How to Elect A Representative Committee using Approval Balloting

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Abstract

Approval balloting is applied to the problem of electing a representative committee. We demonstrate several ways to identify such a committee, paying particular attention to weighting methods that can reduce the influence of voters whose views are very different from those of other voters. We show that a general class of voting systems based on approval ballots can be implemented through analysis of an appropriate table. A by-product of this procedure is clarification of the complexity of these systems.

1 Introduction

Approval voting is a well-known voting procedure applicable to single-winner elections. Voters approve of as many candidates as they like, and the candidate with the most approvals wins [Brams and Fishburn, 1978, 1983]. But this method of aggregating approval votes is not the only one possible, as Merrill and Nagel [1987] argue. It is therefore useful to distinguish between *approval balloting* (each voter submits a ballot that identifies which candidates are approved) and *approval voting* (the method, indicated above, by which approval ballots are tallied to determine the winner).

In this paper, we discuss how approval ballots can be used to select a committee—a subset of the candidates—that represents, in some sense, all voters. In such an election, voters would be instructed to indicate on their approval ballots the subsets of candidates that best represent them on the committee.

Our procedures for identifying a most representative subset of the set of candidates capitalize on the fact that each ballot also specifies a subset of this set. Of course, different voters will typically specify different subsets. We view the problem of identifying the most representative committee as that of identifying the subset that is "closest" to the collection of subsets specified by the voters. Our procedures can be adapted to reflect restrictions on the size or composition of the committee to be elected.

Based on an appropriate measure of distance, we discuss two criteria of closeness to the collection of subsets specified by the voters, *minimax* (a representative subset should minimize the maximum distance to the subsets of all voters) and *minisum* (a representative subset should minimize the sum of distances or, equivalently, the average distance to the subsets of all voters). Elsewhere we offer a broader discussion of criteria of fairness in electing committees. [Brams, Kilgour, and Sanver, 2005]

2 Terminology and Notation

There are $n > 1$ voters. The set of $k > 1$ candidates is denoted $C = \{1, 2, \ldots, k\}$. We represent a subset of the candidates $S \subseteq C$ by a (row) vector $p = (p_1, p_2, \ldots, p_k)$, where $p_h = 1$ if $h \in S$ and $p_h = 0$ otherwise. (Usually we will write subsets in vector notation without punctuation—for example, 1001101 designates the subset comprising candidates 1, 4, 5, and 7.)

The n voters' ballots are $p^1, p^2, \ldots, p^n$, which we write as rows of a 0-1 matrix, P, called the *ballot data matrix*. Note that P has n rows and k columns; the entry in row i and column h of P, p^i_h, is 1 if voter i approves of candidate h and 0 otherwise.

Example 1 *There are $n = 3$ voters and $k = 3$ candidates. Voter 1's ballot is 100 (i.e. voter 1 approves of candidate 1 only), whereas voter 2's ballot is 110, and voter 3's ballot is 101. The ballot data matrix is*

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

The ballot data matrix records the subset of candidates approved by each voter. Because we wish to construct anonymous voting systems, we need not maintain a record of which voter approved of a particular subset. Moreover, as the number of voters grows (and the number of candidates becomes relatively small), it is increasingly likely that some voters will cast identical ballots—that is, approve of exactly the same subset. If so, the ballot data matrix will contain many identical rows. To simplify the data, we record only the distinct ballots (in any order), and the number of times each is chosen.

More specifically, we associate with the voted-for subsets, $q^1, q^2, \ldots, q^\ell$, corresponding counts $m_1, m_2, \ldots m^\ell$, indicating that $m_j > 0$ different voters approve of exactly the subset q^j. It follows that $\sum_{j=1}^{\ell} m_j = n$ and $q^j \neq q^{j'}$ whenever $j \neq j'$. As before, we write $q^1, q^2, \ldots, q^\ell$ as rows of a 0-1 matrix Q, called the *compressed ballot data matrix.* Note that Q has ℓ rows and k columns, and the entry in row j and column h of Q is denoted q^j_h. Associated with Q is a *count vector*, m, which is a column vector with the count, m_j, as its j^{th} entry. We call (Q, m) *compressed ballot data.*

