The Problem with Majority Rule

Shepsle and Bonchek Chapter 4

Majority Rule is problematic

1. Who's the majority?
2. Sometimes there is no decisive winner
   - Condorcet's paradox: A group composed of individuals with with individually transitive preferences do not necessarily have transitive preferences as a collectivity
3. When the group's preferences are intransitive there is either no stable outcome or the outcome is determined by the rules of the game.
   - Typically, the rule designating an agenda setter is decisive

Today, we're going to explore the implcations of these problems by asking:

1. “How general" a problem is "cyclical" majorities?

2. What's so special about majority rule anyway?

3. What can be done?

Are "intransitive group preferences" a common problem?

- Sure, Andrew, Bonnie, and Chuck ran into trouble deciding, but….

……they had other issues too (Red Sox? You gotta be kiddin’ me!)

Are groups of normal, canoli-eating, Yankee game watchin’ people likely to have the same problem?

It depends….

Probability of group intransitivity=$f(m,n)$

where

m is the number of alternative and

n is the number of voters

Specifically….

$$p(\text{intransitivity}) \; ? \; \frac{\#\,of\ \text{"}\,problem\,\text{"}\ preference\ configurations}{(m!)^{n}}$$

$$p(\text{intransitivity}) \; ? \; \frac{\#\,of\ \text{"}\,problem\,\text{"}\ preference\ configurations}{(m\ ?\ (m\ ?\ 1)\ ?\ (m\ ?\ 2)\ ?\ \text{.......}\ ?\ 2\ ?\ 1)^{n}}$$

Probability of a cyclical majority, *f(m,n)*

(m)	*Number of voters (n)*					
	3	5	7	9	11	limit
3	.056	.069	.075	.078	.080	.088
4	.111	.139	.150	.156	.160	.176
5	.160	.20	.215			.251
6	.202					.315
limit	?1.00	?1.00	?1.00	?1.00	?1.00	?1.00

Example: Divide the Dollars

- Suppose there are three regions in a town and they've just been given $1000 dollars to divide - *if* they can agree how to divide it.

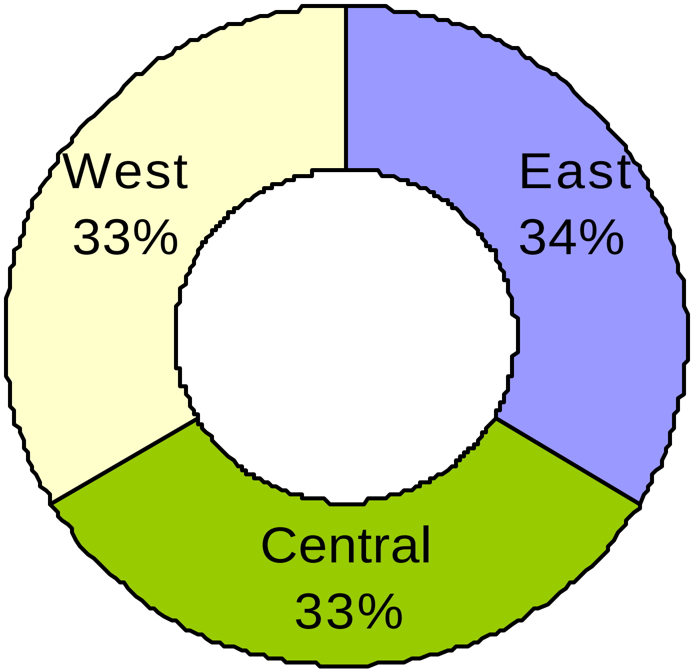
Divide the Dollars - details

- let s(E), s(C), and s(W) be the shares going to East, Central, and West, respectively.
- a sharing scheme (strategy combination) (s(E); s(C); s(W)) is feasib**le** if each component is non-negative, and the components sum to something less than $1000.
- A sharing scheme is efficient if the values sum to $1000 (nothing is wasted).
- Representatives make alternating offers until they settle on a division of the pie that defeats all additional proposals

