

## The Problem with Majority Rule

Shepsle and Bonchek Chapter 4

## Majority Rule is problematic

1. Who's the majority?
2. Sometimes there is no decisive winner
  - Condorcet's paradox: A group composed of individuals with with individually transitive preferences do not necessarily have transitive preferences as a collectivity
3. When the group's preferences are intransitive there is either no stable outcome or the outcome is determined by the rules of the game.
  - Typically, the rule designating an agenda setter is decisive

Today, we're going to explore the implications of these problems by asking:

1. "How general" a problem is "cyclical" majorities?
2. What's so special about majority rule anyway?
3. What can be done?

## Are "intransitive group preferences" a common problem?

- Sure, Andrew, Bonnie, and Chuck ran into trouble deciding, but....
- .....they had other issues too (Red Sox? You gotta be kiddin' me!)

Are groups of normal, canoli-eating, Yankee game watchin' people likely to have the same problem?

## It depends....

Probability of group intransitivity= $f(m,n)$

where

$m$  is the number of alternative and

$n$  is the number of voters

## Specifically....

$$p(\text{intransitivity}) = \frac{\# \text{ of "problem" preference configurations}}{(m!)^n}$$

$$p(\text{intransitivity}) = \frac{\# \text{ of "problem" preference configurations}}{(m \times (m-1) \times (m-2) \times \dots \times 2 \times 1)^n}$$

### Probability of a cyclical majority, $f(m,n)$

| $(m)$ | Number of voters ( $n$ ) |                |                |                |                |                |
|-------|--------------------------|----------------|----------------|----------------|----------------|----------------|
|       | 3                        | 5              | 7              | 9              | 11             | limit          |
| 3     | .056                     | .069           | .075           | .078           | .080           | .088           |
| 4     | .111                     | .139           | .150           | .156           | .160           | .176           |
| 5     | .160                     | .20            | .215           |                |                | .251           |
| 6     | .202                     |                |                |                |                | .315           |
| limit | $\approx 1.00$           | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ |

### Example: Divide the Dollars

- Suppose there are three regions in a town and they've just been given \$1000 dollars to divide - *if* they can agree how to divide it.

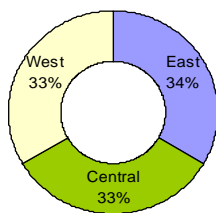
### Divide the Dollars - details

- let  $s(E)$ ,  $s(C)$ , and  $s(W)$  be the shares going to East, Central, and West, respectively.
- a sharing scheme (strategy combination)  $(s(E); s(C); s(W))$  is feasible if each component is non-negative, and the components sum to something less than \$1000.
- A sharing scheme is efficient if the values sum to \$1000 (nothing is wasted).
- Representatives make alternating offers until they settle on a division of the pie that defeats all additional proposals

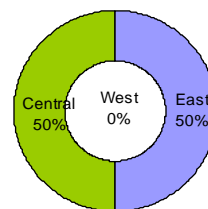
### What happens?

- Well, we can say that the outcome will be efficient, but we can't say much more than that. Why?

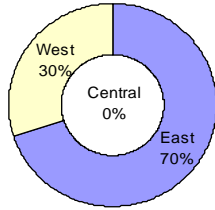
### Divide the Dollar: Proposal 1 "To share is fair"



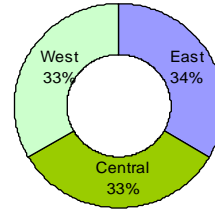
### Divide the Dollar: Proposal 2 "Go @\$%! West Man"



Divide the Dollar: Proposal 3 “West says to East: “I’m easy, I don’t want alot”



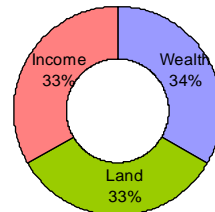
Divide the Dollare: Proposal 4 “Can’t we find a “fair” solution?”



