The Importance of the Median Voter

According to Duncan Black and Anthony Downs
Committee Decisions

utility

\[ 0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad 100 \]
Single-Peakedness Condition

- The preferences of group members are said to be *single peaked* if the alternatives under consideration can be represented as points on a line, and each of the utility functions representing preferences over these alternatives has a maximum at some point on the line and slopes away from this maximum on either side.
Ms. 5 has single-peaked preferences, Mr. 1 does not
Ms. 5’s “preferred to y” set

utility

0

y=80

z=92

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P_5(y)

x_5

u_5
Each actor has single peaked preferences
And a “preferred to y” set
The "winset" of $y$ contains the alternatives that can "win" a contest against $y$. 

![Diagram showing utility levels and decision points.]

- $P_1(y)$
- $P_2(y)$
- $P_3(y)$
- $P_4(y)$
- $P_5(y)$

Utility levels $u_1, u_2, u_3, u_4, u_5$. x-axis represents decision points $x_1, x_2, y, x_3, x_4, x_5$.
Under majority rule a winset is the set of outcomes preferred by a majority.
In this case, the winset of $y$ is the same as $P_3(y)$.
So, if Ms.3, Mr.4, or Ms.5 are allowed to make a proposal, what will they propose?
Black’s Median Voter Theorem

If members of a group $G$ have single-peaked preferences, then the ideal point of the median voter has an empty winset
Ms. 3’s ideal point has an empty winset

\[
P_3(x_3) = \emptyset
\]
A Model of an Election

• An electorate has to elect a chief executive and
  – There are 2 candidates
  – Voters have single-peaked preferences that can be arrayed along a one-dimensional ideological continuum that is “the proportion of economic activity left in the hands of the private sector.” And they vote for candidate with expected policy closest to their ideal point.
  – Candidates “only want to get elected”
Order of Moves

1. Candidate 1 announces her policy position \((y_1)\)
2. Candidate 2 announces his policy position \((x_1)\)
3. All voters vote

WHAT HAPPENS?
Lets examine a case where there are 5 (equally sized) groups of voters, such that all members of a group have the same preferences.
What does Candidate 1 do? Let's say she is the incumbent and her position is viewed by the electorate as the weighted average of all her prior actions, and this happens to be at $y_1$. 

![Diagram showing utility and various points labeled G1, G2, G3, G4, G5 with utility levels u1, u2, u3, u4, u5 and values at specific points labeled (125) for each.]
What is candidate 2’s best response to 1’s record?
A position just to the right of the incumbent will get candidate 375 votes, leaving 250 for candidate 1.
But in our model, Candidate 1’s policy is up to her, so she would choose her policy to make the two group of voters on each side of her the same size.
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When she chooses a policy position (or, as the incumbent, *implements a policy*) that is equal to the median voter, there are 250 voters on each side of her
Candidate 2’s best response is also to adopt the median voter’s position, at which point, the elections is, effectively, tied.
This result does not depend on the distribution of voters into these groups. Different distributions will change where the median voter is and what policy gets implemented, but will not alter the centrifugal tendencies of the system.
If a third party candidate observed this and was allowed to enter the race, she would adopt a position just to the left of the median voter, she’s get 250 votes and 1 and 2 would each get $\frac{372}{2}$ utility.
But 1 and 2 might foresee this and adopt policies that are just far enough from median voter to deter entry.
For example, if $y_1$ chose $G_2$’s ideal point and $y_2$ chose $G_4$’s ideal point, there’s no where $y_3$ to enter with any chance to win.
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Let's say 1 is a “socialist” and 2 is a “capitalist”. 1 wants policy 1*, 2 wants policy 2*
If 1 implements 1*, 2 can win the next election by proposing a policy in G₃’s “preferred to 1*” set
This sets off a logic that leads these socialists and capitalist to both propose moderate policies.
When politics is single-dimensional, the median voter rules

- We get some separation because of “entry deterrence”
- It doesn’t matter whether candidates are policy-oriented or office-seeking.
- These results depend crucially on the assumption that everybody votes