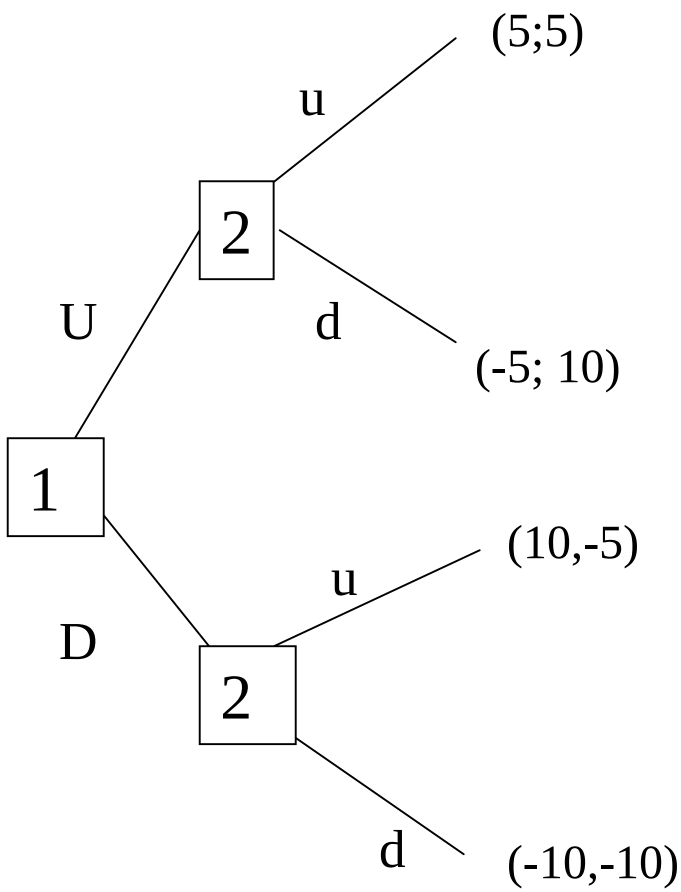
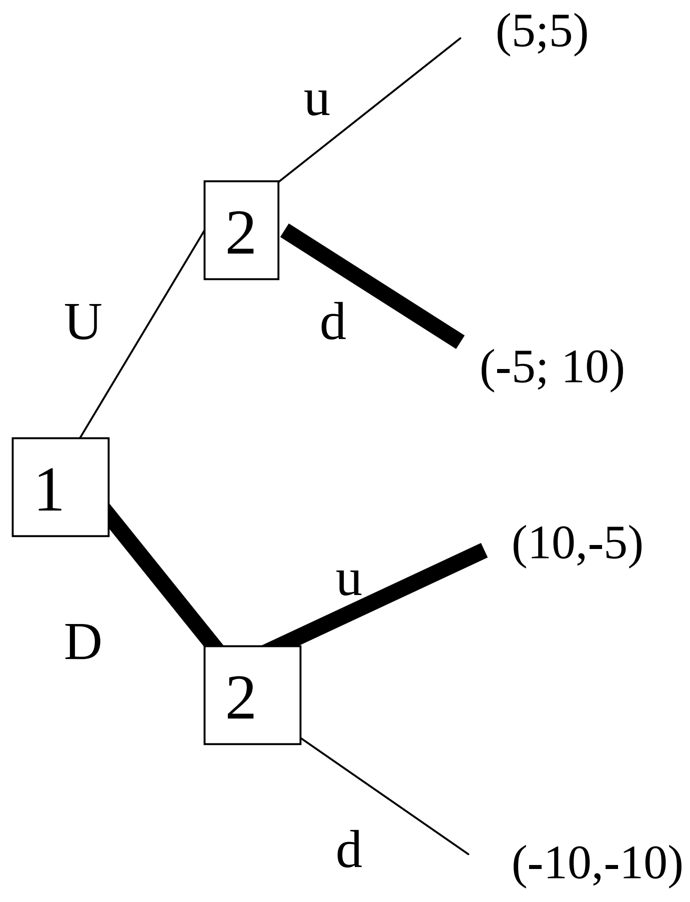
Game Theory Review

Solving for Nash and Subgame Perfect Equilibria in Extensive and Strategic Form Games

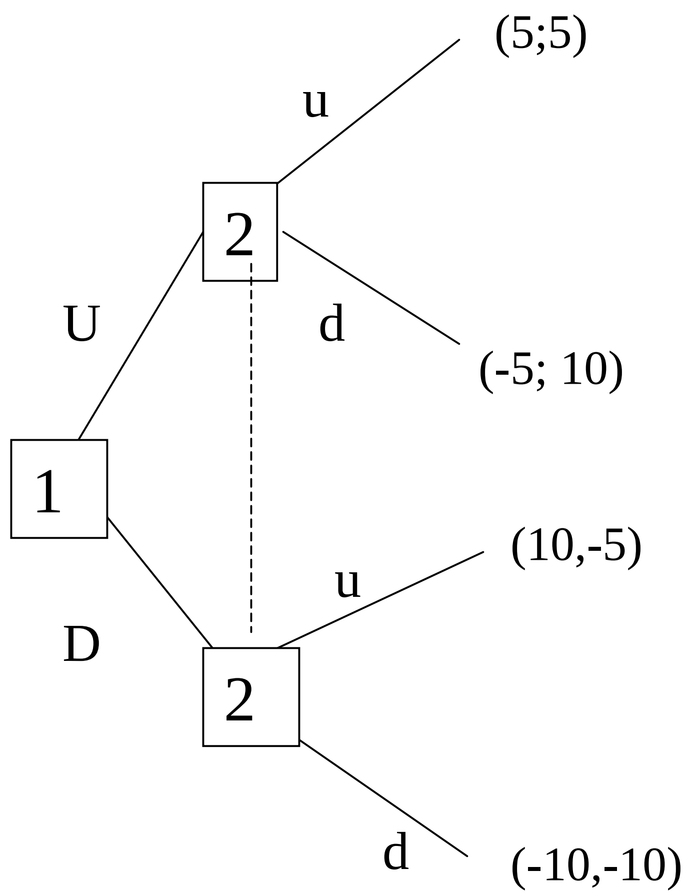
A Game



(D;d,u) is subgame perfect



What if player 2 can't observe what player 1 chose?



Strategic form game

Player 2

Player 1

	u	d
U	5,5	-5,10
D	10,-5	-10,-10

Player 1 asks "What if player 2 plays "u"?

Player 2

Player 1

	u	d
U	5,5	-5,10
D	**10**,-5	-10,-10

Player 1 asks
“what if player 2 plays “d”?