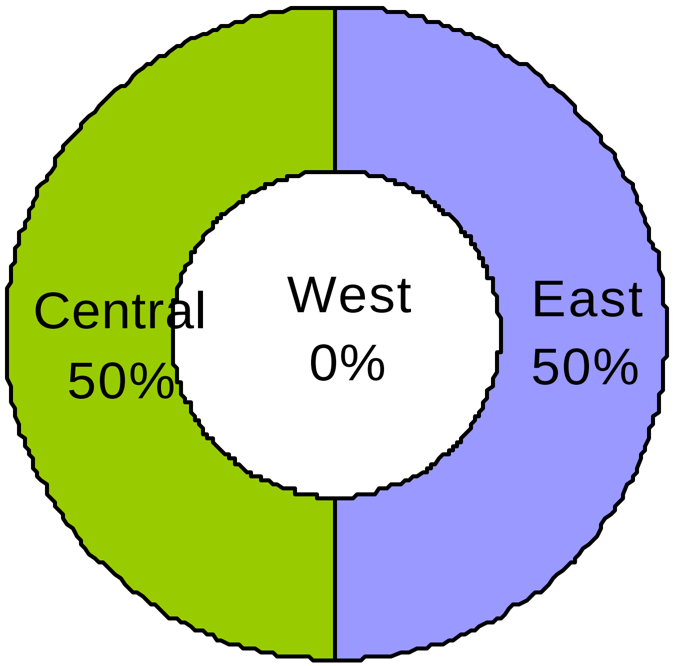
What happens?

- Well, we can say that the outcome will be efficient, but we can't say much more than that. Why?

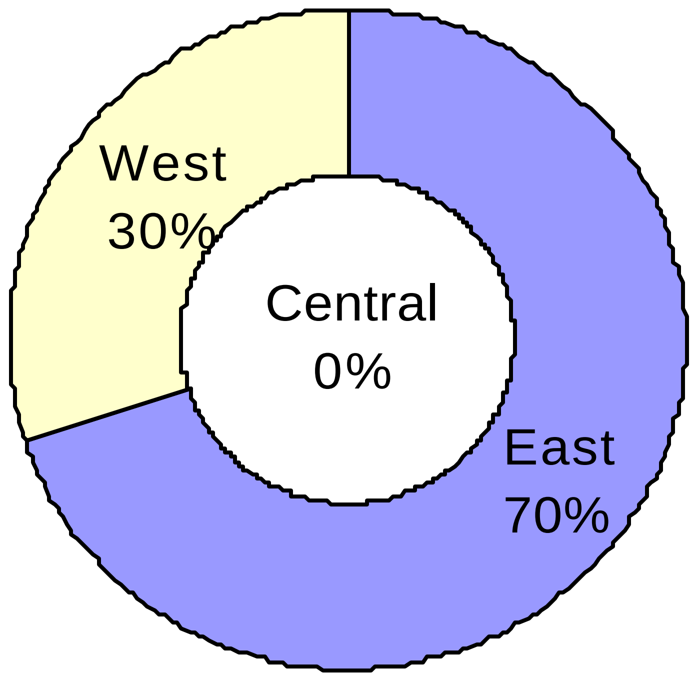
Divide the Dollar: Proposal 1 “To share is fair”



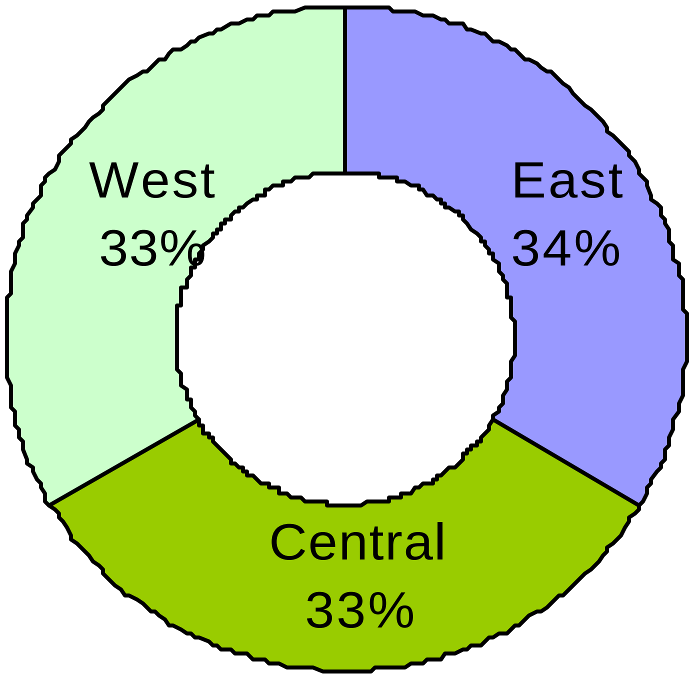
Divide the Dollar: Proposal 2 “Go @#$%! West Man”



