one of outlast. Both agents attempt to claim the good by committing their time to the "conflict" over it; the conflict ends when one of the agents surrenders her claim to the prize by devoting her time to something else.[2] The remaining player wins the prize and can then also devote her time to another activity (e.g. production).

I posit as the prize an input to production that actually enables agents to use an entirely different (and usually superior) technology for producing the consumption good. Because the capital input is durable and affects agents' productive opportunities, and hence income, the evolution of the distribution of the capital good across the (heterogeneous) population changes the incentives that agents face over time. Positing conflict over an input rather than an output is desirable at a later stage of the analysis because it allows us to examine the role of institutions in matching inputs to production as well as in securing property. Lastly, because the prize and the agent's labor are complementary inputs to production, and because the value of the prize is derived solely from its role in the production of the consumption good, the value of the prize to the contestants - in a departure from the extant literature on war-of-attrition - actually decreases as they vie for it: more time committed to the conflict means less time to allocate to production, which means a lower marginal productivity of the land.

To study the effects of the economic incentives produced by the inherent comparative and absolute advantages of individuals with different skills or aptitudes, the population of agents is assumed to be heterogeneous in two different types of abilities.[3] These abilities are relevant to different modes of production, one of which

[1]I choose to call this input land because of the contemporary relevance of land as an input and of conflicts over the possession and use of land in developing economies (see, for example, de Soto (1989)) and because of its historical importance in the development of English Common Law.

[2]Note that, although the game has been dubbed the "war" of attrition, the opponents are not posited to be in actual physical conflict. They are simply devoting their time to the means of winning the dispute.

[3]The introduction of types that vary on two dimensions rather than one complicates the usual incomplete information war of attrition in a new way. In their model of firm exit, Fudenberg and Tirole (1986) assume that firms vary in the sum of their opportunity costs and their fixed costs of production. This factor affects both the magnitude of the firm's losses as a duopoly and the firm's monopoly profits, thus the costs of engaging in conflict and the value of winning are inversely related. This assumption is suited to the study of duopoly competition because the cost of conflict and the rewards of winning are determined by the same activity: producing the same good with the same technology. Assuming such a relationship between the costs of conflict and the rewards of winning is unwarranted here, because in the state of nature, the means of acquiring land is entirely independent of the means of using it. Thus the model presented here is more general in the sense that the costs of engaging in conflict need not be correlated to the value of the prize; indeed, here they are taken to be independent. Additionally, because the prize and the agent's labor are complementary inputs to production, and because the value of the prize is derived solely from its role in the production of the consumption good, the value of the prize to the contestants actually decreases as they vie for it: more time committed to the conflict means less time to allocate to production, which means a

requires a complementary input ("land") and one of which requires only labor. Thus the model here provides the potential for gains from trade and from the specialization of economic activity, enabling the study of the development of institutions governing exchange.

An agent's type is known only to herself, although the distribution of types from which the population is drawn is common knowledge. The introduction of types that vary on two dimensions is important, as it serves as the basis of potential gains from trade and captures difference between the means of acquiring land and the means of using it.[4] The costs of engaging in conflict need not be correlated to the value of the prize; indeed, here they are taken to be independent.

In order to study the connection between primitive economic parameters and the feasibility of different kinds of decentralized institutions, it is necessary to provide a repeated game, and preferable to provide an infinitely repeated one. Repeating the sequence of play of the one-shot game discussed above produces a dynamic infinite sequential game, where agents' payoffs, and thus their optimal strategies, change from period to period in response to the evolution of the distribution of agent types among those who possess land and those who do not. The evolution of the distribution of land across agent types is, however, itself a consequence of the strategies agents choose. Thus the game that results from repeating the basic sequence of play of the one-period game provides a richer notion of the macroeconomic efficiency and distributional consequences of the state of nature than could be obtained from repeated games that lack this kind of dynamic component.

The initial stage of this dynamic model is characterized by commonplace conflict and the uncertainty of future possession of land. I show, however, that in the steady-state equilibrium of this game, attained in finite time, the population of heterogeneous agents separates into two permanent groups (of the resource "haves" and "have-nots") in accordance with the known function of their primitive types. Moreover, once the separation is sufficiently complete, no agent finds it in her interest to challenge another agent for possession of the resource. Thus, in the steady-state equilibrium, there is no conflict and the agents enjoy *de facto* security of property.

Although this security has some of the desirable characteristics of a socially legitimated and enforced right to the security of property, such as providing incentives to engage in productive activities and even to invest, it has a critical short-coming: it does not necessarily support voluntary exchange. Because the basis for the security of property is the landholders' group reputation for their willingness to fight, combined with the inability of landless agents to distinguish between landholders who are more and less willing to fight, security can be undermined by weakening the group reputa-

lower total productivity of the land. This constitutes an economically relevant departure from the literature on war-of-attrition games.

[4]Consider the example of rationing by waiting discussed at length by Barzel (1989). The cost of waiting in line is not necessarily related to the value an individual places on the good being distributed; it is determined primarily by the opportunity costs of her time.

tion or by identifying those landholders who are less likely to fight for their continued possession of land. Reallocating land through voluntary exchange may weaken the group reputation by replacing landholders who succeeded in obtaining and retaining land through conflict with agents who lost such conflicts. Landholders who attempt to engage in exchange may also endanger their own security of property because, by indicating a willingness to exchange, they are revealing themselves to be of low types relative to other landholders. Thus the pattern of ownership that emerges spontaneously in the state of nature differs from a property right not only in its formal, legal status but in its immediate economic consequences. Inefficiency persists in the presence of a stable ownership pattern because factor goods are not allocated efficiently. Essentially, the distribution of land that comes about through conflict in the state of nature does not always favor those agents who have the greatest ability to make use of it. Agents' attempts to remedy this misallocation through trade are complicated by their inability to commit credibly to refrain from expropriating the goods they have just "sold." When an agent loses her land through conflict, she has no incentive to return to challenge the victor again; she has just discovered that that particular agent is willing to commit more time than she is to securing the land. However, when an agent agrees to cede possession of land to another agent without conflict, she may find it in her interest to renege by re-expropriating the land. Thus it is apparent that inefficiency arises not only from the allocation of time to unproductive activities but also from the inefficient allocation of factor goods. Agents have incentives to find ways to realize gains from trade.

1.2 The Emergence of Institutions of Exchange

In the model below I consider the possibility of efficiency-enhancing self-enforcing contracts in the steady-state of the state of nature between the resource- "haves" and "have-nots." In the stage of the game following the attainment of the steady-state, agents can contract for the use of a capital input for a more productive technology. I present a characterization of the equilibrium contracts for given combinations of primitive economic parameters and identify the conditions under which the security "rights" of the state of nature may be supplemented with partial alienability (understood as *protected lending* or use rights). I also prove results that characterize the relationship between the dynamics of the state of nature conflict and the efficiency potential of the feasible steady-state trades.

Individual agents have incentives to create institutions that guarantee alienable property rights because, by doing so, they can generate additional units of the consumption good by better matching land and labor inputs. The process of creating institutions can be viewed as the efforts of self-interested agents to constrain themselves (i.e., own potential demands for re-negotiation) and others (potential challengers) so that they are able to engage credibly in voluntary trade.

Consider the different incentives that agents face in the equilibrium described in the previous section. In the steady-state, landholders are never challenged and thus have no need to create further deterrence. However, landholders with relatively low productivity in the land technology could benefit from voluntarily alienating their land to agents with high productivity in exchange for payments in the consumption good, if they were able to create an institution that could guarantee the contract (or if landless agents were able to accumulate goods and pay the seller up front).

Short of the ability to exchange land outright (which creates a set of unique challenges not formally treated in this paper but discussed in some detail in the conclusion), agents may exchange labor or consumption goods and the use of land: a landholding agent may offer a landless agent the use of her land in exchange for a payment at the end of each period. Intuitively, the ability to realize gains from voluntary exchanges of this kind depends crucially on the fact that the payments to the landholding agent are distributed over time, in an infinite stream of rents. This payment arrangement guarantees the lessee's right to use the land by creating incentives for the lessor not only to refrain from evicting her but also to continue to repel challenges from third parties, incentives that the lessor would not have if she were paid in advance or if she completely alienated all residual claims to the land. Provided that the landholder believes that her tenant will pay the agreed sum at the end of the period, she respects her tenant's claim to the use of the land because evicting her would deprive the landholder herself of the agreed payment. The landholder also continues to defend her possession of the land against any challengers: because the landholder voluntarily agreed to the contract, her utility from having land and a tenant must be greater than her utility from having land without a tenant, and hence her willingness to commit time to conflict in order to maintain possession of the land must be greater when she has a tenant than when she does not. A side effect of her defending her own claim to the land (and rents proceeding from it) is her defense of the tenant's use of the land against encroachment by a third party. This feature of the arrangement constitutes an important aspect of a right, namely the social aspect of its enforcement.

As already mentioned, the landholder's incentives to behave in the way just described is conditioned on the expectation that the landless agent will actually meet her contractual obligations at the end of the period. The model below shows that her willingness to pay is induced by a costless and credible promise from the landholder to offer her progeny contracts in the future if and only if she behaves "honestly," i.e. if she fulfills the terms of any contract she accepts and, conversely, if she declines any contracts she would not ultimately fulfill.[5]

The incentives to fulfill contracts induced by this promise are not necessarily sufficient to support all potentially profitable exchanges in a given economy. The model below gives rise to several observations regarding the ability to engage successfully in exchange based on numerical estimations of the derived equilibrium behavior. In

[5]In Chapter 3, agents live for one period but care about the welfare of their progeny, whereas in Chapter 2 they are infinitely-lived and have no offspring.

economies in which land is less abundant or the maximum productivity with land is greater, a larger proportion of landholding agents enjoy effective transferability of their use rights.

2 The Model

2.1 Stage 1: Conflict and Production in the State of Nature

Before play begins, Nature selects a finite population of agents, $N + L$, from a two-dimensional continuum of types distributed with constant density. Agent i's type is characterized by the vector (α_i, β_i), where α_i is the marginal productivity of the agent's labor using technology A, and β_i is the marginal productivity of the agent's labor in technology B.Nature then chooses $L < N$ of these $N + L$ agents and endows them with land. Agent i's possession of land is indicated by $l_i = 1$, where $l_i \in \{0, 1\}$. Henceforth, agents who possess land will be said to be members of set L, and the remaining agents will be called members of set N. Agents' skill types do not change, but agents can move from set L to set N and vice versa.

Agents know the distribution of types from which the population is drawn, but they do not know the distribution of types in the realized finite population. Agents believe that the distribution of types in the population mirrors the distribution of types from which the population was drawn: α and β are assumed to be independently distributed over the intervals $[a, b]$ and $[c, d]$, respectively, with constant probability densities

$$\begin{aligned} p(\alpha) &= \frac{1}{b-a} \\ p(\beta) &= \frac{1}{d-c}. \end{aligned} \tag{1}$$

On the basis of these densities, agents can form expectations about the distributions of types in N and L in each period.

Each agent is endowed with T units of time per period, t of which can be allocated to appropriating or securing land, and $T - t$ of which to producing the consumption good. Two production technologies are available. Only technology A requires land; both A and B require labor, although they require labor of different kinds, as signified by the distinction between α and β.

$$\begin{aligned} A(\alpha, t, l) &= \alpha l(T - t) \\ B(\beta, t) &= \beta(T - t) \end{aligned} \tag{2}$$

I assume that the lowest (highest) productivity with technology A is at least as great as (greater than) the lowest (highest) with technology B (formally, $a \geq c$ and $b > d$). Thus, although ranges of productivity overlap for the two technologies, technology A

is generally more productive than technology B. I assume in what follows that it is also the case that $\frac{d}{c} < \frac{b}{a}$, which, roughly speaking, insures that the range of agents' productivities with respect to technology A is greater than with respect to technology B.[6]

At the beginning of each period, Nature randomly matches agents who do and do not possess land, i.e., agents in set L with agents in set N. Note that $N - L$ agents will not be matched with opponents; these agents have no opportunity to acquire land that period and allocate the entire period to producing the consumption good with technology B. The agent who does not possess land may attempt to acquire it by taking it from the other agent; in the absence of any institutions to enforce the claim of one or the other to the land, the agents must settle the dispute through open conflict, here modeled as a war of attrition, or second-price all-pay auction. Each agent chooses her strategy t, the maximum amount of time she is willing to commit to an effort to obtain land, i.e. her bid for the land. The agents invest their time in the conflict starting at the beginning of the period. When one agent quits, the other agent is able to observe her opponent's exit and stops devoting time to the conflict; thus the winner i pays her opponent's bid t_j, which is less than her own, $t_j < t_i$. After the possession of land is settled, agents devote their remaining time to production. Because technology $A(\alpha, t, l)$ requires land, only the winner can use it[7]. Note that only agents for whom $\alpha > \beta$ have incentives to engage in contests for land; agents with $\alpha \leq \beta$ concede defeat immediately.[8] If both agents commit the entire period to conflict, the landholder retains possession of the parcel.[9] It is assumed that parcels of land cannot be divided among agents and that there are no further challenges to the winner's possession of the land until the next period, when Nature randomly matches landholders and landless agents again.[10]

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[6]The substantive results are robust to relaxing this assumption, which is made purely for the sake of mathematical tractability.