Example 2 *There are $n = 4$ voters and $k = 3$ candidates. Voters 1, 2, and 3 vote as in Example 1; voter 4 votes exactly as voter 3. The compressed ballot data is (Q, m) where*

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad m = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

To measure the distance between two subsets of C, p and q, we will use the *Hamming distance*, $d(p, q) = \sum_{h=1}^{k} |p_h - q_h|$. Thus, $d(p, q)$ equals the number of components of p and q that are different, or the number of candidates who are in one of p or q but not the other. Note that for any subsets p and q of C, $0 \leq d(p, q) \leq k$ and, of course, $d(p, q) = 0$ iff $p = q$. To illustrate using Examples 1 and 2, $d(100, 110) = 1$ (the two subsets differ only on candidate 2), and $d(110, 101) = 2$ (the subsets differ on candidates 2 and 3).

3 Minisum and Minimax Criteria

We begin by addressing the problem of selecting a representative subset of candidates, p, given a ballot data matrix, P. We assume that there are no restrictions on the subset to be selected.

This problem was considered by Brams, Kilgour, and Sanver [2004]. One solution they proposed was majority voting (MV), which can be implemented on P by summing each column to obtain the total vote for each candidate. An MV committee is a committee comprising only candidates who receive at least $\frac{n}{2}$ votes.[1] There is always at least one MV committee; in the extreme case in which every candidate receives fewer than $\frac{n}{2}$ approvals, the MV committee contains no members, that is $00\ldots0$.

Formally, for ballot data P, the number of votes for candidate h is $n_P(h) = \sum_{i=1}^{n} p_h^i$. The set of all MV committees is then

$$MV(P) = \left\{ p \subseteq C : p_h = 1 \text{ if } n_P(h) > \frac{n}{2} \text{ and } p_h = 0 \text{ if } n_P(h) < \frac{n}{2} \right\}. \quad (1)$$

For instance, in Example 1, the numbers of votes for candidates 1, 2, and 3 (the column sums of P) are $n_P(1) = 3$, $n_P(2) = 1$, and $n_P(3) = 1$, respectively. Because only candidate 1 receives more than $\frac{n}{2} = \frac{3}{2}$ votes, and no candidate receives exactly $\frac{3}{2}$ votes, the unique MV committee is 100 (i.e. it includes only candidate 1).

Brams, Kilgour, and Sanver [2004, Proposition 4] proved that a subset of the candidates, p, is an MV winner if and only if it minimizes $\sum_{i=1}^{n} d(p, p^i)$. Thus, the MV winners are the subsets of candidates that are at minimum total distance (or, equivalently, at minimum average distance) from the voters' ballots. For this reason, they referred to an MV committee as a *minisum* committee. We define

$$minisum(P) = MV(P).$$

There are two or more committees in $minisum(P)$ whenever at least one candidate receives precisely $\frac{n}{2}$ votes. If so, the total distance from the ballots to a committee containing such a candidate is exactly equal to the total distance from the ballots to the same committee without the candidate, rendering both of these committees members of $minisum(P)$.

[1] We adopt this definition, rather than the standard requirement of "more than $\frac{n}{2}$ votes," for technical reasons that will become apparent shortly. Our definition is equivalent to the conventional one whenever n is odd; when n is even, differences are unlikely unless n is small. Our definition implies that the MV committee is not unique (two or more subsets are tied for winning) if and only if n is even and at least one candidate receives precisely $\frac{n}{2}$ votes. In this case, an MV committee includes all candidates who receive more than $\frac{n}{2}$ votes, plus any subset of the set of candidates who receive exactly $\frac{n}{2}$ votes.