Divide the Dollar: Proposal 3 “West says to East: “I’m easy, I don’t want alot”



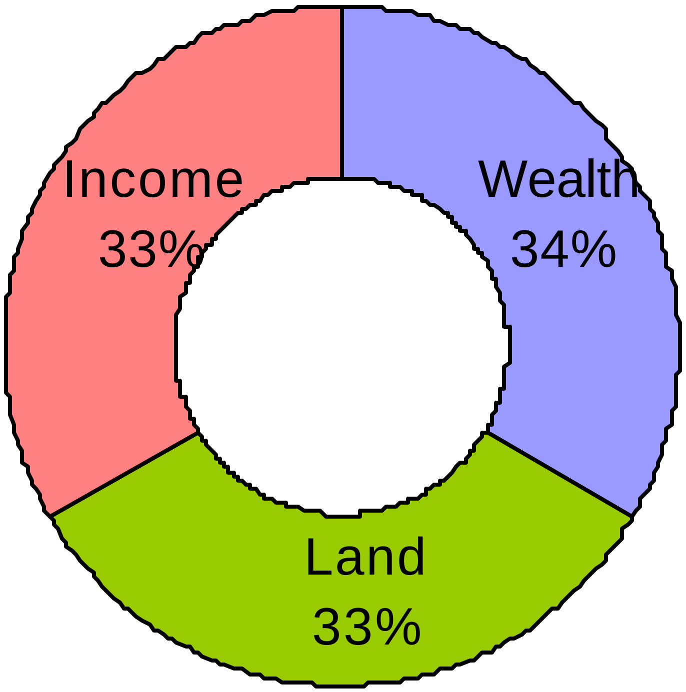
Divide the Dollare: Proposal 4 “Can’t we find a “fair” solution?”



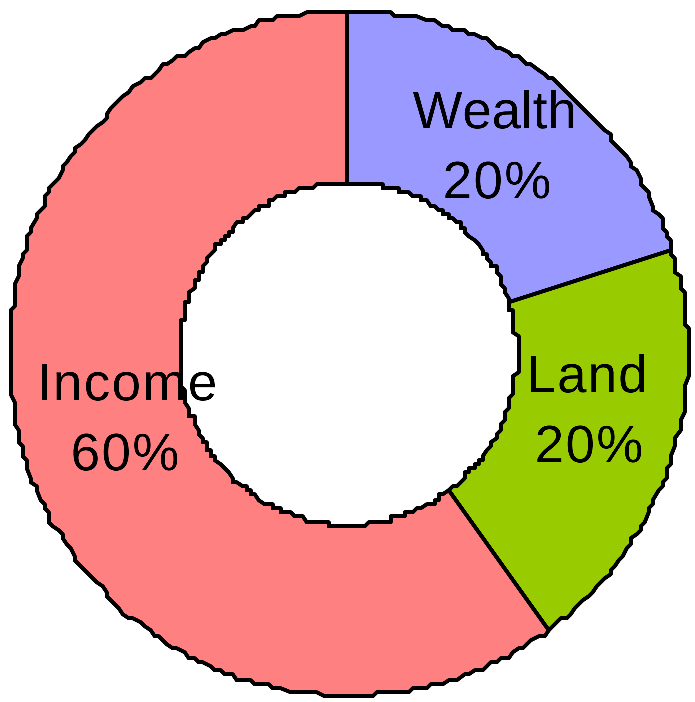
Majority Cycle in “Divide the Dollar Game”

2. (500,500,0) P_{EC} (333 1/3; 333 1/3; 333 1/3)
3. (700,0,300) P_{EW} (500,500,0)
4. (333 1/3; 333 1/3; 333 1/3) P_{CW} (700,0,300)
5. (500,500,0) P_{EC} (333 1/3; 333 1/3; 333 1/3)
6. (700,0,300) P_{EW} (500,500,0)
7. (333 1/3; 333 1/3; 333 1/3) P_{CW} (700,0,300)….etc