[7]Recall that any agent may use technology B; possessing land does not bar an agent from choosing technology B over technology A. Only agents with $\beta \geq \alpha$ would make such a choice, however, and they are not likely to possess land after the first period, since it is not in their interest to engage in conflict for it.

[8]If Nature endows an agent for whom $\alpha < \beta$ with land and then matches that agent with another individual for whom $\alpha < \beta$, the land lies fallow for the first period. Both agents use technology B(.). The final distribution of the land between them is irrelevant.

[9]This assumption captures the notion that the capital good is already in some agent's possession; a challenger must actually wrest control of the resource from its current holder in order to make use of it herself. The common assumption of a probabilistic tie-breaker (flipping a coin) is less appropriate in this context because it treats the resource as if it were entirely in the public domain.

The results presented in this chapter are robust to allowing conflicts to continue into subsequent periods. The continuation of conflict is made possible by matching contestants who fight until the end of the period with each other again in the next period, rather than with new opponents. Because this extension complicates the recursive definitions of the probability distributions of agent types considerably, while yielding the same substantive results as the basic model, the simpler assumption is used here.

[10]The assumption that these parcels are of a fixed size should be understood in conjunction with

Agents obtain utility only from consumption; agents desire land only as a means of producing the consumption good. No saving is possible. The per period payoff for agent i when matched against agent j is

$$u_i = \begin{cases} \beta_i(T - t_i) \text{ if } t_i < t_j \\ \alpha_i l_i(T - t_j) \text{ if } t_i > t_j \end{cases} \tag{3}$$

Agents consider the sums of their discounted expected future payoffs when allocating their time between production and conflict, not just their single-period payoffs, i.e., the utility of i is the sum of discounted future u_i across all periods.

Repeating this sequence of play produces a dynamic infinite sequential game, where agents' payoffs, and thus their optimal strategies, change from period to period in response to the evolution of the expected distribution of agent types among those who possess land and those who do not. The evolution of the distribution of agent types is, however, itself a consequence of the strategies agents choose. Modeling this dynamic element explicitly permits the direct analysis of the long-term effects of the absence or failure of rights-enforcing institutions on the level of conflict and on the distributions of both capital goods and income. This kind of dynamic element also provides a natural way to explore the emergence of *de facto* property rights, as speculated by Skaperdas and Syropoulos (1995) and Sugden (1986).

Let $t(\alpha, \beta)$ denote the period (sub-)strategy for a player of type (α, β) in the symmetric Bayesian equilibrium of this game, i.e. the solution to the following optimization problem for every agent in the economy. Each agent i, when matched with agent j, must choose the amount of time t_i she will devote to conflict over the possession (and use) of land. Recall that she does not know j's type; thus, she faces an expected utility maximization problem:

$$\begin{aligned} \max(\Pr\left((\alpha_j, \beta_j) \mid t(\alpha_j, \beta_j) > t_i\right) \left[\beta_i(T - t_i) + E[U^N(\alpha_i, \beta_i)]\right] \\ + \int_0^{t_i} \Pr(\left(\alpha_j, \beta_j\right) \mid t(\alpha_j, \beta_j) = t)[\alpha_i l_i(T - t) + E[U^L(\alpha_i, \beta_i)]]dt). \end{aligned}$$

The first order condition is

$$\begin{aligned} \Pr(\left(\alpha_j, \beta_j\right) \mid t(\alpha_j, \beta_j) = t_i) \left(\alpha_i l_i(T - t_i) + E[U^L(\alpha_i, \beta_i)]\right) \\ + \Pr(\left(\alpha_j, \beta_j\right) \mid t(\alpha_j, \beta_j) = t_i) \left(\beta_i(T - t_i)E[U^N(\alpha_i, \beta_i)]\right) \\ - \beta_i \Pr(\left(\alpha_j, \beta_j\right) \mid t(\alpha_j, \beta_j) > t_i) = 0. \end{aligned}$$

the form of the production functions. The implicit idea embedded in the form of these assumptions is that having more land reduces the amount of labor per unit of land, producing the same outcome. A single individual with a limited amount of labor farms her plot; increasing the size of the plot reduces the amount of labor she can devote to each unit of land, reducing the per unit productivity of land. These assumptions are consistent with the economic activities described by Umbeck (1977), de Soto (1989) and others.

A key step in solving this problem is finding the expression that relates α to β along level curves of t. These level curves are the ratio of the additional product per unit of labor with land to the opportunity cost of obtaining land.

$$k_i = k(\alpha_i, \beta_i) = \frac{\alpha_i - \beta_i}{\beta_i}$$

For simplicity of notation, let $s_t^L(k)$ denote the equilibrium strategy, i.e. the amount of time devoted to conflict, of an agent of type k in group L in period t (and, similarly, $s_t^N(k)$ is the strategy for the agent in group N). The equilibrium strategy is increasing in k, so agents with a higher ratio of their additional product per unit of labor with land to their opportunity cost of obtaining land are willing, in equilibrium, to devote more time to obtaining (or keeping) land.

Initially, the expected distributions of agent types are identical in N and L, but after the first round of contests, both the expected and the actual distributions of types are different among agents who possess land (the winners), L, than they are among those who do not, N. Consequently, agents of the same type in different groups have different optimal strategies after the first round of play, since they assign different probabilities to their opponents' being of any given type. An agent without land who challenges a landholder knows that her opponent won in the previous period; similarly, the landholder knows that, if her opponent fought in the previous period, she must have lost. Thus landless challengers expect their opponents to be relatively "tough," and are less likely to find it worthwhile to devote time to conflict as, with each round of conflict, their landholding opponents get (on average) tougher and tougher. Likewise, landholding agents expect their opponents to give up relatively quickly, on average, and thus they find it more worthwhile to devote a little more time to conflict. To avoid the proliferation of unnecessary notation, I assume that agents remember neither the identities nor the actions of past opponents.[11]

The principal results are summarized in the following theorem:

Theorem 1 *(Hafer 2003)*

1. *Both $s_t^L(k)$ and $s_t^N(k)$ exist, are unique, and are increasing in k.*
2. *In a finite number of periods, L and N converge to L^* and N^* such that, $\forall i \in L^*$ and $\forall j \in N^*$,*

   $s^L(k_i) > s^N(k_j)$.

   $\forall k \in N^*$, $s^N(k) = 0$.
3. *For combinations of primitive parameters such that L and N converge to L^* and N^* more quickly, agents are sorted into L^* and N^* less completely.*

[11]Relaxing this assumption to allow perfect recall does not affect the main substantive results.

2.2 The Allocation of Factor Goods

Note first that agents with the same ratios of net marginal productivity with land to opportunity cost of conflict, $k = \frac{\alpha-\beta}{\beta}$, earn different payoffs from production because of the differences in their underlying productivities (α and β). Because agents' success in conflict, and hence the possession and use of land, depends on $\frac{\alpha-\beta}{\beta}$ rather than $\alpha - \beta$, there is a discrepancy between the steady-state equilibrium allocation of land and labor and the efficient allocation of land and labor. Note that, although the form of k is a result of this particular model, e.g. of modeling conflict as a war of attrition and of the particular forms of the production and utility functions, the substantive conclusion is robust. For any means of determining who will use a resource that does not depend *solely* on the agents' valuations of that resource, i.e. any means that does not systematically direct the resource to its highest value use, allocational inefficiencies will result.

It is intuitive that agents' optimal strategies in any contest in which their participation is costly will depend on both their valuation of the prize and the costs of their participation. In the model presented above, their valuations of the prize depend on their ability to derive enjoyment from consumption (their utility functions), the technology that transforms resources into consumption (the production functions), and their particular productivities with those technologies (their types α and β). The costs of their participation in the contest are determined by the manner in which participation consumes resources and the costs of those resources for the agents. The war of attrition requires the participants time, and the cost of that time for an agent is the utility she would derive from the next best use of that time, in this case, the utility she would derive from the consumption good she could produce using the B-technology with her productivity β. It is the costliness of the conflict over the resource that causes the discrepancy between the ultimate distribution of the resource that results from conflict and the efficient distribution of the resource. Note that the addition of a third dimension to the agents' primitive types to capture any differences in their ability to use the conflict technology, e.g. differences in their primitive abilities to fight, would not affect the conclusion that inefficiencies are the natural result of allocating goods through conflict. It would simply introduce another factor, the differences in the rates at which agents' participation in conflict consumes their resources, in addition to the differences in their opportunity costs of using that resource, into the determination of the costliness of conflict. The existence of agents' optimal strategies for engaging in conflict is all that is necessary to imply that conflict will result in the sorting of agents into groups of "haves" and "have-nots" according to some composite type, a function of the primitive types, that determines their choice of strategy in conflict. It follows that allocational inefficiencies of the sort that result from the model presented here are robust to a wide range of conflict and production technologies and to the heterogeneity of the population in aspects relevant to them.

These allocational inefficiencies are exacerbated by the fact that, ultimately, agents do not sort completely even with respect to the composite type, k. Incomplete sorting exacerbates inefficiency in expectation because, as discussed above, agents' strategies in conflict do depend (positively) on their valuation of the prize. Thus the allocation of the resource through conflict does correlate, however imperfectly, with the efficient allocation, whereas the assumed initial allocation was completely uncorrelated with it. The extent to which agents are sorted according to the composite type (k) in the steady-state depends on how quickly the gap between the conflict strategies of identical types in L and N, $s^L(k)$ and $s^N(k)$, widens, and hence on how quickly the system converges. The more quickly the gap widens, the more quickly the system converges and the more incomplete the sorting; thus the less severe are the losses due to the allocation of time to conflict and the more severe are the losses associated with the misallocation of land across agents.

Figure 2 illustrates the case where agent types (α, β) are evenly distributed on $[1, 4] \times [1, 3]$. For any given agent $(\alpha, \beta)_i$, we can identify the level curve $k(\alpha_i, \beta_i)$ on which she lies. To the left of this level curve lie all the agents who, in expectation, will lose to agent i in conflict, i.e. the portion of the population over which i would be victorious in the first-period conflict. To the right of this contour are all the agents who will defeat agent i.

To see that there are potential gains from trade, look at the areas bounded by the level curve of k and the level curve of $(\alpha - \beta)$ through some $(\alpha, \beta)_i$, where i is, in expectation, equally likely to become a member of L^* and N^*. In Figure 2, these level curves are distinguished from other level curves of k and $(\alpha - \beta)$ by darker lines. $(\alpha_i - \beta_i)$ is the additional quantity of the consumption good that individual i produces with technology A (instead of technology B). Land is most valuable to agents who have the highest differences $(\alpha - \beta)$. In the diagram, it is clear that there are agents with $(\alpha - \beta) > (\alpha_i - \beta_i)$ whom agent i defeats. Likewise, there are also agents with $(\alpha - \beta) < (\alpha_i - \beta_i)$ to whom i loses. Thus it is apparent that inefficiency arises not only from the allocation of time to unproductive activities but also from the inefficient allocation of factor goods. Agents have incentives not only to secure property and enforce peace, but also to find ways to realize gains from trade by constraining themselves to abide by their agreements.

This brief example shows some of the potential of this model for analyzing the impact of basic economic parameters on the security of property and the allocation of resources in the absence of true property rights. Two sources of potential gains in an individual agent's production, and thus in utility, have been identified: reducing the time allocated to conflict (before convergence to the steady-state equilibrium) and engaging in mutually beneficial voluntary exchanges. The magnitude and commonality of these incentives depends directly on the primitive features of the economy (the technologies, the amount of land, the number of agents, and the distributions and ranges of α and β).