As a second approach to finding a representative committee, Brams, Kilgour, and Sanver [2004] adapted the unanimity version of the Fallback Bargaining procedure [Brams and Kilgour, 2001]. They proposed an iterative procedure that takes place in discrete time, $t = 0, 1, 2, \ldots$. At time t, voter i is modeled as willing to be represented by any subset of candidates in his or her *acceptable set*

$$A^i_P(t) = \{p \in C : d(p^i, p) \leq t\}. \tag{2}$$

For example, at time $t = 0$, voter i's only acceptable subset is p^i, the subset specified on his or her approval ballot. At time $t = 1$, $A^i(t)$ includes p^i and any subset at Hamming distance 1 from p^i —any subset that differs from p^i in exactly one candidate, who might be a member of p^i now excluded, or a non-member of p^i now included. At time $t = 2$, voter i's acceptable set includes all subsets at Hamming distance at most 2 from p^i, and so on. This fallback process continues until there is a subset that is acceptable to all voters.

Formally, the fallback process ends at time t^*_P defined by

$$t^*_P = \min\{t = 0, 1, \ldots : \bigcap_{i=1}^{n} A^i_P(t) \neq \emptyset\}. \tag{3}$$

The fallback (FB) winners are all subsets acceptable to all voters at time t^*_P. Formally,

$$FB(P) = \left\{p \subseteq C : p \in A^i_P(t^*_P), i = 1, \ldots, n\right\}. \tag{4}$$

For instance, in Example 1, the three voters' acceptable subsets at times 0 and 1 are given in the following table:

	$A^1(t)$	$A^2(t)$	$A^3(t)$
$t = 0$	$\{100\}$	$\{110\}$	$\{101\}$
$t = 1$	$\{100, 000, 110, 101\}$	$\{110, 010, 100, 111\}$	$\{101, 001, 111, 100\}$

Observe that the acceptable sets at time $t = 0$ are disjoint, while the acceptable sets at time $t = 1$ have exactly one common member, namely 100. It follows that $t^*_P = 1$ and $FB_P = \{100\}$. Hence the unique FB committee corresponding to P is 100, according to this iterative procedure in which acceptable sets for each voter become larger and larger over time until there is at least one subset in common.

Based on a result of Brams and Kilgour [2001, Theorem 3], Brams, Kilgour, and Sanver [2004] concluded that a subset, p, is an FB winner if and only if it minimizes $\max_{i=1} d(p, p^i)$. Thus, the FB winners are the subsets of candidates for which the maximum distance to any voter's ballot is a minimum. For this reason, they referred to an FB committee as a *minimax* committee. We define

$$minimax(P) = FB(P).$$

Brams, Kilgour, and Sanver [2005] next asked whether duplication of ballots could change the set of winning committees. In our more general setting, we consider how the minisum and minimax procedures can be applied to compressed ballot data, (Q, m), as opposed to ballot data, P.

It is immediate that, for compressed ballot data, the number of votes for candidate h is $n_{Q,m}(h) = \sum_{j=1}^{\ell} m_j q_h^j$. Given this emendation, the definition of the set of minisum committees,

$$minisum(Q, m) = \left\{ p \subseteq C : p_h = 1 \text{ if } n_{Q,m}(h) > \frac{n}{2} \text{ and } p_h = 0 \text{ if } n_{Q,m}(h) < \frac{n}{2} \right\}, \quad (5)$$

is essentially unchanged from (1).

When the minimax procedure is applied to ballot data, P, (2) defines the acceptable set of voter i at time t. For compressed ballot data, the acceptable set at time t for a voter who voted for q^j is, analogously,

$$A^j_{Q,m}(t) = \{p \in C : d(q^j, p) \leq t\}. \quad (6)$$

The set of minimax winners can then be determined using

$$t^*_{Q,m} = \min\{t = 0, 1, 2, \ldots : \bigcap_{j=1}^{\ell} A^j_{Q,m}(t) \neq \emptyset\};$$
$$minimax(Q, m) = \left\{ p \subseteq C : p \in A^j_{Q,m}(t^*_{Q,m}), j = 1, \ldots, \ell \right\},$$

which is essentially unchanged from (2), (3), and (4).

As Brams, Kilgour, and Sanver [2005] noted, the minimax winners are not altered by the duplication of ballots. In our terms, $minimax(Q, m)$ does not depend on the count vector, m, because FB committees reflect only which subsets were voted for, not how many votes each one received. While this property is consistent with some approaches to fairness (the FB committees are the committees that for which the worst-represented voter is best represented, which recalls Rawls's [1971] approach to justice), it makes the outcome highly sensitive to outliers and thus not representative of any tendency of voters to cast similar ballots.