Player 2

Player 1

	u	d
U	5,5	-5,10
D	10,-5	-10,-10

Player 2 asks "what if player 1 plays "U"?

Player 2

Player 1

	u	d
U	5,5	-5,10
D	10,-5	-10,-10

Player 2 asks "what if player 1 plays "D"?

Player 2

Player 1		u	d
	U	5,5	-5,10
	D	10,-5	-10,-10

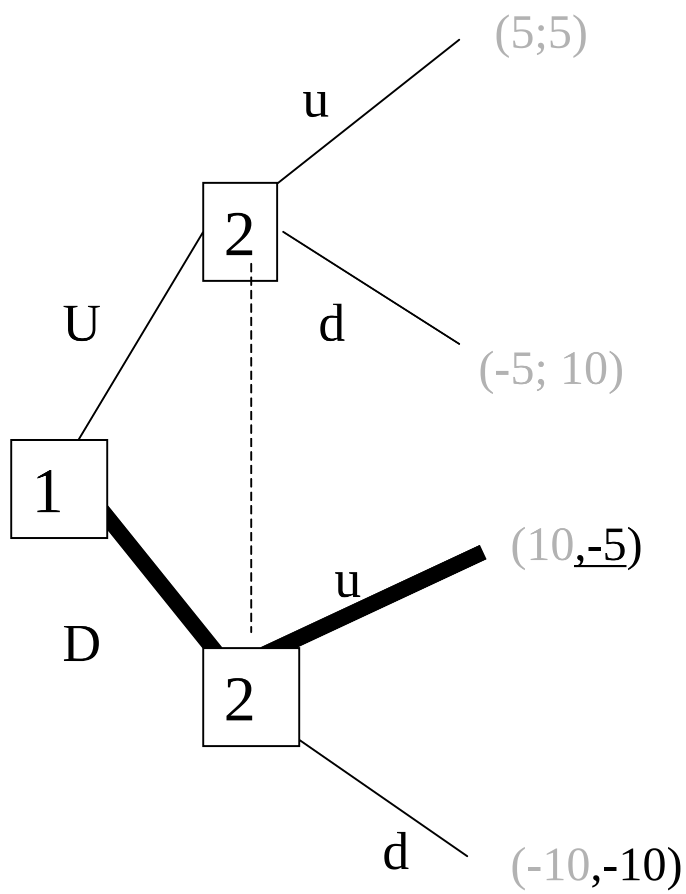
There are two Nash equilibria to the game (D;u) and (U;d)

Player 2

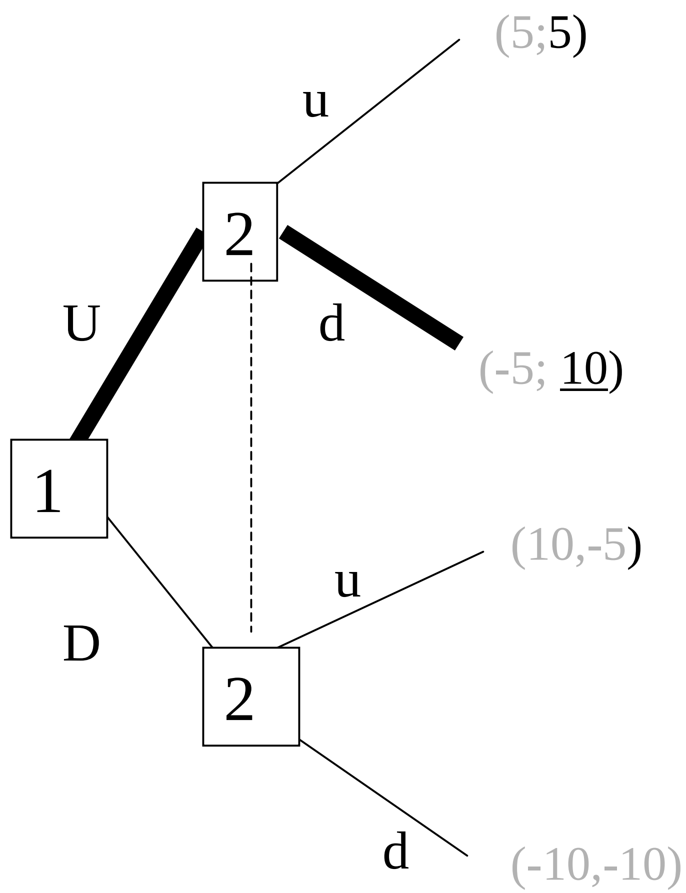
Player 1

	u	d
U	5,5	-5,10
D	10,-5	-10,-10

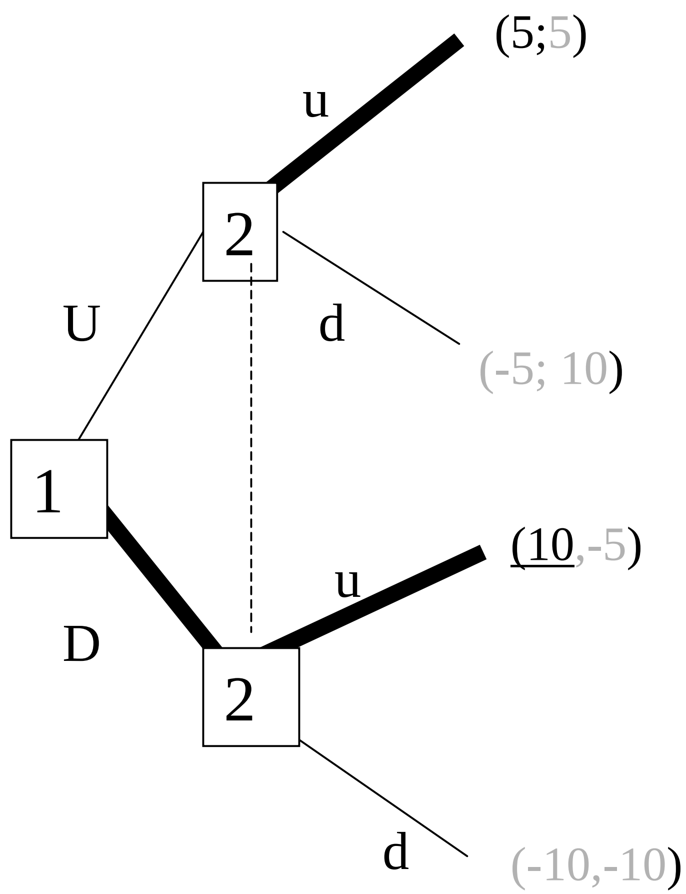
Now, lets return to extensive form; player 2 asks… what if player 1plays D?