2.3 Stage 2: Self-Enforcing Contractual Exchange

This stage begins in the steady-state equilibrium of the conflict and production game; that is, through successive rounds of conflict, agents have sorted by a function of their primitive types into groups L^* and N^*. Two features of the steady-state equilibrium are critical in motivating the following analysis: no conflict occurs, giving rise to a de facto system of secure possession; and the allocation of the input to production across agents is inefficient, creating potential gains from trade. The following model assumes an allocation of land that is both inefficient and stable in the sense that no agent can benefit from attempts to expropriate land from another. The interesting questions are, then, "to what extent can agents realize these gains from trade in a decentralized setting?" and "how does this ability vary with changes in the economic primitives of the model?" Rather than supposing that agents are infinitely lived, I assume that each agent lives for one period and has one offspring, whose utility enters the parent's utility additively with discount $\delta < 1$. Formally, let u_i^g be the consumption of the gth generation progeny of agent i, with the understanding that the gth generation lives in period $g + t$, where t is the period in which i lives. As before, agents' utility is derived solely from consumption. Thus, agent i's expected utility

$$E[U_i] = u_i + \delta E[u_i^1] + \delta^2 E[u_i^2] + ...$$

I assume that the type of i's progeny is a realization of a random variable distributed according to the expected distribution of types at the end of the previous period in the group (L or N) into which he is born (i.e., the group in which the "parent" ends up as a result of conflict and production decisions in his own period). Thus, the probability of i's progeny, i^g, being of type (α, β), $\Pr((\alpha, \beta)_{i^g} = (\alpha, \beta))$,is the probability density of (α, β) in i's group, $p^L(\alpha, \beta)$ or $p^N(\alpha, \beta)$. Agents' types are, thus, assumed to be a function of "nurture" rather than "nature."[12] It bears emphasizing that the assumptions on the timing and nature of reproduction imply that agents' expectations about each other's types and about the types of their own and others' progeny are always commonly known (though the knowledge of their own type is certain and private).

Unless otherwise noted, I abstract below from the possibility of incomplete sorting and focus on the consideration of the baseline case where $\forall i \in L$, $j \in N$, $k_i > k_j$. In

[12] The model in Chapter 2 can be interpreted as the case where the types of offspring are assumed to be a function of nature alone (i.e., of the parent's type - indeed, fully induced by the parent's type). Which assumption is more appropriate is, of course, contestable. If one takes a Hobbesian interpretation of life in the state of nature, then reaching a no-conflict steady-state may be plausibly seen as bringing about a qualitative change in the reproductive regime: enabling the memetic transmission of productive "skills" which is effectively impossible under the conditions of active conflict.

There are also methodological reasons for making the present assumption. As will be seen momentarily, it buys a simplification (common expectations of successive generation types) that allows us to focus the analysis on the primary issues of concern.

addition to increasing the tractability of the characterization of equilibrium contracts offered in this economy, this assumption has also a distinct substantive rationale. Incomplete sorting allows for the possibility that in making a contract offer to a potential renter, a landholder reveals enough information about her type to induce interest in challenging her for the possession of the land, whereas certainty of being able to retrieve the leased resource has been identified as a critical factor responsible for the viability of customary exchange institutions and use rights (see, e.g., Oaul 1993). It is convenient to denote the lowest type k in L, or, equivalently, the highest type k in N, by k^*, i.e. k^* such that $\Pr(k < k^*) = \frac{N}{N+L}$.

The sequence of actions is as follows. At the beginning of the first period, Nature randomly matches a landholding agent, i^1, and a landless agent, j^1(in what follows I suppress the generational superscripts when it creates no ambiguity). The excess landless agents who are not matched with landholding agents engage in production with technology B for the entire period. Each pair of agents may then engage in a one-sided, take-it-or-leave-it bargaining procedure over a simple, non-binding contract. Agent i may offer j a rental contract $r \in R^+$. The terms of this contract are the following: i allows j the use of the land in the current period in exchange for j's payment of rent r at the end of the period. If j accepts, agent i devotes the period to production with technology B. Agent j may challenge i, or, if offered a contract r, j can accept or reject it. If j rejects the contract and chooses not to fight, she returns to production without land (i.e., production with technology B). Given the assumption of complete sorting and the definition of k, j anticipates losing and thus will prefer not to challenge. (By the assumption on the nature of reproduction, this will always be the case, and so in what follows, I suppress the possibility of a renter's challenging a landlord in the interests of brevity). Therefore, j's only relevant options are accepting the contract and producing with land or rejecting the contract and producing without it. If j accepts the contract, then, at the end of the period, she chooses whether or not to pay r. Following payment or non-payment, i and j consume the goods in their possession at the time, and j chooses whether to "move," which in effect amounts to deciding whether j's progeny, j^2, will make herself available to i's progeny, i^2, for another possible offer in the following period. An alternative (but logically equivalent) assumption is that following j's decision to pay or not to pay, i chooses whether to attack j (with immediate success guaranteed) and to force her off the land.

At the beginning of the next period ($t = 2$), agents who choose not to stay return to the pool of agents to be matched randomly by Nature. Let M be the total number of landless agents who have chosen to stay with their respective landholders. Nature, then, randomly matches $\min\{L, N - M\}$ agents from the pool with landholders so that at most one agent from the pool is matched with each landholder, no agent is matched with more than one landholder, and matches are "as even as possible" (i.e., new matches go to the unmatched landholders until all landholders have at least one match, after which they go the landholders who have only one match, etc.). After i^2 is matched with any additional agent h^2, she chooses whether to make a contract offer and to whom (h^2 or j^2 if j had not moved at the end of $t = 1$). The game then continues as in period 1 (see Figure 3). The sequence of events in each subsequent period is identical to that in $t = 2$.

Although the bargaining procedure[13] and form of contract are assumed, it is evident that some of the more obvious alternatives would be less feasible from the standpoint of the actors. Outright purchase of land is problematic because the seller cannot credibly commit to refraining from re-expropriating the land after receiving the payment, and hence potential buyers would be unwilling to engage in such a transaction. The same commitment problem plagues rental agreements in which payment is made up front: the landholder could accept payment and proceed to use the land herself. The ability to realize gains from voluntary exchanges of the kind posited in the game depends crucially on the fact that payments to the landholding agents (L) are made at the end of the period in which the lessee uses the land. It follows that a lessor cannot benefit by coercively removing a lessee from the land before the end of the period, since, if she did so, she would not receive her payment r. This payment arrangement also preserves the security of property and peace in the steady-state equilibrium because it enhances the incentives of each lessor to defend her land, and now also her lessee's use of the land, against any attempt by another agent to take that land through conflict. Whereas the landholding agent receives per period utility $\alpha_i T$ without a lessee, she obtains $\beta_i T + r > \alpha_i T$ if she is party to a successful contract. It follows that a given landless agent's expected rate of success in obtaining land through conflict can only become worse in the presence of such contracts; thus, given that she was deterred in their absence, she is certainly deterred in their presence. This willingness of the landholder to secure the lessee's use of the land obviates the sorts of concerns that a buyer would have about her ability to secure her newly acquired property herself.

For similar reasons, auctioning the use of land to the highest bidder, or any bargaining procedure that treats past lessees on an equal footing with other landless agents, is, paradoxically, undesirable to the landholders. Because the lessee's opportunity to use land in the future is not contingent on her fulfilling her current contract, such a bargaining procedure would not provide her with an incentive to pay. But in the absence of a credible commitment to pay, the landholder would prefer not to engage in the transaction and use the land herself. In contrast, in the procedure assumed above, the lessor is able (in equilibrium, discussed below) to guarantee the honest lessee's progeny's ability to use the land in future periods, and thus the lessee is willing to pay an amount less than or equal to the discounted difference between

[13]The procedure assumed here is the paradigmatic principal-agent model, in which one party (the principal) has all the bargaining power and makes a single take-it-or-leave-it offer, which the other party (the agent) can only accept or reject, without making any counter-proposal. Note that the designations "principal" and "agent" are not meant to imply, in this context, that one party is employed by the other or is in any way acting on the other's behalf. For further discussion of this model and its use in the literature on contracts, see Salanié (1997).

the expected utility of her progeny when the landholder's progeny shows a preference for contracting with them and their expected utility when the landholder's progeny does not.

The solution concept relied on below is the stationary Perfect Bayesian equilibrium, which ensures that individual decisions are sequentially rational and allows us to restrict attention to the strategies that depend only on the state-variable description of the current-period environment.[14]

We proceed by backward induction. First note that i is indifferent between agents and so, in order to induce the "right" incentives (in this case, to pay the rent and to decline if paying would not be in the renter's interest), she can credibly commit to choosing an agent whose parent behaved in the "right" way (note that all the relevant information for implementing these strategies is captured by a state variable $Z \in N \times \{\lambda, \nu\}$ which identifies last-period tenants as agents whose progeny is to be "rewarded," i.e. to be offered access to land, or to be "punished.") In particular, i's progeny can favor a landless agent j whose parent behaved honestly by choosing her over others to be the recipient of an offer r; I shall refer j as the preferred recipient of i

Let $E[u_j|\lambda]$ represent the expected utility of an agent j in N whose parent or ancestor dealt faithfully with a particular landholder and who, therefore, expects to be the recipient of any offer that landholder's progeny chooses to make; similarly, $E[u_j|\nu]$ is the expected utility of an agent whose family has had no past interaction of this kind or who reneged on the last such interaction. If j has accepted a contract r, j pays if and only if both

$$\begin{array}{rrcl} & r & \leq & \alpha_j T \\ \text{and} & r & \leq & \delta(E[u_j|\lambda] - E[u_j|\nu]), \end{array} \tag{4}$$

where the first line expresses the budget constraint, i.e., her ability to pay, and the second her willingness to pay. If either of these conditions is violated, then j reneges. Given this sequentially rational behavior, if both these conditions (4) hold, j's choice to decline or accept a contract reduces to a choice to decline or to accept and fulfill it. She prefers accepting the offer, knowing that she will pay, if $\alpha_j T - r + \delta E[u_j|\lambda] > \beta_j T + \delta E[u_j|\lambda]$, i.e. if

$$\left(\alpha_j - \beta_j\right) T > r. \tag{5}$$

If conditions (4) are violated, j chooses to accept (and subsequently renege) rather than to decline if $\alpha_j T + \delta E[u_j|\nu] > \beta_j T + \delta E[u_j|\lambda]$, i.e. if

$$\left(\alpha_j - \beta_j\right) T > \delta\left(E[u_j|\lambda] - E[u_j|\nu]\right).$$

[14] In the interests of not proliferating notation, I am not formalizing these definitions in terms of players' strategies in this game.

If $r \leq \delta(E[u_j|\lambda] - E[u_j|\nu])$, then, letting $\Pr\left((\alpha,\beta)_j : \left(\alpha_j - \beta_j\right)T > r\right)$ represent the probability of an agent j in N of the type (α, β) that would (from (5)) accept contract r, i's expected utility from making offer r is given by

$$E[u_i(r)|\alpha_i, \beta_i] = \alpha_i T + \delta E[u_i] + (r - (\alpha_i - \beta_i)T) \Pr\left((\alpha,\beta)_j : \left(\alpha_j - \beta_j\right)T > r\right), \tag{6}$$

which is greater than her expected utility from using the land herself, $\alpha_i T + \delta E[u_i]$, if and only if

$$\begin{cases} r > (\alpha_i - \beta_i)T \\ \Pr\left((\alpha,\beta)_j : \left(\alpha_j - \beta_j\right)T > r\right) > 0. \end{cases} \tag{7}$$

If $r > \delta(E[u_j|\lambda] - E[u_j|\nu])$, then, from (4), j will renege if she accepts the offer, and therefore i's expected utility from making such an offer is no more than her utility from not making any offer at all:

$$\begin{aligned} &E[u_i(r)|\alpha_i, \beta_i] = \alpha_i T + \delta E[u_i] \\ &- (\alpha_i - \beta_i)T \Pr\left((\alpha,\beta)_j : \left(\alpha_j - \beta_j\right)T > \delta(E[u_j|\lambda] - E[u_j|\nu])\right) < \alpha_i T + \delta E[u_i]. \end{aligned}$$

Hence, no $i \in L$ will offer $r > \delta(E[u_j|\lambda] - E[u_j|\nu])$.