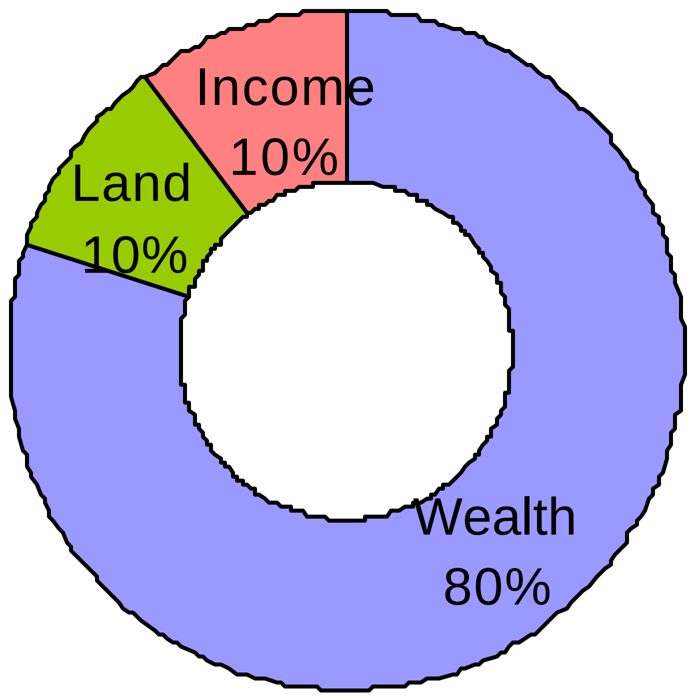
Shift the tax burden: Proposal 1 “share the love”



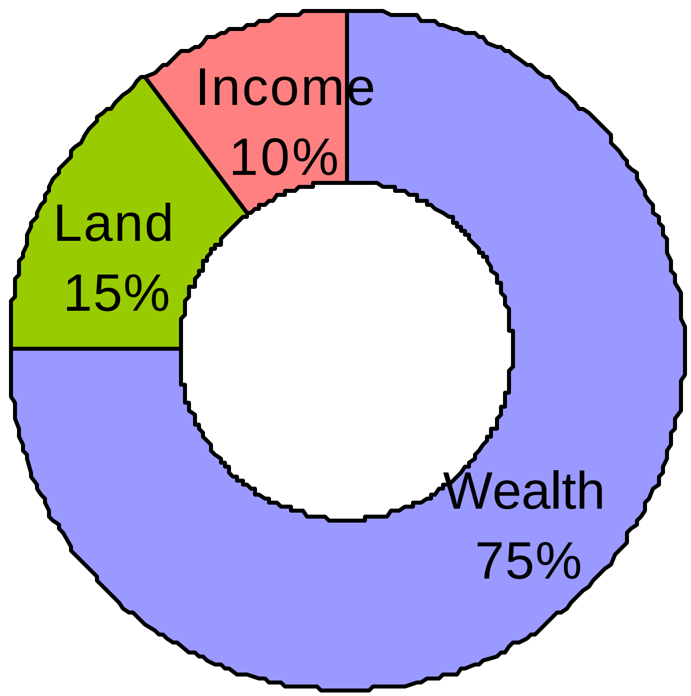
Shift the tax burden: Proposal 2 “Family values: Protect inheritance, and protect our nation’s farms”