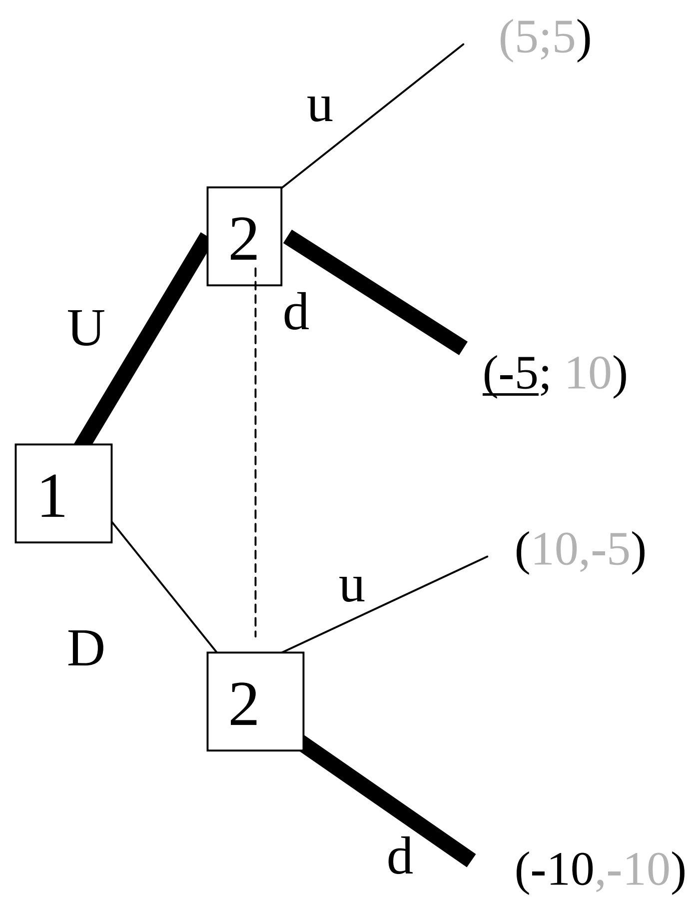
And, what if player 1 plays U?



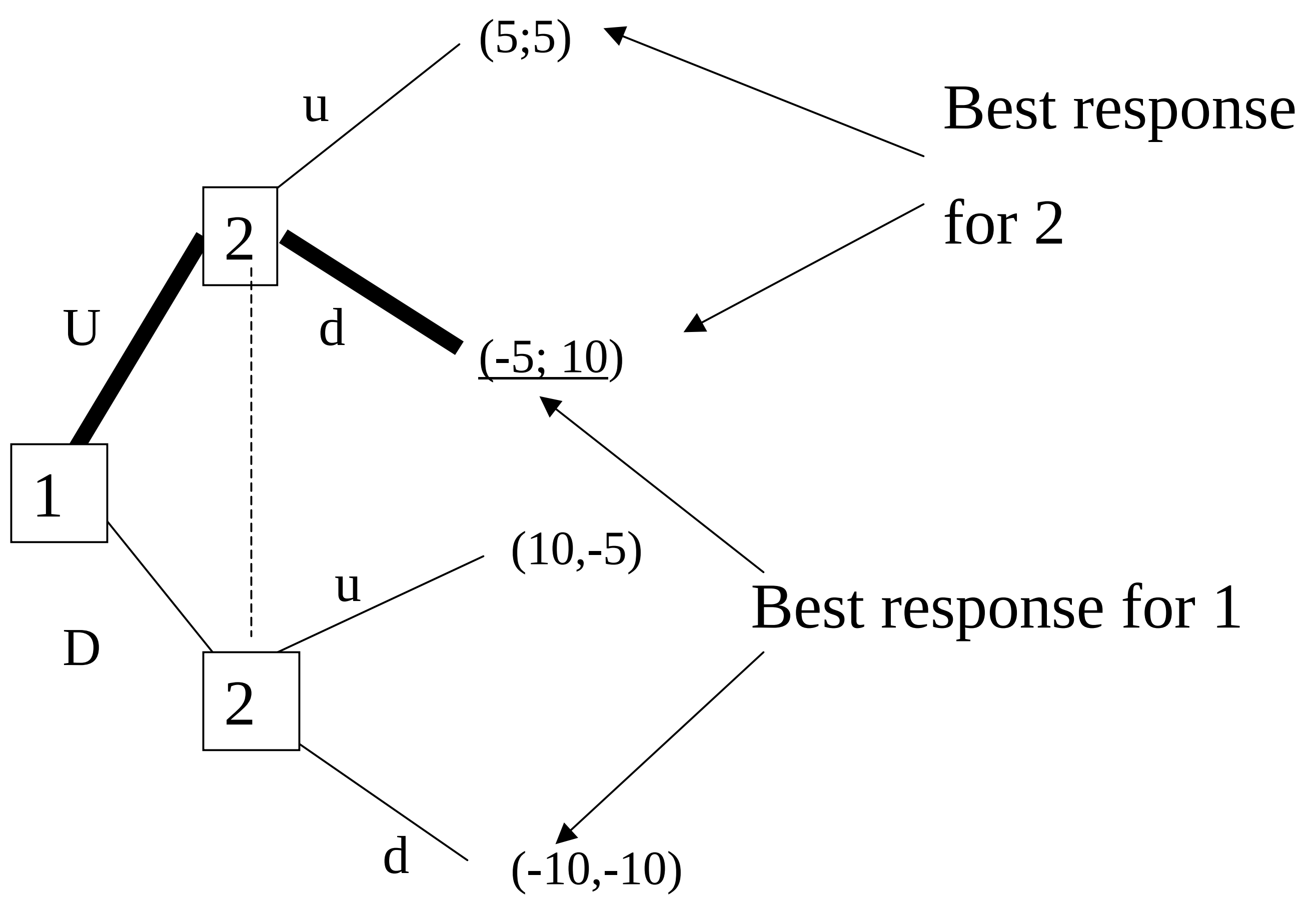
And player 1 asks what if player 2 plays u?



What if player 2 plays d?