When she makes an offer, agent i chooses r by the following program:

$$\begin{aligned} &r \in \arg\max E[u_i(r)|\alpha_{i,}\beta_i] \\ \text{s.t. } &r \leq \delta\left(E[u_j|\lambda] - E[u_j|\nu]\right). \end{aligned} \tag{8}$$

From (6) and (8), the FOC of the optimization problem for r is given by:

$$\begin{aligned} &-\Pr((\alpha,\beta)_j \in N : (\alpha_j - \beta_j)T = r)(r - (\alpha_i - \beta_i)T) \\ &+ \Pr((\alpha,\beta)_j \in N : (\alpha_j - \beta_j)T > r) = 0. \end{aligned} \tag{9}$$

The constraint $\delta(E[u_j|\lambda] - E[u_j|\nu])$ is derived as follows. Recall that agents do not know the types of their progeny, only the distribution of their types; thus their expected utilities are their expected utilities conditioned on type, weighted by the probability of that type, integrated over all possible types. To integrate over all possible types, it is helpful to recall from the definition of k that the inverse of function k that returns a type α is $\beta(k-1)$, and the inverse of k that returns a type β is $\frac{\alpha}{k-1}$. The lowest type k is $\frac{a-d}{d}$, and the value of k such that proportion $\frac{N}{N+L}$ of agents in the entire society have a lower type k is k^*. The expected utility conditioned on type (α, β) for a preferred recipient can be written as a weighted sum of their utility when they decline or do not receive an offer, accept and fulfill a contract, and accept and renege upon a contract, where the weights are the probabilities of each of these events occurring. Note that the probabilities of these events depend not only on j's type but also on the distribution of landholder types and the offers made by different landholder types. Taking $\beta_j T$ and $\delta E[u_j|\lambda]$ out of the integrals, the expected utility of a preferred recipient (i.e. the future utility associated with behaving "honestly") can be written in the following form:

$$\begin{aligned}E[u_j|\lambda] &= E[\beta_j]T + \delta E[u_j|\lambda] \\ &+ \int_{\frac{a-d}{d}}^{k^*} p^N(k_j = k) \int_{\max\left\{c,\frac{a}{k-1}\right\}}^{\min\left\{d,\frac{b}{k-1}\right\}} \Pr(\beta_j = \beta) \\ &\cdot [\int_{r:\ j \text{ accepts, pays}} \Pr\left((\alpha,\beta)_i \in L : r\right)\left((\beta(k-1)-\beta)T - r\right)T dr \\ &+ \int_{r:\ j \text{ accepts, reneges}} \Pr\left((\alpha,\beta)_i \in L : r\right) \\ &\cdot \left((\beta(k-1)-\beta)T - \delta E[u_j|\lambda] - E[u_j|\nu]\right)\right) dr] d\beta dk.\end{aligned} \tag{10}$$

Note that the left-hand side of the expression appears on the right as well. Recalling that M is (in equilibrium) the number of preferred recipients, and hence the number of landholders who will not make offers to landless agents newly matched to them, the expected utility of a landless agent who is not a preferred recipient can be written similarly:

$$\begin{aligned}E[u_j|\nu] = E[\beta_j]T + \delta E[u_j|\nu] + \frac{L-M}{N} \int_{\frac{a-d}{d}}^{k^*} p^N(k_j = k) \int_{\max\left\{c,\frac{a}{k-1}\right\}}^{\min\left\{d,\frac{b}{k-1}\right\}} \Pr(\beta_j = \beta) \\ \cdot [\Pr\left((\alpha,\beta)_i :\ r_i \notin [0,(\alpha_j - \beta_j)T]\right) \delta\left(E[u_j|\lambda] - E[u_j|\nu]\right) \\ + \int_{r:\ j \text{ accepts, pays}} \Pr((\alpha,\beta)_i \in L : r)((\beta(k-1)-\beta)T - r + \delta\left(E[u_j|\lambda] - E[u_j|\nu]\right))dr \\ + \int_{r:\ j \text{ accepts, reneges}} \Pr\left((\alpha,\beta)_i \in L : r\right)(\beta(k-1)-\beta)T dr] d\beta dk.\end{aligned}$$

The constraint on the landholder's optimal offer (8) can now be obtained by combining and simplifying these expressions, using the equilibrium behavior derived thus far, e.g., that no landholder i in L offers a contract such that some landless agent j would accept it and renege:

$$E[u_j|\lambda] - E[u_j|\nu] = \frac{1}{1-\delta} \int_{\frac{a-d}{d}}^{k^*} p^N(k_j = k) \int_{\max\left\{c, \frac{a}{k-1}\right\}}^{\min\left\{d, \frac{b}{k-1}\right\}} \Pr(\beta_j = \beta)$$

$$\cdot [\int_{0}^{\min\{(\beta(k-1)-\beta)T,\ \delta(E[u_j|\lambda]-E[u_j|\nu])\}} ((\beta(k-1)-\beta)T - r) \Pr((\alpha,\beta)_i : r) dr$$

$$+ \int_{\delta(E[u_j|\lambda]-E[u_j|\nu])}^{\max\{(\beta(k-1)-\beta)T,\ \delta(E[u_j|\lambda]-E[u_j|\nu])\}} ((\beta(k-1)-\beta)T - r) \Pr((\alpha,\beta)_i : r) dr] d\beta dk.$$

Note that the discounted difference in the lessee's expected utilities from being in the states λ and ν, $\delta(E[u_j|\lambda] - E[u_j|\nu])$, appears in the boundaries of the integrals on the right side of the last equation. This quantity represents a constraint on the highest offer that any landholder would make (8), and it is derived from the fact that any landless agent who accepted a contract higher than this amount would certainly fail to fulfill it (4). This constraint is binding if and only if it is less than the highest contract that any landless agent would accept if she knew that she would have to fulfill it, which, from (5), is $(\beta(k-1) - \beta)T$.

I restrict my attention here to economies in which $\frac{a-c}{c} \leq k^* \leq \frac{b-d}{d}$. This is a condition on, simultaneously, the maximum and minimum agent productivities with each technology and the scarcity of land. This condition requires that, when the agents are sorted fully by k, all agents who have the maximum productivity with land have land and all agents who have the minimum productivity with land do not have it, regardless of their productivities without land.[15]

Define $W(a, b, c, d, N, L, \delta) = \delta\left(E[u_j|\lambda] - E\left[u_j|v\right]\right)$; because W is a function solely of parameters of the economy, i.e. it is constant with respect to agent types, I will suppress its arguments. In equilibrium, W represents the highest contract that a landless agent would choose to fulfill, if able, given that she had already accepted it; for higher contracts, landless agents will only decline or accept and renege. Call i's offer *potentially successful* if i believes that there exists a non-zero probability of the contract's acceptance and fulfillment. In particular, given the information available to the agents in the present model, requiring that only potentially successful offers be made excludes the making of offers that are higher than the highest offer accepted by any type that could occur in N. Given (5), the definition of k^*, and the assumption of complete sorting, the highest offer that could be accepted is $V = dk^*T$. In cases

[15] For the highest values of k^*, such that $\frac{b-d}{d} < k^* \leq \frac{b-c}{c}$, there are agents who have the maximum productivity with land do not own it, and, similarly, for $\frac{a-d}{d} \leq k^* < \frac{a-c}{c}$ there are agents who have the minimum productivity with land who do. For $k^* \leq 0$, there is actually a surplus of land and some is left fallow, obviating any need for rights to the use of land.

where landless agents' propensity to renege does not infringe on landholders' ability to offer potentially successful contracts, i.e. $W > V$, there are landholders who are indifferent between making offers that will never be accepted, $W > r > V$, and not making any offer at all. Because such offers are never accepted, they have no substantive relevance, and, for the sake of brevity, in what follows I restrict attention to potentially successful offers.

The following proposition summarizes agents' equilibrium behavior, i.e. who makes offers, what offers they make, who accepts them and who does or does not fulfill them.

Proposition 2 *For this economy, the Perfect Bayesian equilibrium path of play is characterized by the following (sub-)strategies:*

1. *landholding agent i makes a potentially successful offer r if and only if the difference in her per period productivity with and without land is less than both the highest corresponding difference among the landless agents and the discounted difference in expected utilities for landless agents who are preferred recipients of contracts and for landless agents who are not, i.e. if both $\alpha_i - \beta_i < \frac{V}{T}$ and $\alpha_i - \beta_i < \frac{W}{T}$ hold;*

2. *the optimal potentially successful offer r^* is uniquely defined by*

$$r^* = \begin{cases} W \ \ if \ \frac{3W-V}{2T} < \alpha_i - \beta_i < W < V \\ \frac{2}{3}\left(\alpha_i - \beta_i\right) T + \frac{1}{3} V \quad else; \end{cases}$$

3. *landless agent j who receives an offer accepts and fulfills the contract if and only if it is less than the difference in her per period productivity with and without land, i.e., $r < (\alpha_j - \beta_j)T$; otherwise, she declines;*

4. *landless agent j remains with landholding agent i.*

Proof. Parts 1, 3 and 4 are proven in the text above. See Appendix 4.1 for the proof of part 2. ■

Note that any contract that is accepted is fulfilled; there is no observable contract violation, and hence property rights appear to be enforced completely. This appearance may be misleading, however. When contract enforcement is inadequate, landholders, foreseeing that landless agents would not fulfill certain contracts that they might nonetheless accept, simply do not offer those contracts. Put differently, there is no observable cheating because the threat of cheating is enough to deter landholders from placing themselves at risk. However, because the threat of cheating is confined to a particular class of contracts, $r > W$, landholding agents are able to offer contracts $r < W$ without fear of being cheated. In equilibrium, landless agents prefer declining such offers to accepting them when they do not intend to fulfill them.

See Figure 4 for a graph of the optimal offer curve $r^*(\alpha, \beta)$ given the values of the parameters. Because W is a constraint on r, it is helpful in the subsequent analysis to consider three separate cases: where W is not binding on any agent's choice of offer, $W \geq V$; where it is binding on some but not all agent's offers, $\frac{2ck^*T+V}{3} < W < V$; and where it is binding on all agents, but some agents do still make an offer. In the first case, no agent offers W. In the second and last cases, some or all, respectively, of the landholding agents offer W, which is characterized by the real root of the fifth-order equations of full rank derived in Appendix 4.1. Although no analytical solution exists for finding such roots, numerical simulations can be used to find W and to examine the relationship between it and primitives of the economy when the values of the parameters a, b, c, d, L, N, δ, and T are specified.

The following four observations and their accompanying graphs are based on simulated data, with parameters $(a, b, c, d, N, L, \delta, T) = (1, 5, 1, 2, 5000, 3000, \frac{3499200}{3500161}, 1)$, unless otherwise specified. The results of such simulations for the relationship between L and W on the interval $L \in (2000, 5000)$, which corresponds to between two-sevenths and one-half of the population having land, are reported in Figure 5, which graphs the trend in the simulated data. They give rise to the following observation:

Observation 1 *The highest offer W made by any landholder is decreasing as land becomes less scarce.*

Although it may seem intuitive that the maximum price commanded by land increases as it becomes more scarce, it is important to remember that this result is not driven by a price-setting market mechanism. In the context of this model, an increase in W indicates an increase in the difference in landless agents' expected utilities when they are and are not given preferential treatment by some landholder in the offering of contracts, and hence an increase in the range of contracts that landless agents will fulfill and landholding agents will offer. Recall that, in equilibrium, no one reneges, and hence $E\left[u_j|v\right]$ is constant. Therefore, the results of the simulation indicate that increasing the supply of land decreases $E[u_j|\lambda]$.

A similar set of numeric simulations can be obtained for the relationship between W and b, which indicates the effect of universal improvements in the technology that requires land on the highest offer that any landholder would make, and hence on which types of landholders make offers. The results for the interval $b \in [3, 11]$ and the trend in the simulated data are graphed in Figure 6. The following observation summarizes these results:

Observation 2 *For the given specification of the vector of parameters, the highest offer W made by any landholder increases as the maximum productivity of land increases.*

Next, consider the effects of the scarcity of land and the productivity of land on optimal offers that are not constrained by W. When W is not binding on any landholder's optimal offer, the highest offer that may be profitable for a landholder is the highest offer that any landless agent might pay, $V \geq \left(\alpha_j - \beta_j\right) T$.