Brams, Kilgour, and Sanver [2005] suggested that it might be appropriate to revise the minimax procedure so that the rate of increase of the acceptable subset centered at q^j depends inversely on m_j. Applying this variation to compressed ballot data would yield, instead of (6),

$$A^{j\,\prime}_{Q,m}(t) = \left\{ p \in C : d(q^j, p) \leq \frac{t}{m_j} \right\}. \quad (7)$$

Then the revised minimax winners would be determined using

$$t^{*\prime}_{Q,m} = \min\{t \in [0,\infty) : \bigcap_{j=1}^{\ell} A^{j\prime}_{Q,m} i(t) \neq \emptyset\};$$

$$minimax'(Q,m) = \left\{p \subseteq C : p \in A^{j\,\prime}_{Q,m}(t^{*}_{Q,m}), j = 1,\ldots,\ell\right\}.$$

Note that the iterative fallback process still starts at time $t = 0$, but it now takes place in continuous (rather than discrete) time. (Since the m_j are integers, the threshold times $t^{*\prime}_{Q,m}$ are always rational, but we make no use of this simplification.) Note that the minimax committees produced by this adjusted procedure reflect ballot duplication: the more voters who vote for q^j, the slower the acceptable set centered at q^j grows.

Let us illustrate our results so far, taking P as in Example 1 and (Q,m) as in Example 2. We noted earlier that $minisum(P) = \{100\}$. For (Q,m), $n_{Q,m}(1) = 4$, $n_{Q,m}(2) = 1$, $n_{Q,m}(3) = 2$, and $\frac{n}{2} = 2$, so $minisum(Q,m) = \{100, 101\}$; in other words, both 100 and 101 are minisum committees for (Q,m).

We determined earlier that $minimax(P) = \{100\}$. The calculation that $minimax(Q,m) = \{100\}$ is identical. But the adjusted procedure gives the following table of acceptable sets $A^{j\,\prime}_{Q,m}(t)$:

	$A^{1\,\prime}(t)$	$A^{2\,\prime}(t)$	$A^{3\,\prime}(t)$
$t=0$	$\{100\}$	$\{110\}$	$\{101\}$
$t=1$	$\{100,000,110,101\}$	$\{110,010,100,111\}$	$\{101\}$
$t=2$	$\{100,000,110,101$ $010,001,111\}$	$\{110,010,100,111$ $000,011,101\}$	$\{101,001,111,100\}$

Note that at time $t = 1$ the three acceptable sets have no common member, as they did in the previous table (100). But at time $t = 2$, there are three common members; formally, $t^{*\prime}_{Q,m} = 2$ and $minimax'(Q,m) = \{100, 101, 111\}$.

4 Weighted Distances

The main objective of this paper is to show how different weightings—which take into account, for example, similarities among ballots—can affect the determination of representative committees. The first observation is simple: If we write $A_j = A^{j\,\prime}_{Q,m}$, then (7) is equivalent to

$$A_j(t) = \left\{p \in C : w_j d(q^j, p) \leq t\right\}, \tag{8}$$

provided $w_j = m_j$. Thus, whether a particular subset belongs to the acceptable subset centered at q_j can be understood to depend on weighted distances.

It follows from (8) that increasing the weight of q_j tends to draw the winning subset closer to q_j, in the sense that at any particular time, t, fewer alternatives are acceptable to a voter who supported q^j. One weighting of interest, the *count weight*, is based on the count vector; the weight of a voted-for subset, q^j, is $w_j = m_j$, the number of voters who voted for q^j.