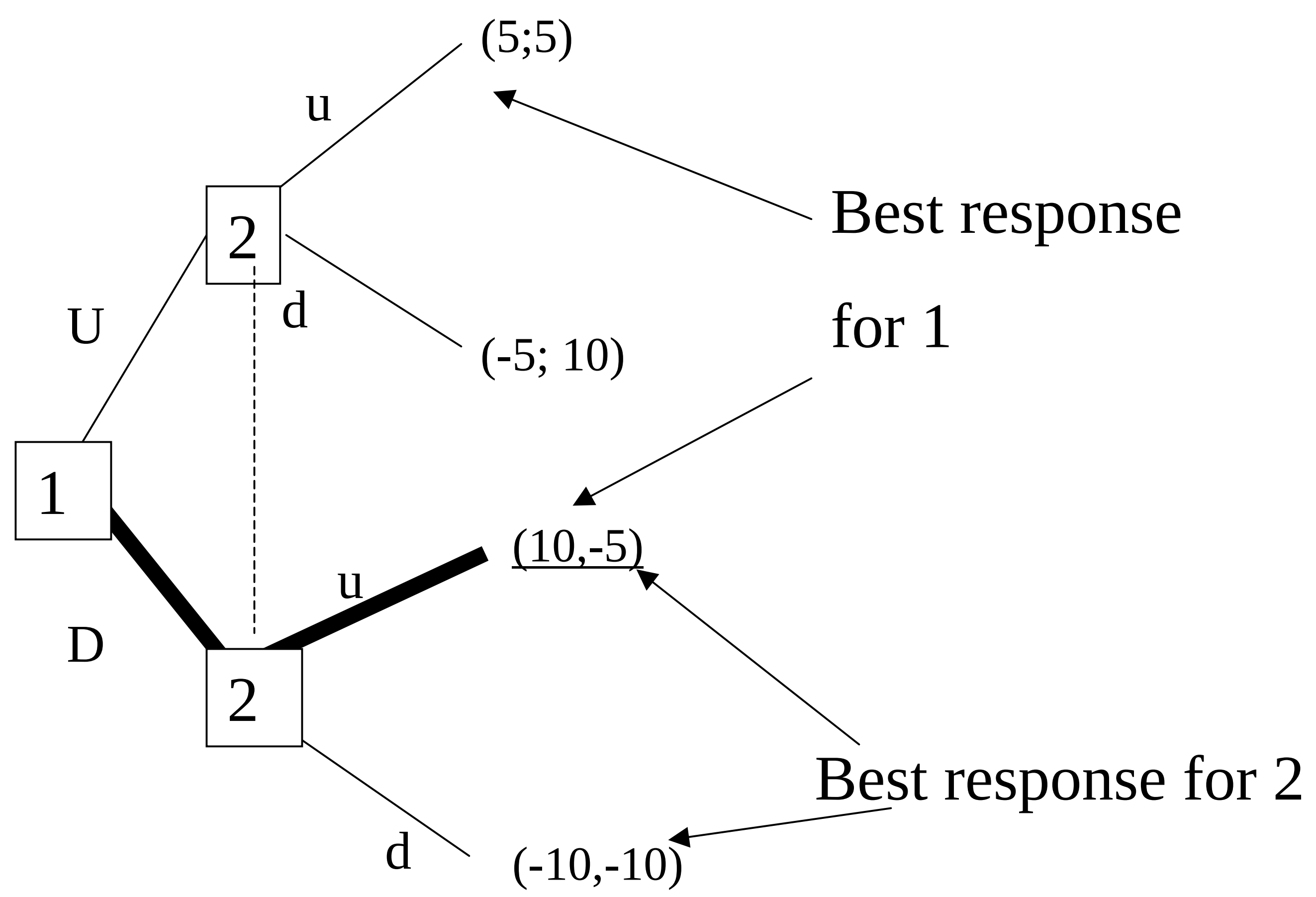


(U,d) is Nash

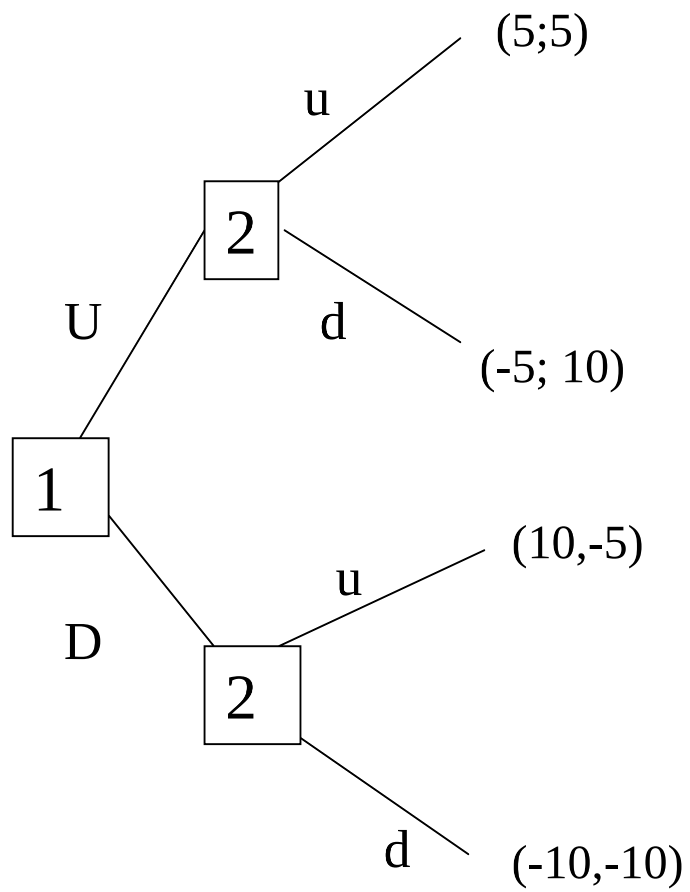


(5;5)
u
Best response
for 2
2
U
d
(-5; 10)
1
(10,-5)
u
Best response for 1
D
2
d
(-10,-10)