Proposition 3 *When W is not a binding constraint on the landholder's offer r^*,*

1. *r^* increases as the cost to the landholding agent of providing the land increases (i.e. increases in α and decreases in β);*

2. *r^* increases as technology A becomes globally more productive relative to technology B (i.e. increases in a and b and decreases in c and d) if $\frac{aL+bN}{N+L} < \frac{(c+d)^2}{2c}$;*

3. *r^* increases as land becomes more scarce (i.e. decreases in L and increases in N);*

4. *V increases as technology A becomes globally more productive relative to technology B (i.e. increases in a and b and decreases in c and d);*

5. *V increases as land becomes more scarce (i.e. decreases in L and increases in N).*

Proof. See Appendix 4.2. ■

2.4 Transferable Property Rights and Aggregate Efficiency

Allocational inefficiencies persist even when agents sort completely through conflict because the determinant of agents' success in conflict is not perfectly correlated with the productive value of land for them, i.e. with the difference in their marginal productivities with and without land. The model presented here shows that, for some economies, some agents are able to exploit the potential gains from trade that are necessarily created when land is allocated through conflict. But how many of the agents who could in principle benefit from voluntary exchange are actually able to do so? How do economic primitives, such as the scarcity of land, affect the feasibility of potentially profitable trades?

The ability to transfer the use of property is equivalent to the ability to execute a voluntary contract. Successfully executing a contract requires that the landless agent have a sufficient incentive to pay the landholder after she has used the land and that the landholder have sufficient incentive to guarantee the tenant's claim to the use of the land, both against encroachment from third parties and against her own aggression. The satisfaction of the latter requirement hinges on the evident willingness of the landholder to defend her possession of the property in the absence of a contract and the fact that her participation in exchange is voluntary. Because the landholder offered the contract, she must receive a payment higher than the difference in what she herself could produce with and without land; and if she evicted the tenant, she would lose this payment. From the fact that she found it in her interest to defend her possession of land against incursion before engaging in exchange, and the fact that having a paying tenant increases the utility she obtains from the land, it follows that she is at least as willing to defend her possession of land when she has a tenant as she was when she did not. In defending her land, she also protects her tenant's claim to the use of the land, effectively creating a use right that she herself enforces. It follows that, as long as the tenant actually fulfills the contract, it is in the landholder's interest to protect the tenant's claim to the use of the land.

The landholding agent's willingness to perform this function does, however, depend on her expectation that the tenant will actually keep her end of the bargain. As discussed above, a landholding agent creates an incentive for the landless agent to fulfill the contract by credibly committing to a sustained exchange relationship between their families: as long as the landless agent behaves honestly, either declining offers or fulfilling those contracts which she accepts, the progeny of the landholder will make the landless agent's progeny preferred recipients of contract offers. The effectiveness of this incentive is limited by the discounted present value of such future arrangements to the landless agent, W. All landless agents have an incentive to renege on any contract greater than W; thus W may impose a limit on effective transferability.

Whether or not W does, in fact, impose such a limit depends on whether any landholding agent would offer a contract as high as W if it were enforceable, i.e. if any landless agent who accepted it were certain to fulfill it. If enforcement were certain, the highest potentially successful offer a landholder could make would be the highest offer that any landless agent would be willing to accept, V. When $W > V$, no landholder is deterred from making a potentially successful offer by the inadequacy of contract enforcement.

When $W < V$, however, there are landholding agents who could, in principle, benefit from trade, but who are not able to do so because of the inability to enforce the payment of contracts greater than W. Likewise, there are landless agents who would, if contracts were enforced, benefit from accepting an offer greater than W, who will not receive such offers because of their inability to credibly commit to paying them in the absence of such enforcement. Thus, when $W < V$, the threat of landless agents' cheating reduces not only the maximum offer made but also the number of offers made, and hence it reduces the ability of both the landless and landholding agents to reap gains from trade. The relationship between the number of potentially successful offers made and the realization of gains from trade suggests the following measure of transferability. Let τ represent the proportion of landholding agents for whom there are potentially successful trades[16] who choose to make offers in equilibrium. Letting

[16] Again, I restrict attention, unless otherwise noted, to potentially successful offers. This limits the space of relevant landholding agents to those for whom real potential gains from trade exist, i.e., those of type $(\alpha, \beta)_i$ such that $\alpha_i - \beta_i < \frac{V}{T}$.

$x = \alpha_i - \beta_i$ and denoting the probability density of x in L, $p^L(x)$:

$$\tau = \frac{\int_{ck^*}^{\min\{\frac{W}{T},\frac{V}{T}\}} p^L(x)dx}{\int_{ck^*}^{\frac{V}{T}} p^L(x)dx} = \frac{\frac{N+L}{L(b-a)(d-c)} \int_{ck^*}^{\min\{\frac{W}{T},\frac{V}{T}\}} (\frac{x}{k^*} - c)dx}{\frac{N+L}{L(b-a)(d-c)} \int_{ck^*}^{\frac{V}{T}} (\frac{x}{k^*} - c)dx}$$

$$= \frac{(d-c)^2(W - ck^*T)^2}{T^2} \tag{11}$$

Using the parameter specification and the simulated data from the previous subsection, we can now examine the effects of changes in the scarcity and productivity of land on the effective transferability of use rights as measured by τ. Figures 7 and 8 depict the relationship between τ and L and b respectively, giving rise to the following observations:

Observation 3 *The greater the scarcity of land, the greater the proportion of landholding agents with potential gains from trade who enjoy effective transferability in the economy.*

Observation 4 *The greater the maximum productivity of land, the greater the proportion of landholding agents with potential gains from trade who enjoy effective transferability in the economy.*

These observations suggest that, for some range of parameter values, greater effective transferability of use rights is associated with better productive technologies that use that input and with the scarcity of that input. Thus, although greater scarcity of land diminishes the productive capacity of the economy, it also has a growth-enhancing effect through its positive impact on transferability. Similarly, improvements in the land technology promote growth not only by increasing the productivity of land directly but also by increasing the efficiency with which land is allocated, i.e. by improving transferability. Because the minimum productivities associated with each technology are equal for the simulation reported here, decreasing the maximum productivity associated with land, b, is equivalent to reducing the difference between the two technologies. Thus Observation 4 suggests that the more similar (in productive capacity) the two technologies are, the lower the level of effective transferability.[17]

Observe that, from the construction of τ, no transfer will occur in economies such that $W < ck^*T$. Given that $W = \delta(E[u_j|\lambda] - E[u_j|\nu])$, if no transfers occur, then $W = 0$; thus W cannot have a value between 0 and ck^*T. As may be anticipated, no transfers occur unless the landless agents value the future sufficiently highly, i.e. unless δ is sufficiently high.

[17] Note that similarity in the productive capacities of the two technologies does not eliminate allocational inefficiency; agents still sort through conflict in accordance with k but value land in accordance with $\alpha - \beta$.

Proposition 4 *The more highly landless agents value the welfare of their progeny, the greater the transferability of use rights, the higher the production path and the greater the average welfare of the individual agents.*

Proof. See Appendix 4.3. ■

Note that a similar measure of transferability based on the proportion of landless agents who can engage in trade would depend on the lowest offer that any landholding agent makes. From Proposition 3, the landholder with the lowest value of $\alpha - \beta$, which, given complete sorting, is ck^*, makes the lowest offer. From Proposition 2, when $\frac{2ck^*T+V}{3} < W$, the lowest offer is $\frac{2ck^*T+V}{3}$. Increases in the lowest offer reduce the number of landless agents who can successfully engage in trade; hence, as land becomes more scarce (Part 5 of Proposition 3) and as the technology that uses land becomes more productive (Part 4 of Proposition 3), fewer of the landless agents who could potentially reap benefits from trade are actually able to do so.

Finally, recall that the sorting of agents by type is likely to be incomplete, and that that incompleteness is a consequence of the nature of the transition path (Proposition 1). It is interesting, then, to ask how the incompleteness of sorting affects transferability. Intuitively, when sorting is less complete, fewer landholders will offer potentially successful contracts because the revelation of a landholder's type through the offer she makes carries with it a greater risk of being challenged for possession of the land. Agent i's offering some contract r reveals information about her type; at the least, it reveals that $(\alpha_i - \beta_i) \leq \frac{r}{T}$ (from (7)). In an economy in which agents' common belief is that sorting is incomplete, a landholder's revealing information about her type that causes her opponent to place more weight on the probability of her being of a relatively low k type increases the likelihood that her opponent will challenge her in an attempt to acquire her land. It follows that her expected utility from making such an offer, and hence her willingness to make it, is lower in economies in which sorting is expected to be less complete. From Proposition 1, these economies are the ones with shorter transition paths. It follows that, in economies that experience greater conflict on the transition path (i.e., longer transition paths), more of the landholders who have relatively low differences in their marginal productivities with and without land make offers. The lower bound on the size of the offers made is lower, and more landless agents are able to engage in exchange. Such economies are thus better able to alleviate allocational inefficiencies on the steady-state production paths, ceteris paribus, and enjoy a higher production path.

3 Conclusion

The project is to offer a framework for the analysis of the emergence of the institutions of property rights in a decentralized political and economic environment. Its distinguishing feature is the causal and analytical prominence of the state of nature determination of control rights over the scarce inputs to production. Analytically, this determination supplies two critical pieces of the framework: the nature of the conflict and production equilibrium and the allocation of physical inputs across agents as a function of their primitive skills. Perhaps the most general conclusion of the model of the state of nature is skepticism about the claims made on behalf of the conditions that would prevail in that state. Models of property rights institutions that depend in some form on its initial (even if imaginary) existence and that present it as a state of perpetual "war of all against all" appear to require considerable reassessment.

This paper presents an investigation of the efficiency-increasing trades in the steady-state of the state of nature that do not rely on the external enforcement of property rights. The trades are modeled as rental contracts between the agents who in the steady-state have (endogenous) control rights over the productive inputs and agents whom the equilibrium of the game that includes only conflict and production consigns to producing with the lower-level (subsistence) technology. The model shows the possibility of a non-cooperative, fully decentralized transition from the minimal, control, aspect of property rights which exist in the pre-trading steady-state of the state of nature to rights that include at least partial (rental) alienability of property.

Interestingly, the arrangement supporting the equilibrium of this model bears strong resemblance to a protection racket. Agent i and her progeny demand periodic payments from j and j's progeny, who make these payments under threat of coercion: if j fails to pay, i attacks and drives her from the land. In return, i provides "protection" for agent j's productive pursuits against encroachment by other agents, but, in equilibrium, this protection is never actually needed. Potential third-party attackers are deterred by j's association with i.

The more important point, however, is that the causal mechanism that sustains security of property in the steady-state of the state of nature becomes fundamental for explaining the possibility of contractual exchange. The ability and willingness of agents who are the steady-state landholders to assure the renters' progeny's status as the preferred claimant of the next period's contract enables the meaningful exercise of exchange (i.e., suppresses reneging) and transferability of use elements of property rights. Critically, the nature of these use rights is distinctly social: they are enforced by somebody other than the user herself. Their exercise, thus, acquires one of the central features of the modern conception of rights: recognition of the validity of one's claim on the part of others (see, e.g., Waldron 1984).

The logic of the no-conflict steady-state of the state of nature also offers another important lesson for the very possibility of exchange in that setting. The attainment of security of property does not necessarily lead to the ability to engage in trade or to the increase in allocative efficiency trade implies; nor does the ability of agents acting in their own self-interest to create institutions that constrain their behavior necessarily promote the development of a system of property rights that enables trade or signals an improvement in macroeconomic performance.

Because the losses due to conflict are greater in economies where convergence is slower, the incentives to inhibit or to prevent voluntary exchange are greater. Similarly, in economies that converge more slowly, the misallocation of land is less severe, and hence the aggregate potential gains from trade are smaller. Thus the relationship between the primitive parameters of the economy and the speed of convergence captures the relationship between the primitive parameters of the economy and the importance, for aggregate production, of institutions that enable voluntary exchange in addition to securing property.

The results on the relationship between the extent of conflict on the transition path and the steady state growth have also somewhat unexpected implications for the efficiency/distributive advantage divide that characterizes the institutionalist theories (including theories of property rights). Whereas such authors such as Demsetz (1967) and Davis and North (1971) argued that property rights emerge in response to the needs of increasing certainty (and decreasing transaction costs) of economic exchanges, Knight (1992) argued forcefully that institutions are best seen as responses to the distributive competition.[18] As Proposition 1 and the results of the present model indicate, however, the causal divide between efficiency and distributive advantage may be overdrawn. Efficiency may result *because* of the conflict (as distinct from a productive competition) over the finite resources.