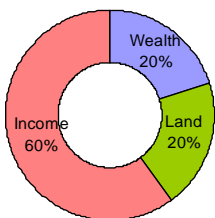
### Majority Cycle in “Divide the Dollar Game”

2. (500,500,0)  $P_{EC}$  (333 1/3; 333 1/3; 333 1/3)
3. (700,0,300)  $P_{EW}$  (500,500,0)
4. (333 1/3; 333 1/3; 333 1/3)  $P_{CW}$  (700,0,300)
5. (500,500,0)  $P_{EC}$  (333 1/3; 333 1/3; 333 1/3)
6. (700,0,300)  $P_{EW}$  (500,500,0)
7. (333 1/3; 333 1/3; 333 1/3)  $P_{CW}$  (700,0,300)...etc

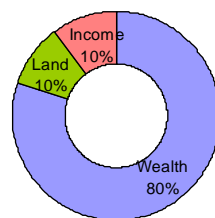
Shift the tax burden: Proposal 1 “share the love”



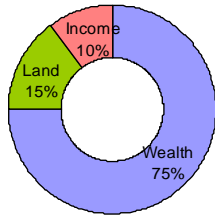
Shift the tax burden: Proposal 2 “Family values: Protect inheritance, and protect our nation’s farms”



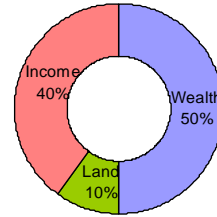
Shift the tax burden: Proposal 3 “Soak the rich!”



Shift the tax burden: Proposal 4 “Save our cities!”



Shift the tax burden: Proposal 5 “Family values: Protect inheritance, and protect our nation’s farms”



Cycling majorities shifting the tax burden

| Proposal | Proposer     | Cut taxes on | wealth, | land   |
|----------|--------------|--------------|---------|--------|
| 2        | Rich         | Cut taxes on | wealth, | land   |
| 3        | Wage-earners | Cut taxes on | income, | land   |
| 4        | Rich         | Cut taxes on | income, | wealth |
| 5        | Farmer       | Cut taxes on | wealth, | land   |
| 6        | Wage-earners | Cut taxes on | ?       | ?      |

## Conclusion

- Majority rule seems to be deeply flawed in handling the “most political” of political problems

**What's so special about majority rule?**

May showed that

Majority rule  $\Leftrightarrow$  A, N, M

So arguing for against majority rule means arguing for or against A,N, or M

### Condition A (Anonymity)

Social preferences depend only on the collection of individual preferences, not on who has which preference.

### Condition N (Neutrality)

changing rank of  $j$  and  $k$  in each group members preferences changes rank of  $j$  and  $k$  in group preferences (i.e. naming the alternatives is arbitrary)

### Condition M (Monotonicity)

- if  $j$  is at least as good as  $k$  from the group's standpoint, and  $j$  becomes more desirable to one of the members, then  $j$  is now strictly better than  $k$  from the standpoint of the group.

### When does majority rule make sense?

- ex. Should grades be determined by majority rule?
- ex. Should what I have for breakfast be decided by majority rule?
- ex. Should amendments to the constitution be decided by majority rule?
- ex. Should students at a public high school be allowed to vote on whether or not to have organized prayer at football games?

### We already saw that

- Majority rule creates practical problems in some situations
- May not be normatively appealing in all situation

So why don't we ditch it?

### Arrows theorem – Majority rule is not special

- The pathologies of majority rule apply to “any” group decision procedure that meets some minimal standards

These minimal standards can be thought of a generalizations of May's conditions for majority rule

| May Condition |   | Arrow Condition   |
|---------------|---|-------------------|
| Anonymity     | → | Dictatorship      |
| Neutrality    | → | Independence      |
| Monotonicity  | → | Pareto Optimality |

A (Anonymity) is a special case of what Arrow called "**Non-Dictatorship**" (D)

There is no distinguished individual  $i^* \in G$  whose preferences dictate the group preference, independent of other members.

N (Neutrality) is a special case of what Arrow called "Independence from Irrelevant Alternatives" (I)

if  $j$  and  $k$  stand in a particular relationship to each other for each member of the group, and this relationship does not change, then neither should the group preference between  $j$  and  $k$

M (Monotonicity) is a special case of what Arrow called "Unanimity" (P) or Pareto Optimality

If every member of  $G$  prefers  $j$  to  $k$  (or is indifferent between them), then the group preference must reflect a preference for  $j$  over  $k$  (or an indifference between them).

Arrow argued that any reasonable procedure for making group choices should involve D, I, and P, and two other criteria:

**Condition U (Universal admissibility)** (each  $i \in G$  may adopt any strong or weak complete and transitive preference ordering over the alternatives in  $A$ )

**Rationality assumption**  $R_G$  is complete and transitive.

### Arrow's theorem

There exists no mechanism for translating the preferences of rational individuals into a coherent group preference that simultaneously satisfies conditions U,P,I, and D

## Conclusion

Arrow showed that if you accept U,P,I as “untouchable” (May shows us that advocating majority rule amounts to making U,P,I untouchable) you have accept either

1. Dictatorship
2. The *potential for* intransitivity