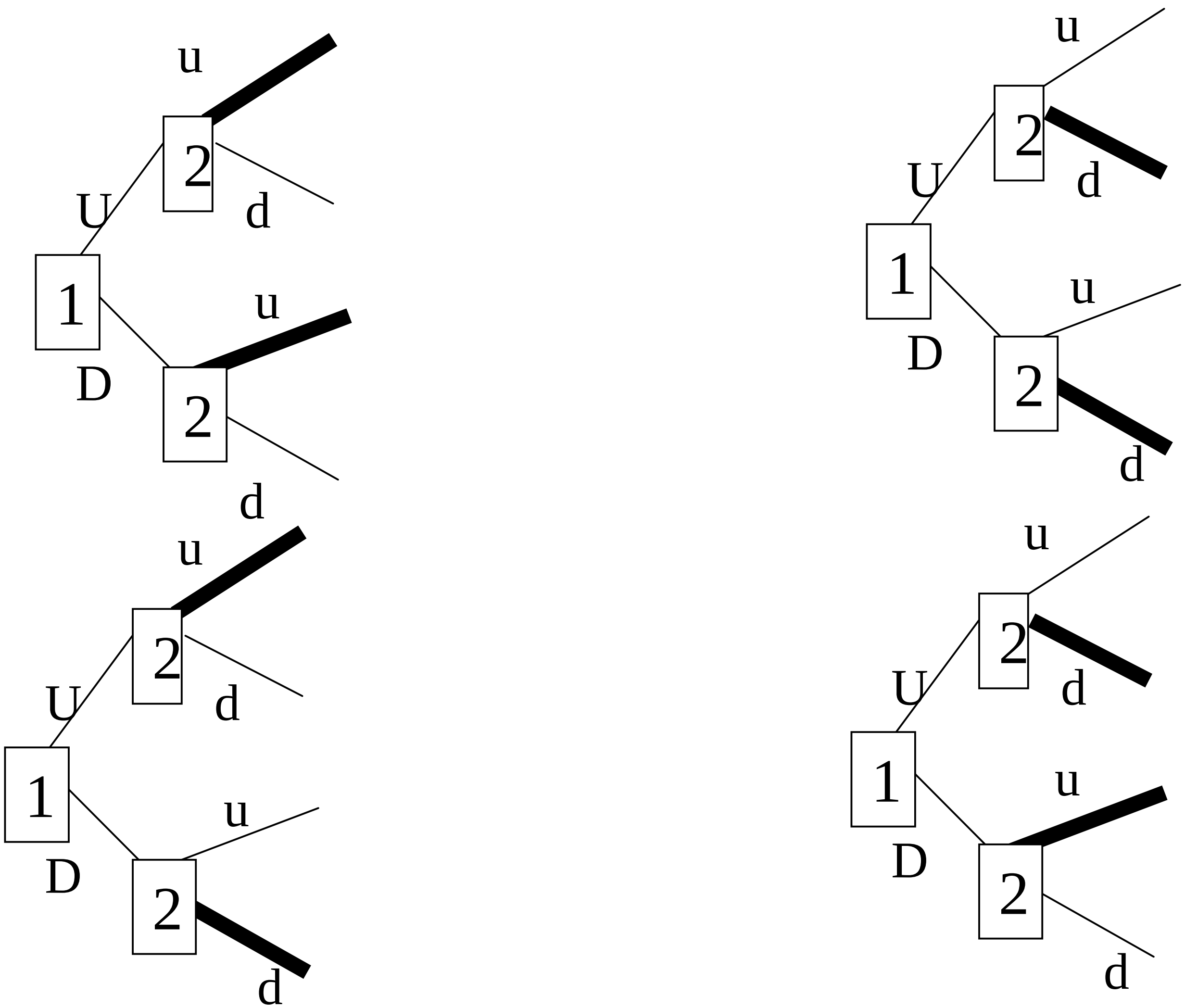
(D,u) is Nash


(5;5)
u
2
d
U
(-5; 10)
1
(10,-5)
u
D
2
d
(-10,-10)
Best response
for 1
Best response for 2

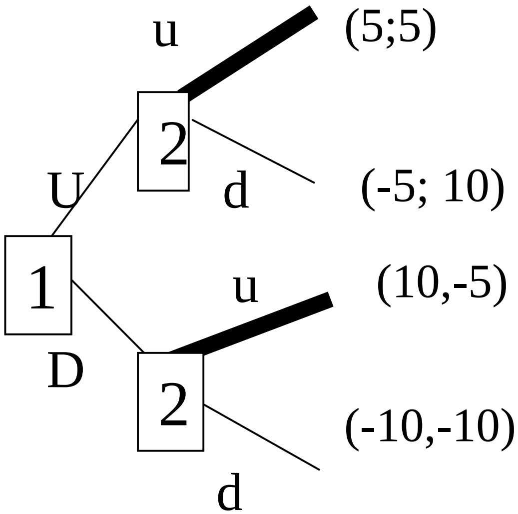
What are the Nash equilibria if player 2 *can* observe player 1's move?

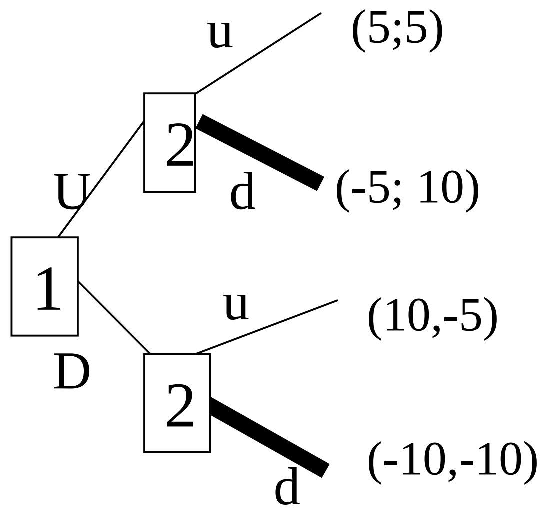


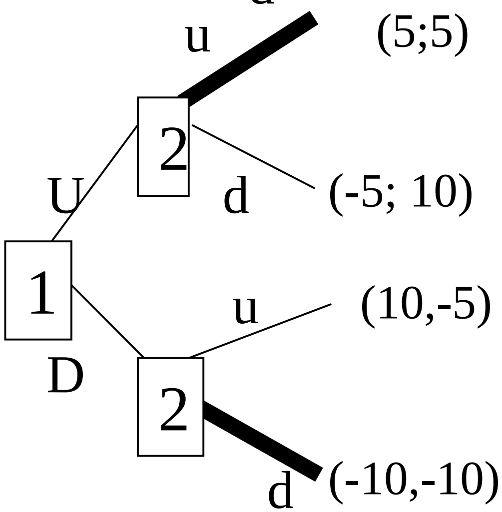
The key to answering this question correctly is realizing that player 2 now has 4 unique strategies (u,u), (u,d), (d,d), (d,u),

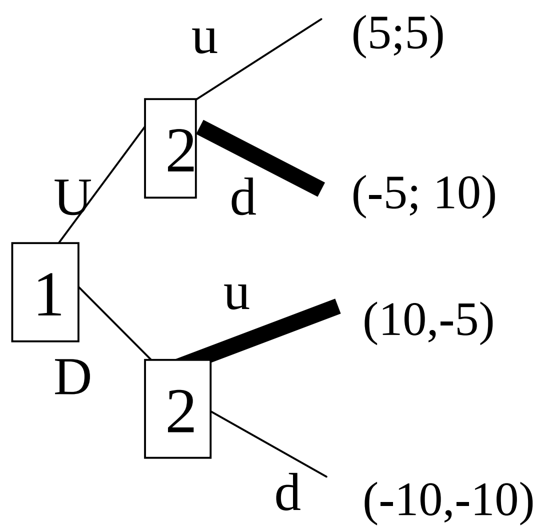


Which strategies are player 2's best responses?