4 Appendix

4.1 Proof of Proposition 2

The optimal offer is the solution of the first-order condition in (9), subject to the constraint $r \leq \delta\left(E\left[u_j|\lambda\right] - E\left[u_j|\nu\right]\right) = W$ (8). An explicit expression for $\Pr((\alpha,\beta)_j \in N : (\alpha_j - \beta_j)T > r)$ is derived in the following manner. By the assumption of complete sorting by type k, all agents of type $k \geq k^*$ have land and all agents of type $k < k^*$ do not. Solving generally for any $x \in [0, dk^*]$,

$$\Pr((\alpha,\beta) \in N : \alpha - \beta > x) = \frac{\Pr((\alpha,\beta) : \beta k^* > \alpha - \beta > x)}{\Pr((\alpha,\beta) : \beta k^* > \alpha - \beta)}. \tag{12}$$

By definition of k^*, the denominator of (12) is $\frac{N}{N+L}$. Recalling that α and β are assumed to be independently and uniformly distributed in the population as a whole,

[18] Although they are not explicitly committed to this position, repeated games models of rights have also focused more on the efficiency-increasing role of institutions.

the numerator of (12) is derived as follows:

$$\Pr((\alpha,\beta):\beta k^{*}>\alpha-\beta>x)$$
$$=\int p(\beta)p(\alpha)(\min\{b,\beta(k^{*}+1)\}-\max\{a,\beta+x\})\partial\beta$$
$$=\begin{cases}\left(\int\limits_{c}^{a-x}(\beta(k^{*}+1)-a)\partial\beta+\int\limits_{a-x}^{d}(\beta k^{*}-x)\partial\beta\right)\cdot\frac{1}{(d-c)(b-a)} & \text{if } 0\leq x\leq a-c\\ \frac{1}{(d-c)(b-a)}\int\limits_{c}^{d}(\beta k^{*}-x)\partial\beta & \text{if } a-c\leq x\leq ck^{*}\\ \frac{1}{(d-c)(b-a)}\int\limits_{\frac{x}{k^{*}}}^{d}(\beta k^{*}-x)\partial\beta & \text{if } ck^{*}\leq x\leq dk^{*}\end{cases}$$

Because the analytical expression for $\Pr((\alpha,\beta)_j\in N:(\alpha_j-\beta_j)T=r)$ is different for each of three intervals on the range of r, there are three different analytical expressions for r^{*}; however, for a given set of values of parameters $\delta,T,L,N,a,b,c,d,$ and of agent i's type, $(\alpha,\beta)_i$, only one of these expressions for r^{*} will fall within the interval of r corresponding to the expression for $\Pr((\alpha,\beta)_j\in N:(\alpha_j-\beta_j)T=r)$ used to obtain it. To see that r must be such that $r>ck^{*}T$, note that from (**??**), and (7), $r>(\alpha_i-\beta_i)T=\beta_i k_i T>ck^{*}T$. Hence, we can restrict our attention to the third interval ($ck^{*}T\leq r\leq V$), for which there are two roots:

$$\begin{aligned} r_1 &= \frac{2}{3}\left(\alpha_i-\beta_i\right)T+\frac{1}{3}V \\ r_2 &= V. \end{aligned} \tag{13}$$

Since $\frac{V}{T}\geq(\alpha_j-\beta_j)$ for every j in N, the expected utility of making offer r_2 is no higher than that of not making one: $E\left[u_i|r_2=\frac{V}{T}\right]=\alpha_i T+\delta E\left[u_i\right]$. Hence, although r_2 is within the interval $ck^{*}T\leq r\leq V$, it can be ruled out by dominance if $E\left[u_i|r_1\right]$ can be higher than $E\left[u_i|r_2\right]$.

Consider now r_1. $r_1\leq V$ iff $\alpha_i-\beta_i\leq\frac{V}{T}$ and $r_1\geq ck^{*}T$ iff $\alpha_i-\beta_i\geq\frac{3ck^{*}T-V}{2T}$. But the latter inequality is always true, since $\alpha_i-\beta_i=\beta_i k_i$ and $d>c$. It follows, then, that $\frac{3c-d}{2}<c$, and since $\beta_i\geq c$, $\beta_i>\frac{3c-d}{2}$. As $k_i\geq k^{*}$, $\alpha_i-\beta_i=\beta_i k_i\geq\frac{3ck^{*}T-V}{2T}$ and hence $r\geq ck^{*}T$. Similarly, $\alpha_i-\beta_i\leq\frac{V}{T}$ $\forall(\alpha,\beta)\in L$ because $\alpha_i-\beta_i\leq b-c\leq ck^{*}\leq\frac{V}{T}$. Therefore, r_1 is also a valid root.

Next, I show that r_1 dominates r_2. To verify that $E\left[u_i|r_1\right]$ is greater than i's expected utility when she makes no offer or makes offer r_2,

$$E\left[u_i|r=\frac{2}{3}\left(\alpha_i-\beta_i\right)T+\frac{1}{3}V\right]=\alpha_i T$$
$$+\frac{2\left(-\left(d+\alpha-\beta\right)\left(c+d\right)\left(N+L\right)+2d\left(aL+bN\right)\right)^{3}T}{27\left(b-a\right)\left(d-c\right)\left(c+d\right)^{2}N\left(N+L\right)\left(2\left(aL+bN\right)-\left(c+d\right)\left(N+L\right)\right)}$$
$$+\delta E\left[u_i\right]>\alpha_i T+\delta E[u_i],$$

From $N > L$ and $b > a$, $2(aL + bN) > (a+b)(N+L)$. From $a \geq c$ and $b > d$, $(a+b)(N+L) > (c+d)(N+L)$. Thus, the denominator of the middle term in the last equation is positive. From $(\alpha_i - \beta_i)T < r \leq dk^*T$, $\alpha - \beta < d\left(\frac{2(aL+bN)}{(c+d)(N+L)} - 1\right)$, hence $(\alpha - \beta + d)(c+d)(N+L) < 2d(aL+bN)$. Thus, the numerator of the middle term also must be positive. Hence, r_1 is the optimal offer, provided that $r_1 \leq W$.

It remains to find $W = \delta(E[u_j|\lambda] - E[u_j|\nu])$. Because $E[u_j|\lambda]$ is a function of the expected offers, it is helpful to consider separately three cases: that in which W is not binding on any landholder's choice of offer, that in which it is binding on some but not all landholders, and that in which it is binding on all landholders. Recall that, from the equilibrium behavior derived thus far, no $i \in L$ offers a contract such that some $j \in N$ would accept and renege.

If $W > V$, $E[u_j|\alpha_j, \beta_j, \lambda] =$

$$\beta_j T + \delta E[u_j \mid \lambda] + \frac{N+L}{L} \int_{\frac{2ck^*T+V}{3}}^{(\alpha_j - \beta_j)T} \left((\alpha_j - \beta_j)T - r\right) \Pr((\alpha,\beta)_{i\in L} : r) dr$$

where $\Pr((\alpha,\beta)_{i\in L} : r) = \int_{c}^{\frac{3r-V}{2k^*T}} p(\beta_i)\, p(\alpha_i)\, d\beta_i$.

If $V > W > \frac{2ck^*T+V}{3}$, $E[u_j|\alpha_j, \beta_j, \lambda] =$

$$\begin{cases} \beta_j T + \delta E[u_j|\lambda] + \frac{N+L}{L} \int_{\frac{2ck^*T+V}{3}}^{W} ((\alpha_j - \beta_j)T - r) \Pr((\alpha,\beta)_{i\in L} : r) dr \\ \qquad + ((\alpha_j - \beta_j)T - W) \Pr((\alpha,\beta)_i : r = W)) & \text{if } (\alpha_j - \beta_j)T > W \\ \beta_j T + \delta E[u_j|\lambda] + \frac{N+L}{L} \int_{\frac{2ck^*T+V}{3}}^{(\alpha_j - \beta_j)T} \left((\alpha_j - \beta_j)T - r\right) \Pr((\alpha,\beta)_{i\in L} : r) dr & \text{else.} \end{cases}$$

Defining a new variable $x = \alpha - \beta$, of which the optimal offer is a function,

$$\Pr((\alpha,\beta)_{i\in L} : r = W)) = \int_{\frac{3W - dk^*T}{2T}}^{\frac{W}{T}} p^L(x) dx = \int_{\frac{3W - dk^*T}{2T}}^{\frac{W}{T}} \frac{N+L}{L} \int_{c}^{\frac{x}{k^*}} p(\beta)p(\alpha) d\beta dx.$$

If $\dfrac{2ck^*T+V}{3} > W > ck^*T$, $E[u_j|\alpha_j, \beta_j, \lambda] =$

$$\beta_j T + \delta E[u_j|\lambda] + ((\alpha_j - \beta_j)T - W) \Pr((\alpha,\beta)_{i\in L} : r = W))$$

where $\Pr((\alpha,\beta)_{i\in L} : r = W)) = \int_{ck^*}^{\frac{W}{T}} \frac{N+L}{L} \int_{c}^{\frac{x}{k^*}} p(\beta)p(\alpha) d\beta dx.$

For all values of W,

$$E\left[u_j|\alpha_j,\beta_j,v\right]=\frac{\beta_j T}{1-\delta}. \tag{14}$$

The results from subtracting (14) from the above expressions for $E[u_j|\alpha_j,\beta_j,\lambda]$ can be written as functions of $x_j=\alpha_j-\beta_j$, denoted $Y(x)$. Multiplying by δ and integrating with respect to x over the appropriate interval yields the following expressions for $W(a,b,c,d,N,L,\delta)=\delta\left(E[u_j|\lambda]-E\left[u_j|v\right]\right)$.

If $W>V$,

$$W=\frac{\delta}{1-\delta}\int_{ck^*}^{\frac{V}{T}}p^N(x)\,\frac{N+L}{L}\int_{\frac{2ck^*T+V}{3}}^{xT}(xT-r)\int_{c}^{\frac{3r-V}{2k^*T}}p\left(\beta_i\right)p\left(\alpha_i\right)d\beta_i dr dx.$$

If $V>W>\frac{2ck^*T+V}{3}$,

$$\begin{aligned}W\;&=\;\frac{\delta}{1-\delta}\int_{ck^*}^{\frac{W}{T}}p^N(x)\,\frac{N+L}{L}\int_{\frac{2ck^*T+V}{3}}^{xT}(xT-r)\int_{c}^{\frac{3r-V}{2k^*T}}p\left(\beta_i\right)p\left(\alpha_i\right)d\beta_i dr dx\\&\quad+\frac{\delta}{1-\delta}\int_{\frac{W}{T}}^{\frac{V}{T}}p^N(x)\,\frac{N+L}{L}\int_{\frac{2ck^*T+V}{3}}^{W}(xT-r)\int_{c}^{\frac{3r-V}{2k^*T}}p\left(\beta_i\right)p\left(\alpha_i\right)d\beta_i dr dx\\&\quad+\frac{\delta}{1-\delta}\int_{\frac{W}{T}}^{\frac{V}{T}}p^N(x)\,(xT-W)\int_{\frac{3W-V}{2T}}^{\frac{W}{T}}\frac{N+L}{L}\int_{c}^{\frac{y}{k^*}}p(\beta)p(\alpha)d\beta dy dx.\end{aligned}$$

If $\frac{2ck^*T+V}{3}>W>ck^*T$,

$$W=\frac{\delta}{1-\delta}\int_{\frac{W}{T}}^{dk^*}(xT-W)\int_{ck^*}^{\frac{W}{T}}\frac{N+L}{L}\int_{c}^{\frac{y}{k^*}}p(\beta)p(\alpha)d\beta dy dx.$$

For the first of these cases, W is defined explicitly; in each of the latter two, it is defined implicitly by fifth-order equations of full rank. If $W>dk^*T$,

$$W=-\frac{\delta}{1-\delta}\frac{(N+L)^2}{NL}\frac{(c-d)^3k^{*3}T^2}{1080(a-b)^2}.$$

If $V > W > \frac{2ck^*T+V}{3}$, then W is a real solution of

$$\frac{(12T-25)}{240k^{*2}T^4}W^5 - \frac{2c(T-2)+d(10T-21)}{48k^*T^3}W^4 + \frac{d(4c(T-2)+d(8T-17))}{24T^2}W^3 - \frac{d^2k^*(6c(T-2)+d(6T-13))}{24T}W^2 + (\frac{cd^3k^{*2}(T-2)}{6} + \frac{d^4k^{*2}(4T-9)}{48} - \frac{(a-b)^2(c-d)^2LN(\delta-1)}{(L+N)^2\delta})W + \frac{k^{*3}T(d^5(405-194T)+10cd^4(162-65T))}{19440} + \frac{k^{*3}T(2c^5T-10c^4dT+20c^3d^2T-20c^2d^3T)}{1215} = 0.$$

If $\frac{2ck^*T+V}{3} > W > ck^*T$, then W is a real solution of

$$\frac{1}{12k^{*2}T^4}W^5 - \frac{(2c+3d)}{12k^*T^3}W^4 + \frac{(c^2+6cd+3d^2)}{12T^2}W^3 - \frac{dk^*(3c^2+6cd+d^2)}{12T}W^2 + (\frac{c^2d^2k^{*2}}{4} + \frac{cd^3k^{*2}}{6} - \frac{(a-b)^2(c-d)^2LN(\delta-1)}{(L+N)^2\delta})W + \frac{c^2d^3k^{*3}T}{12} = 0.$$

For every septuple of permissible parameters $(a, b, c, d, N, L, \delta)$, exactly one of these equations will have a real root W that lies within the specified interval. That root is the upper bound on r^* for every agent $i \in L$, i.e. it is constant with respect to agent type. Note that, when the condition $W > V$ is satisfied (and W is defined explicitly), W is not binding and, from (13), $r^*(\alpha, \beta) = r_1(\alpha, \beta)$. See Figure 4 for a graph of $r^*(\alpha, \beta)$ for an economy in which W is a binding constraint on some landholders' optimal offers but not on all, $V > W > \frac{2ck^*T+V}{3}$.