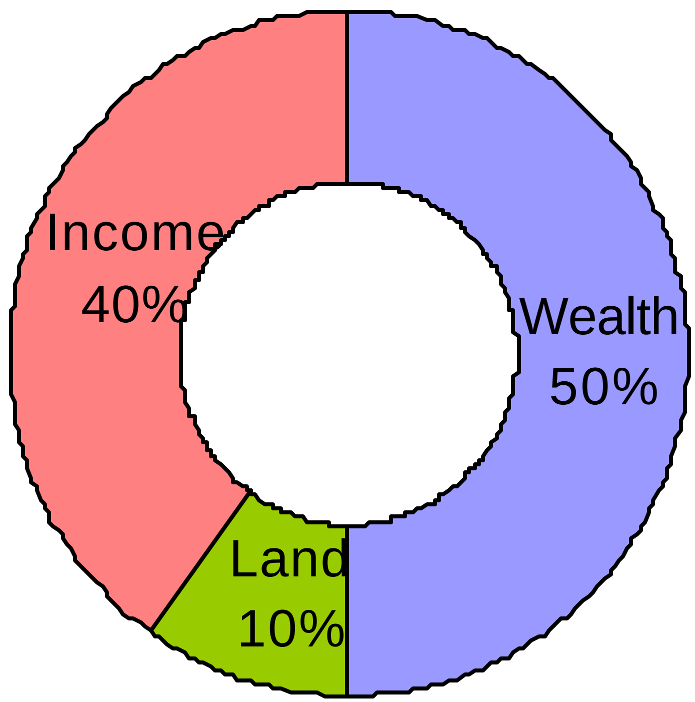
Shift the tax burden: Proposal 3 “Soak the rich!”



Shift the tax burden: Proposal 4 “Save our cities!”



Shift the tax burden: Proposal 5 “Family values: Protect inheritance, and protect our nation’s farms”



Cycling majorities shifting the tax burden

Proposal	Proposer			
2	Rich	Cut taxes on	wealth ,	land
3	Wage-earners	Cut taxes on	income,	land
4	Rich	Cut taxes on	income,	wealth
5	Farmer	Cut taxes on	wealth,	land
6	Wage-earners	Cut taxes on	?	?

Conclusion

- Majority rule seems to be deeply flawed in handling the “most political” of political problems

What's so special about majority rule?

May showed that

Majority rule <=> A, N, M

So arguing for against majority rule means arguing for or agains A,N, or M

Condition A (Anonymity)

Social preferences depend only on the collection of individual preferences, not on who has which preference.

Condition N (Neutrality)

changing rank of *j* and *k* in each group members preferences changes rank of *j* and *k* in group preferences (i.e. naming the alternatives is arbitrary)

Condition M (Monotonicity)

- if *j* is at least as good a *k* from the group's standpoint, and *j* becomes more desirable to one of the members, then *j* is now strictly better than *k* from the standpoint of the group.

When does majority rule make sense?

ex. Should grades be determined by majority rule?

ex. Should what I have for breakfast be decided by majority rule?

ex. Should amendments to the constitution be decided by majority rule?

ex. Should students at a public high school be allowed to vote on whether or not to have organized prayer at football games?

We already saw that

- Majority rule creates practical problems in some situations
- May not be normatively appealing in all situation

So why don’t we ditch it?

Arrows theorem – Majority rule is not special

- The pathologies of majority rule apply to “any” group decision procedure that meets some minimal standards

These minimal standards can be thought of a generalizations of May's conditions for majority rule

May Condition		Arrow Condition
Anonymity	✍	Dictatorship
Neutrality	✍	Independence
Monotonicity	✍	Pareto Optimality

A (Anonymity) is a special case of what Arrow called "**Non-Dictatorship" (D)**

There is no distinguished individual i* ? G whose preferences dictate the group preference, independent of other members.

N (Neutrality) is a special case of what Arrow called "Independence from Irrelevant Alternatives" (I)

if j and k stand in a particular relationship to each other for each member of the group, and this relationship does not change, then neither should the group preference between j and k

M (Monotonicity) is a special case of what Arrow called "Unanimity" (P) or Pareto Optimality

If every member of G prefers j to k (or is indifferent between them), then the group preference must reflect a preference for j over k (or an indifference between them.

Arrow argued that any reasonable procedure for making group choices should involve D, I, and P, and two other criteria:

Condition U (Universal admissibility) (each *i* e G may adopt any strong or weak complete and transitive preference ordering over the alternatives in A)

Rationality assumption R_G is complete and transitive.

Arrow’s theorem

There exists no mechanism for translating the preferences of rational individuals into a coherent group preference that simultaneously satisfies conditions U,P,I, and D

Conclusion

Arrow showed that if you accept U,P,I as “untouchable” (May shows us that advocating majority rule amounts to making U,P,I untouchable) you have accept either

1. Dictatorship
2. The *potential for* intransitivity