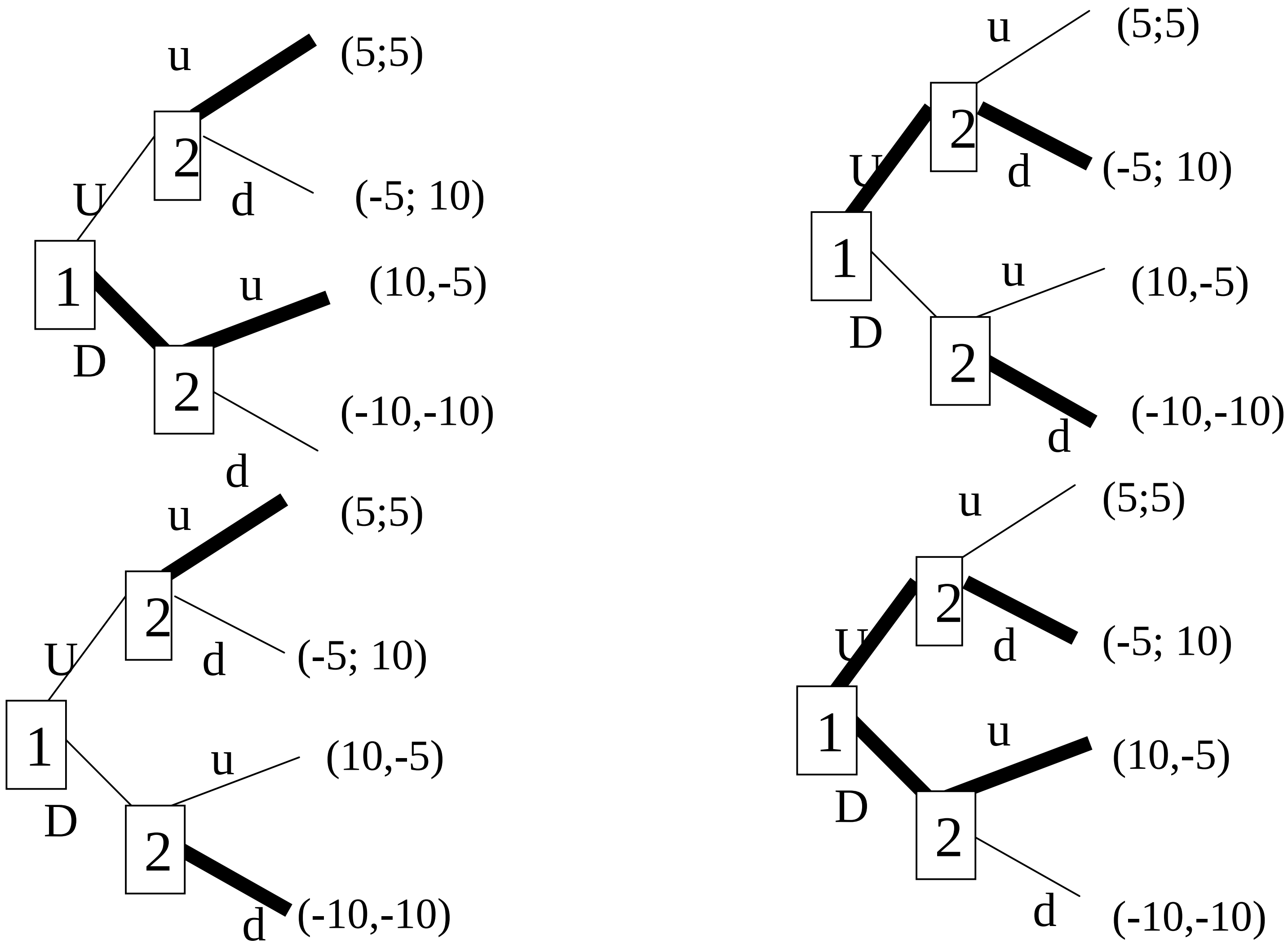








Which strategies are player 2's best responses?



So from player 2's perspective,

- (u,u) is a best response to player 1 playing D
- (d,d) is a best response to player 1 playing U
- (d,u) is a best response to player 1 playing D
- (d,u) is a best response to player 1 playing U

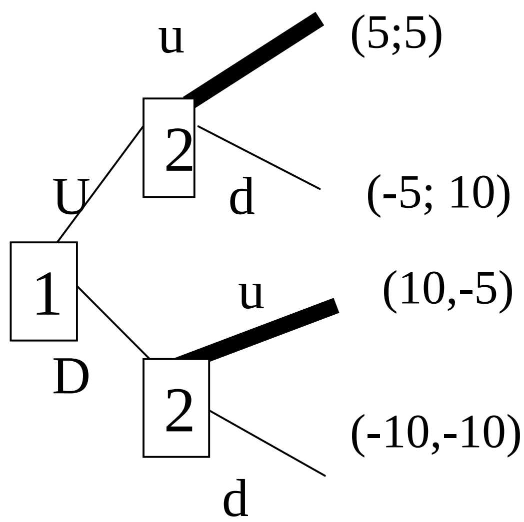
(Note, (d,u) is a player 1's *dominant strategy*, that is, it is a best response "no matter what player 1 does".)

We’re only half-way home

- Because (u,u), (d,d) and (d,u) constitute all the best responses on the party of player 2, they *might* be part of a Nash equilibrium and any equilibrium that has player 2 doing something else (u,d) is *definitely* part of a nash equilibrium.
- But we also need to consider the situation from player 1’s perspective.
  - Specifically, what is player 1’s best response to each of the proposed equilibrium strategies for player 2?

What should player 1 do When 2 plays (u,u)?

(recalling that this was 2's best response to (D))



What should player 1 do when 2 plays (u,u)?