4.2 Proof of Proposition 3

We proceed by signing the relevant first derivatives of (13). For part 1,

$$\frac{\partial r}{\partial \alpha} = \frac{2}{3}T > 0.$$

$$\frac{\partial r}{\partial \beta} = -\frac{2}{3}T < 0.$$

For part 2,

$$\frac{\partial r}{\partial a} = \frac{2dLT}{3(c+d)(N+L)} > 0,$$

$$\frac{\partial r}{\partial b} = \frac{2dNT}{3(c+d)(N+L)} > 0, \text{ and}$$

$$\frac{\partial r}{\partial c} = -\frac{2d(aL+bN)T}{3(c+d)^2(N+L)} < 0.$$

The sign of $\frac{\partial r}{\partial d}$ is negative,

$$\frac{\partial r}{\partial d} = \frac{ck^*T - dT}{3(c+d)} < 0,$$

iff $k^* < \frac{d}{c}$, i.e., given the definitions of k and $k^*$and the uniform distributions of α and β, iff

$$\frac{aL+bN}{N+L} < \frac{(c+d)^2}{2c}.$$

For part 3,

$$\frac{\partial r}{\partial N} = \frac{2dT}{3(c+d)(N+L)^2}(b(N+L) - (aL+bN)) > 0, \text{ and}$$

$$\frac{\partial r}{\partial L} = \frac{2dT}{3(c+d)(N+L)^2}(a(N+L) - (aL+bN)) < 0.$$

For part 4,

$$\frac{\partial V}{\partial a} = \frac{2dTL}{(c+d)(L+N)} > 0,$$

$$\frac{\partial V}{\partial b} = \frac{2dTN}{(c+d)(L+N)} > 0,$$

$$\frac{\partial V}{\partial c} = -\frac{2dT(aL+bN)}{(c+d)^2(L+N)} < 0, \text{ and}$$

$$\frac{\partial V}{\partial d} = -\frac{2dT(aL+bN)}{(c+d)^2(L+N)} < 0.$$

And for part 5,

$$\frac{\partial V}{\partial L} = \frac{2dT(a-b)N}{(c+d)(L+N)^2} < 0, \text{ and}$$

$$\frac{\partial V}{\partial N} = \frac{2dT(b-a)L}{(c+d)(L+N)^2} > 0.$$

4.3 Proof of Proposition 4

Suppose that, for some septuple $(a', b', c', d', N', L', \delta')$, W' satisfies the equation implicitly defining W, which is given at the end of Appendix 4.1. Keeping all other parameters the same, replace δ' with $\delta'' > \delta'$. Let W'' satisfy the same equation for $(a', b', c', d', N', L', \delta'')$. The discount factor appears only in the coefficient of W in this equation, and that coefficient decreases as δ increases. It follows that $W'' > W'$.

From the definition of τ, τ increases as W increases for $W \in [ck^*T, V]$; thus τ increases as δ increases. Because exchange is voluntary, it follows that exchange takes place only when it increases the expected utility of both parties (or, at least, does not decrease the utility of either party). An increase in τ indicates that additional agents are able to engage in exchange, and hence to enjoy greater expected utilities. Because the welfare of other agents in the economy is unaffected, the average welfare increases.

Because exchange can be mutually beneficial only when it increases the total production of the parties to the exchange, creating a surplus to be divided between them, any voluntary exchange that does not affect the production of other members of the economy (i.e. that has no negative externalities) must increase the total production of the economy. It follows that greater transferability elicits greater total production, and hence the higher δ, the higher the production path.

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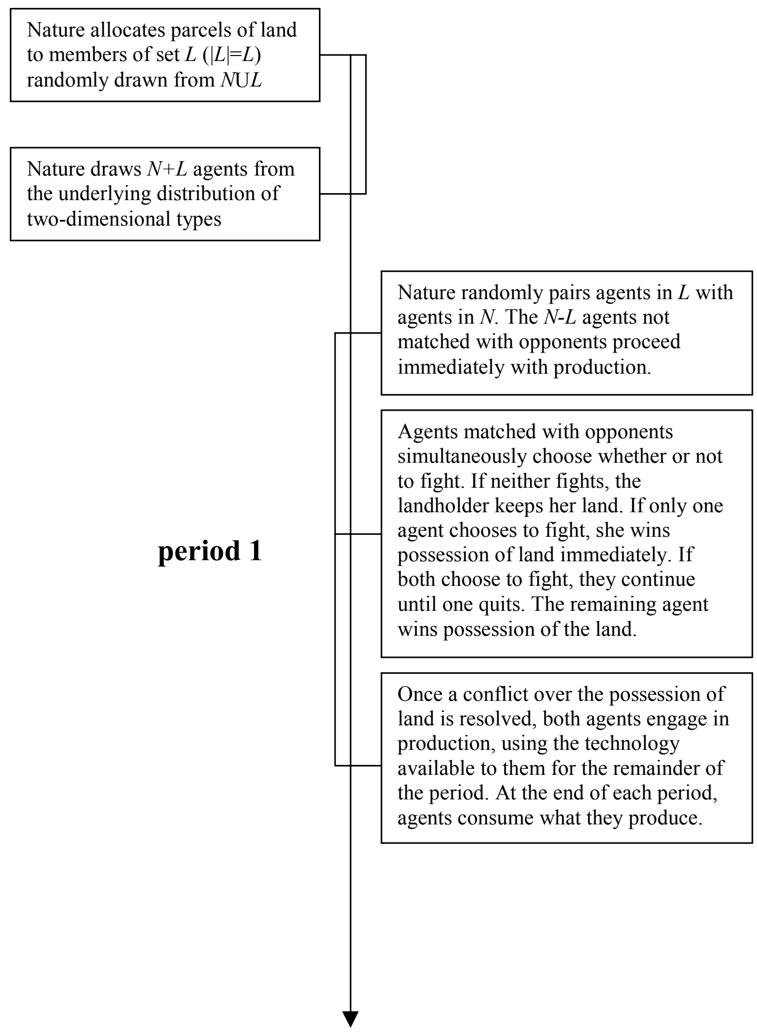