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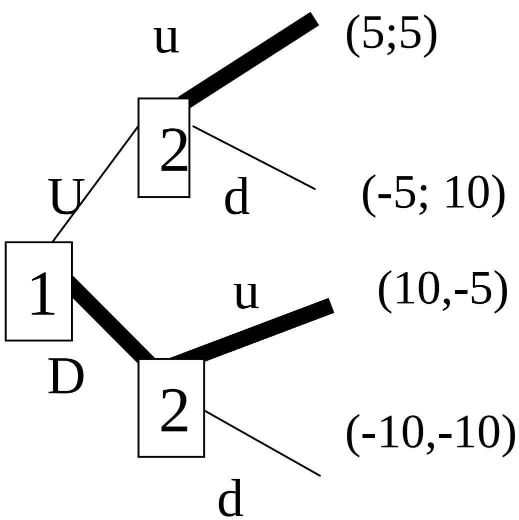
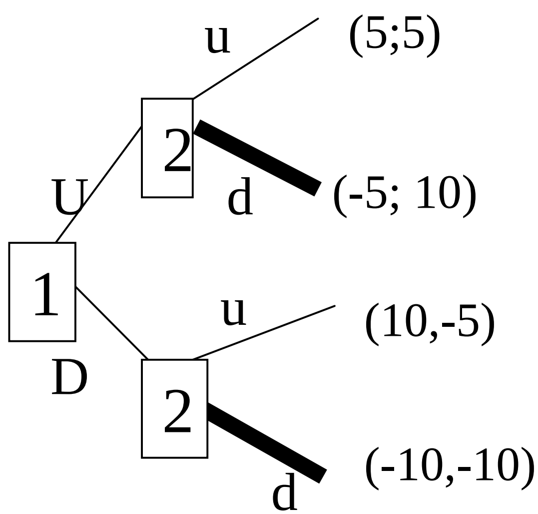


(D;u,u) is a Nash Equilibrium

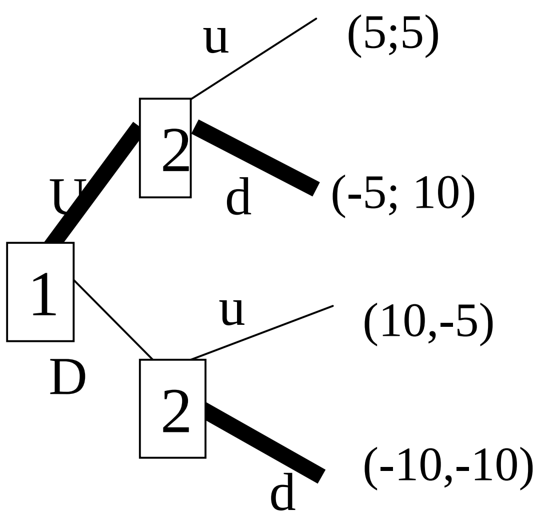
What should player 1 do is player 2 is expected to play (d,d)?

(Recalling that this is a best response to player 2 playing U)



What should player 1 do is player 2 is expected to play (d,d)?

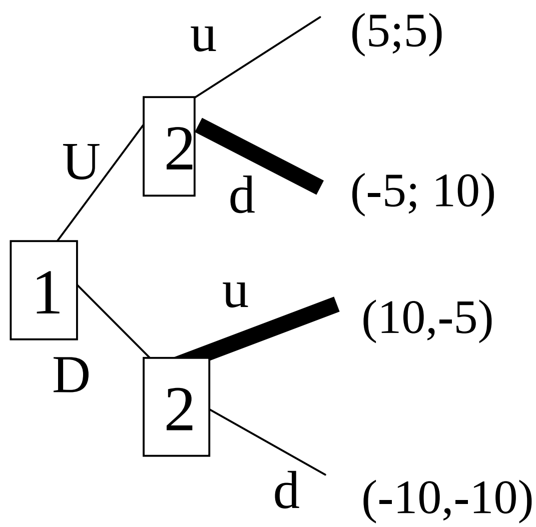
(Recalling that this is a best response to player 2 playing U)



(U;d,d) is a Nash Equilibrium

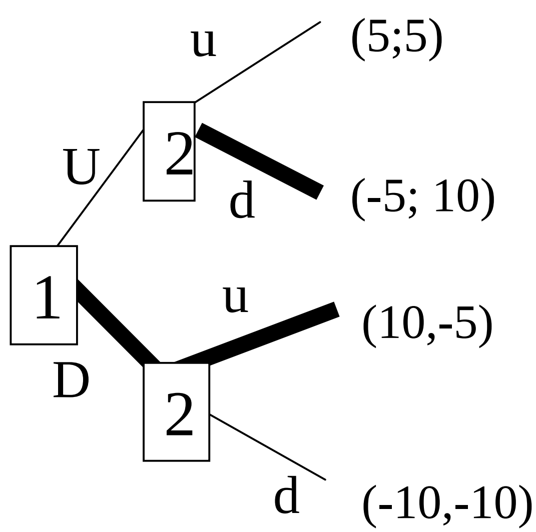
What should player 1 do is player 2 is expected to play (d,u)?

(Recalling that this is a best response to player 2 playing U or D)



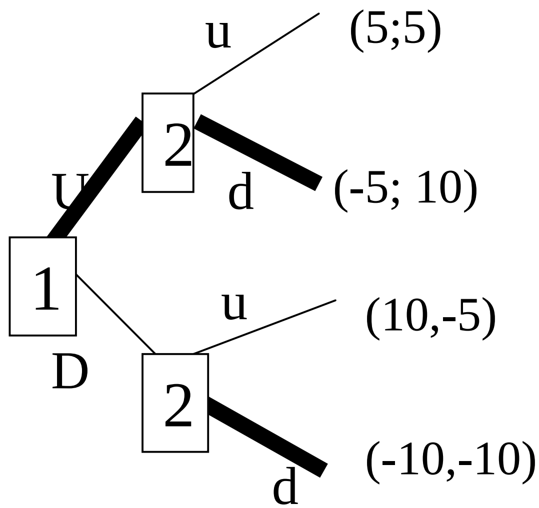
What should player 1 do is player 2 is expected to play (d,u)?

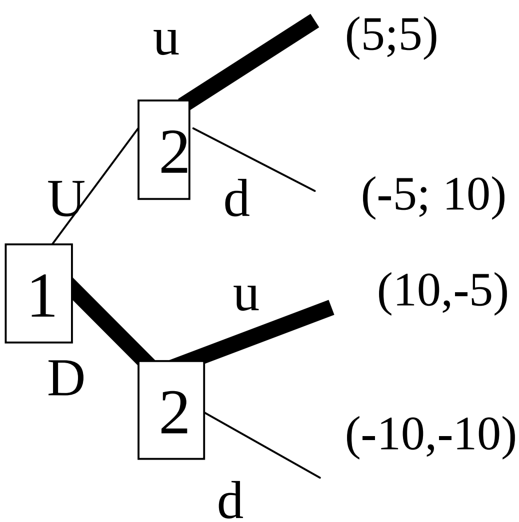
(Recalling that this is a best response to player 2 playing U or D)