Figure 1: Sequence of events I

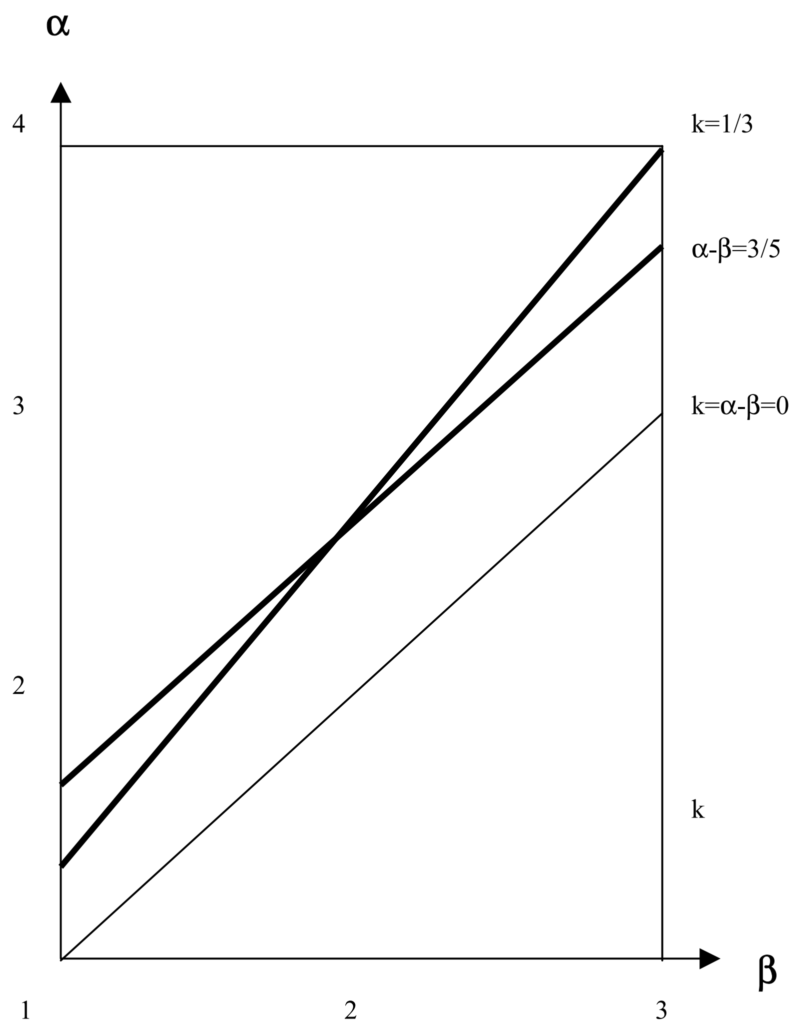

Figure 2: Isoquants of k superimposed on isoquants of $\alpha - \beta$

period t=2

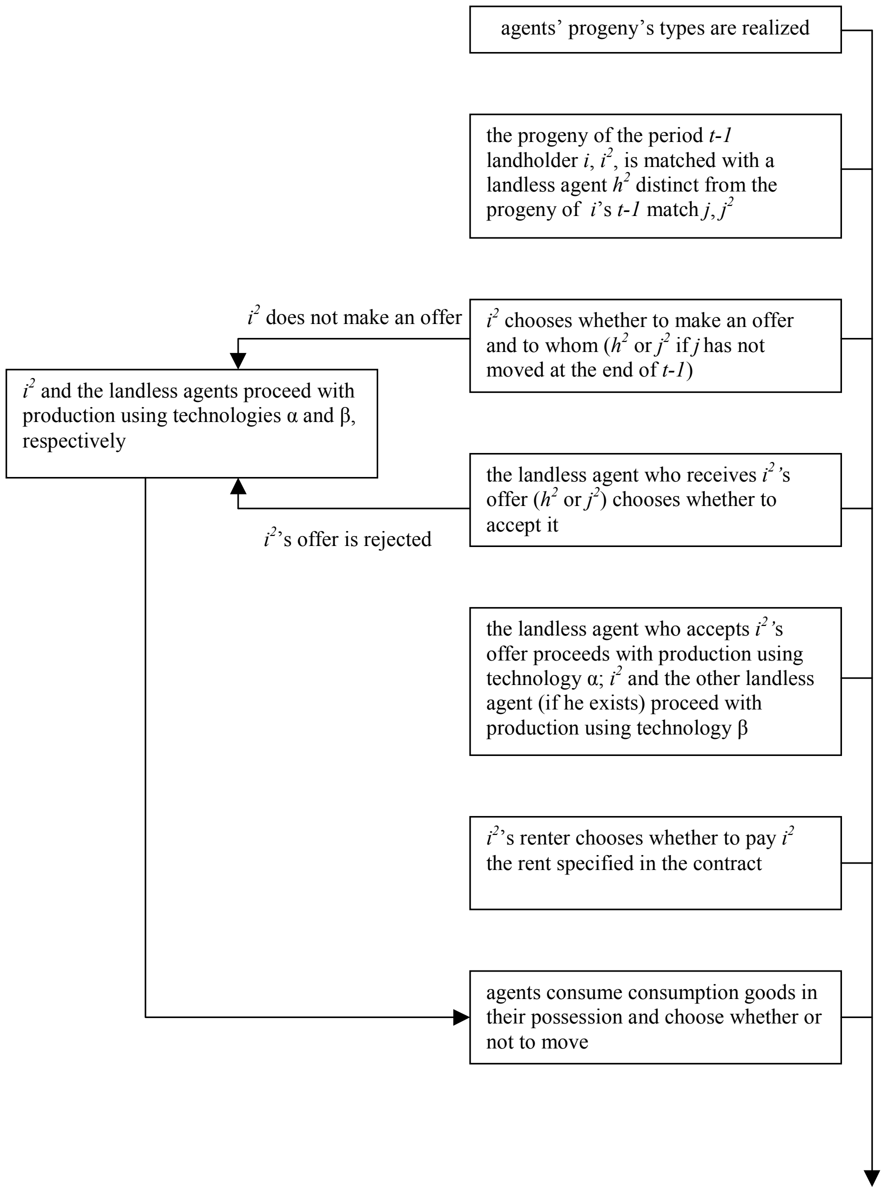

Figure 3: Sequence of events II

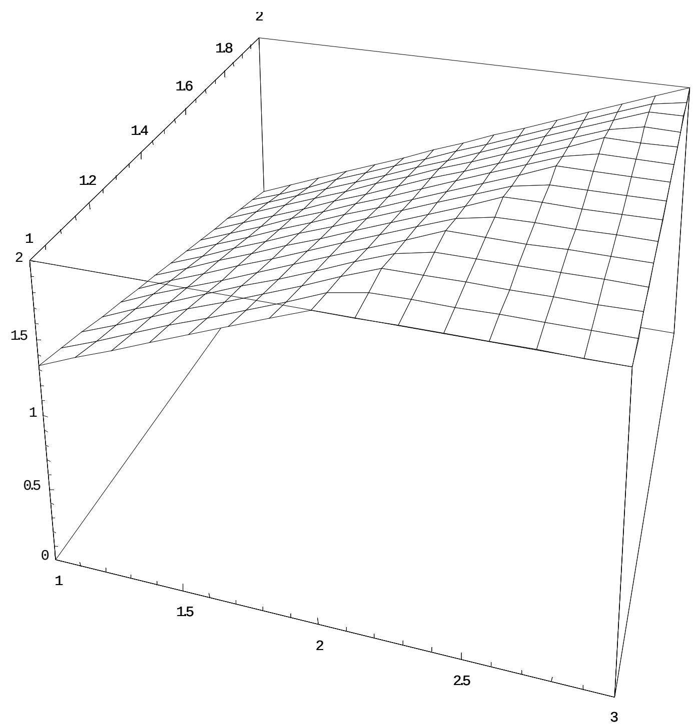

Figure 4: Optimal Contract r as a function of α and β

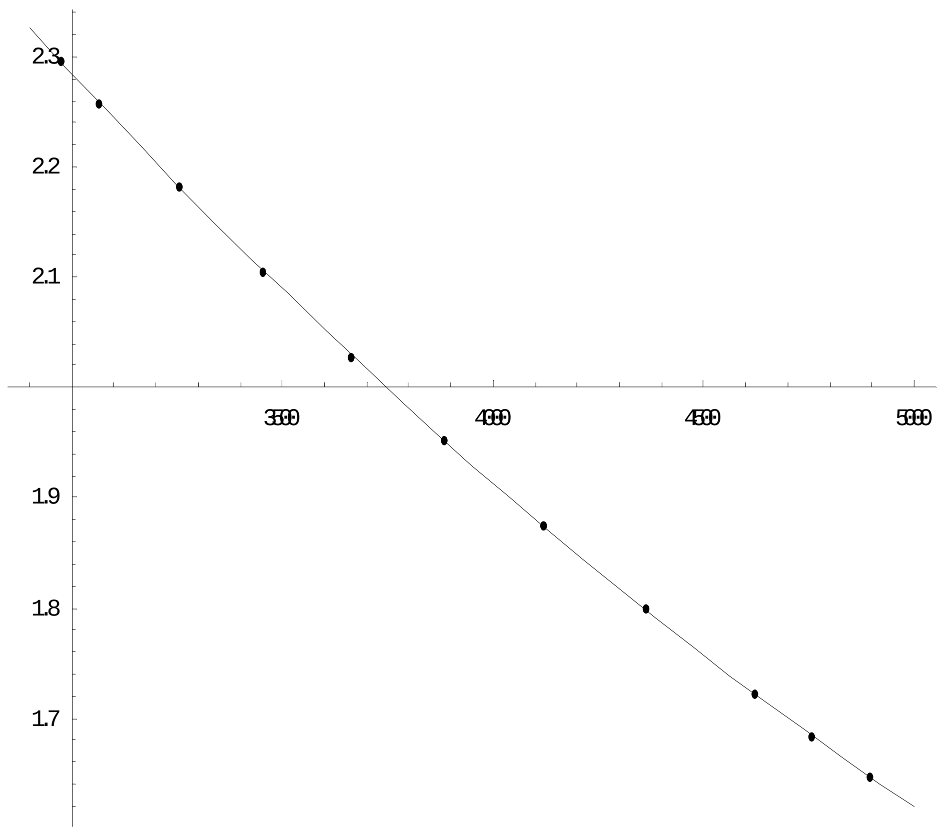

Figure 5: W as a function of L

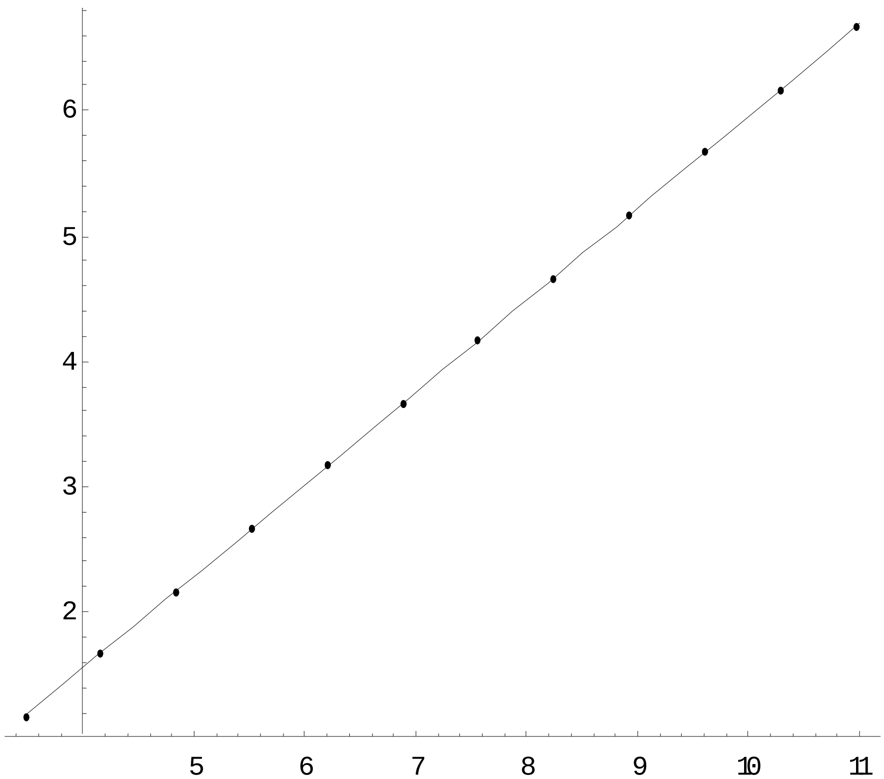

Figure 6: W as a function of b

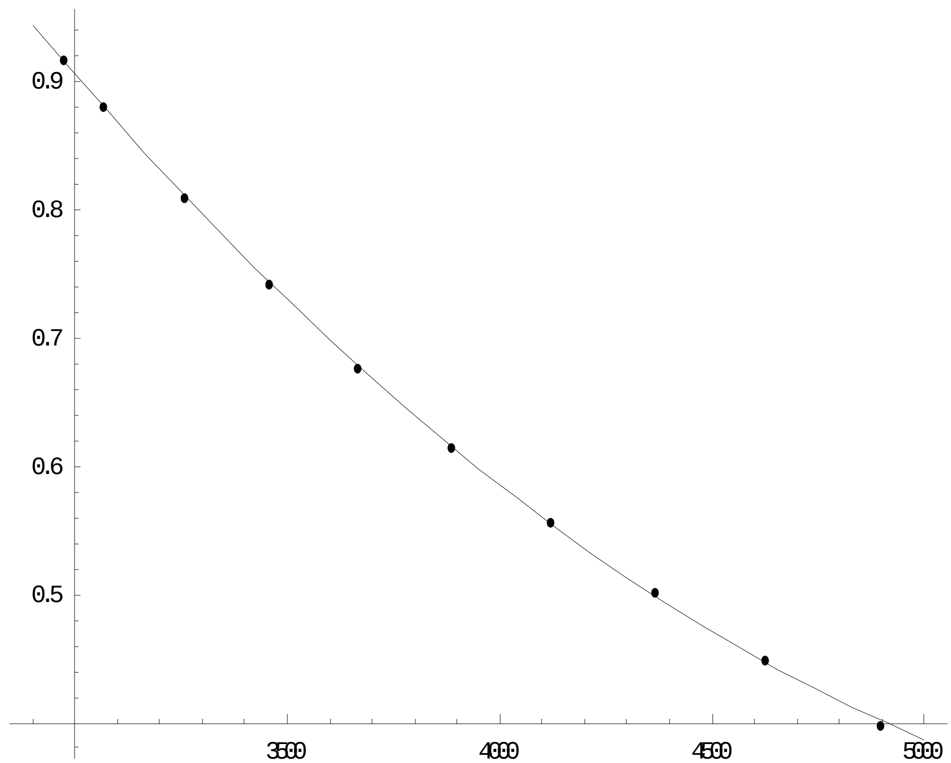

Figure 7: τ as a function of L

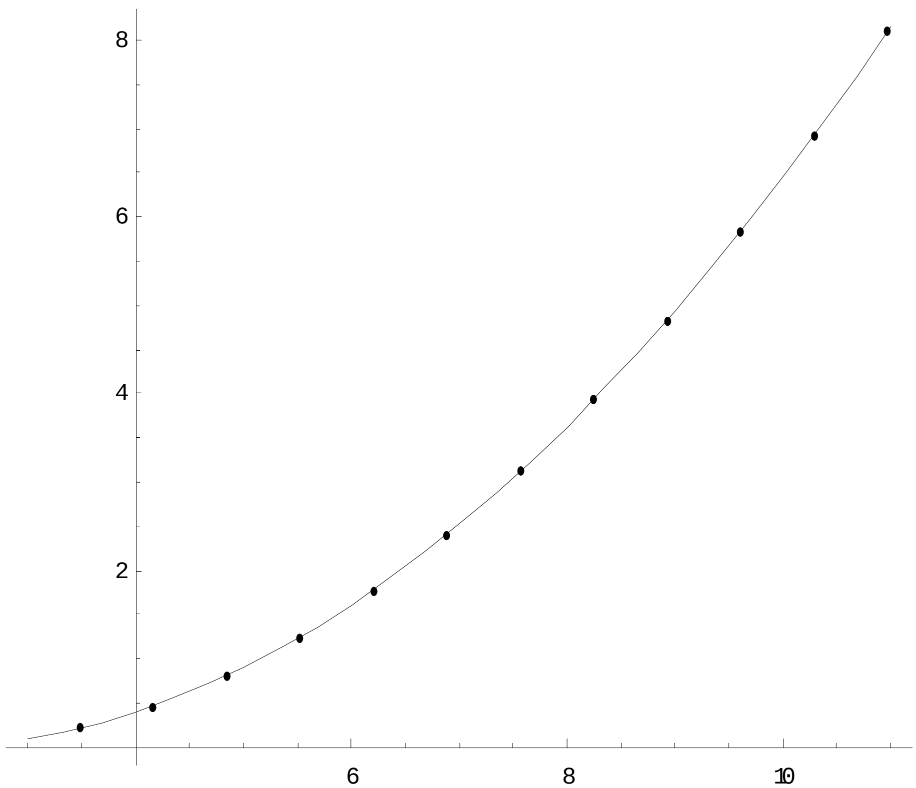

Figure 8: τ as a function of b