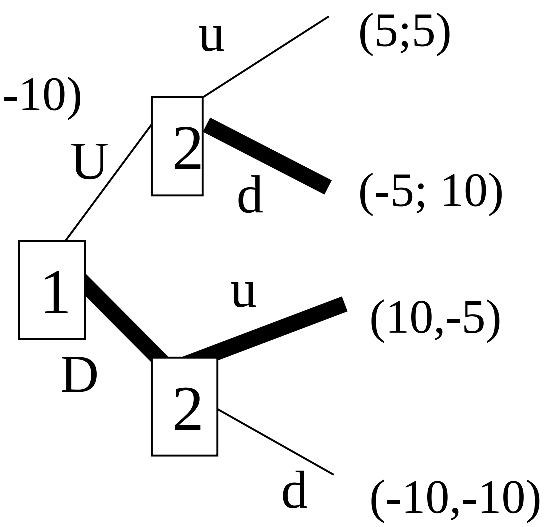


(D; d,u) is a Nash Equilibrium

(D;u,u), (U;d,d) and (D;d,u) are all Nash equilbria.
But which one is our best prediction of behavior?

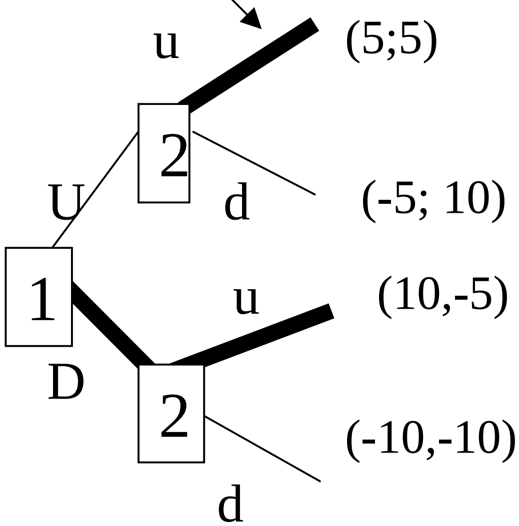


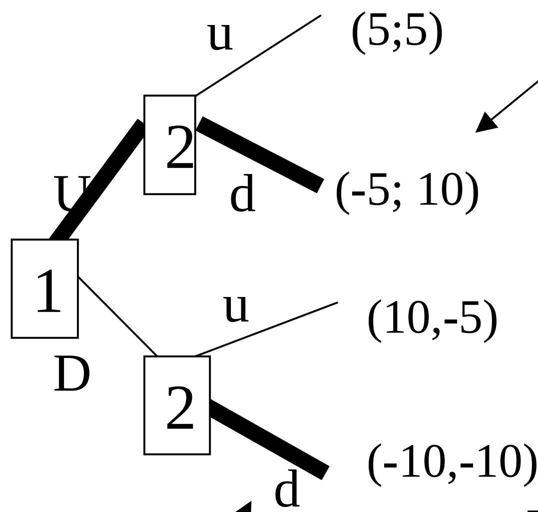




(D;u,u), (U;d,d) and (D;d,u) are all Nash equilibria. But which one is our best prediction of behavior?

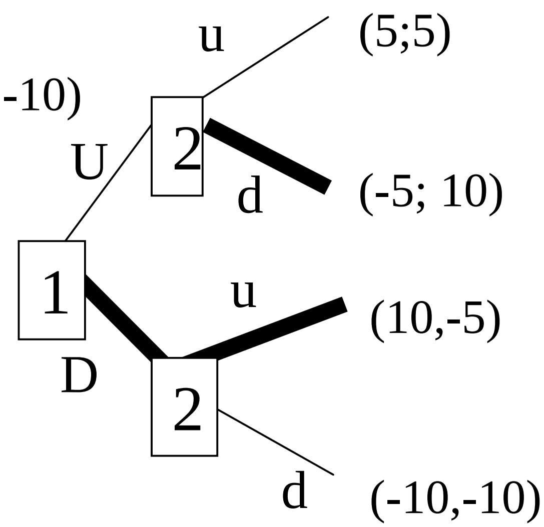
Player 2 makes an incredible threat that actually works against his interest!



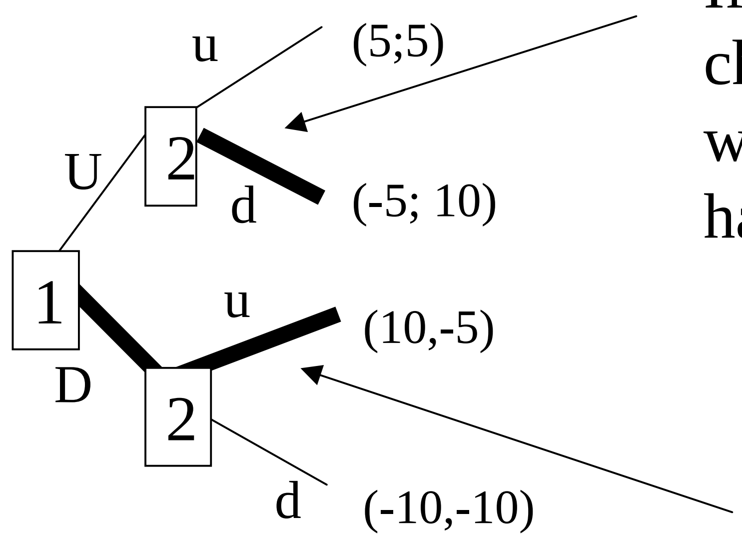


Player 2 gets his best outcome if this equilibria is played

But it depends on an 1 believing an incredible threat



Of the three Nash equilibria, only (D;d,u) is subgame perfect



If only he had the chance, player 2 would be only to happy to play D

Though he wishes he didn’t have to; player 2 plays u in response to D

Conclusions

- Not all Nash Equilibria are sub-game perfect equilibria
  - Those that are not involve a player that moves later making “incredible” threats or promises.
    - I.e. threats or promises that he or she could not be expected to carry out when the time came to implement them.

Conclusions

A very large part of “politics” involves

1. trying to convince other actors that the threats or promises that you are making are “credible” or “self-enforcing”. (Signalling)
2. Trying to determine if other actor’s threats or promises are credible. (Screening)
3. Trying to change the environment so that formerly incredible threats or promises are now credible. (Institutional Design or solving “Commitment problems”.

The benefits of commitment

- In the game where players chose at the same time, there were two Nash equilibria (D;u) and (U;d) and, since there was only one sub-game, they were both sub-game perfect.

Player 2

Player 1		u	d
	U	5,5	-5,10
	D	10,-5	-10,-10

Benefits of commitment, cont…

- But when player 1 was able to move first (I.e. was able to credibly commit to a strategy), then the only subgame perfect strategy was