Another weighting, which we will argue is useful for both minimax and minisum procedures, is *proximity weight*, in which the weight of q^j reflects the extent to which a q^j voter is similar to, or different from, other voters. Specifically,

$$w_j = \frac{m_j}{\sum_{r=1}^{\ell} m_r d(q^j, q^r)}, \qquad (9)$$

so that the weight of q^j is proportional to m_j, the number of voters who voted for q^j. The denominator of the fraction in (9) is the sum of the Hamming distances from q^j to the subsets approved by all voters. (Of course, $d(q^j, q^j) = 0$, so the distance to q^j does not contribute to this sum.) Thus m_j, the weight of q^j, is small when few voters approve exactly q^j or any subset similar to it. By giving them less weight, proximity weighting tends to make the outcome less sensitive to the views of "extreme" voters—that is, voters whose ballots differ substantially from those of most other voters.

In general, consider compressed ballot data (Q, m), and assume that a positive weight, w_j, has been assigned to each q^j. (There is no requirement other than that all weights be positive; in particular, no normalization is assumed—the weights need not sum to 1.) We define the weighted vote for candidate $h = 1, \ldots, k$ to be

$$n_w(h) = \sum_{j=1}^{\ell} w_j q_h^j. \qquad (10)$$

The natural threshold of weighted votes is $S = \frac{1}{2}\sum_{j=1}^{\ell} w_j$, or half the total weight. Thus, a weighted majority rule or MV_w committee is every committee that includes all candidates whose weighted vote is greater than S and no candidates whose weighted vote is less than S.[2] Below, we show that the MV_w committees are precisely the minisum committees in a weighted-distance context.

We think of $w_j d(p, q^j)$ as the weighted distance between a voted-for subset q^j and a possible committee $p \in C$. Then (8) defines the subsets acceptable to a q^j voter at time t. Then the unanimity version of the fallback bargaining procedure can be implemented using

[2]We follow the same convention as discussed in the previous footnote. In particular, a candidate whose weighted vote is exactly S may or may not be included in an MV_w committee.

(8) and

$$t^* = \min\{t \in [0,\infty) : \bigcap_{j=1}^{\ell} A_j(t) \neq \emptyset\}; \tag{11}$$

$$FB_w = \{p \subseteq C : p \in A_j(t^*), j = 1, \ldots, \ell\}\,. \tag{12}$$

The interpretation is as before: The time t^* is the earliest moment that at least one subset is acceptable to all voters, and FB_w is the set of all subsets that are acceptable to all voters at time t^*.

We next characterize the weighted versions of the minisum and minimax procedures—in terms of the weighted distances they minimize—to identify representative committees. At the same time, we develop a simple procedure for computing all winning committees.

Theorem 1 *A subset $p \in C$ is an MV_w committee iff it minimizes*

$$\sum_{j=1}^{\ell} w_j d(p, q^j).$$

A subset $p \in C$ is an FB_w committee iff it minimizes

$$\max_{j=1\ldots\ell} w_j d(p, q^j).$$

Proof: For any fixed $h = 1, \ldots, k$ and any fixed $j = 1, \ldots, \ell$, define

$$\delta_h(p, q^j) = \begin{cases} w_j & \text{if } q_h^j \neq p_h \\ 0 & \text{if } q_h^j = p_h \end{cases}$$

from which it follows that

$$\sum_{j=1}^{\ell} \delta_h(p, q^j) = \begin{cases} \sum_{j=1}^{\ell} w_j q_h^j & \text{if } p_h = 0 \\ \sum_{j=1}^{\ell} w_j (1 - q_h^j) & \text{if } p_h = 1 \end{cases}$$

Therefore, using (10),

$$\sum_{j=1}^{\ell} \delta_h(p, q^j) = \begin{cases} n_w(h) & \text{if } p_h = 0 \\ 2S - n_w(h) & \text{if } p_h = 1 \end{cases}$$

But $\sum_{j=1}^{\ell} w_j d(p, q^j) = \sum_{j=1}^{\ell} \sum_{h=1}^{k} \delta_h(p, q^j) = \sum_{h=1}^{k} \sum_{j=1}^{\ell} \delta_h(p, q^j)$. It follows that $p \in C$ minimizes $\sum_{j=1}^{\ell} w_j d(p, q^j)$ iff (i) $p_h = 0$ whenever $n_w(h) < S$ and (ii) $p_h = 1$ whenever $n_w(h) > S$. This proves the first statement of the theorem.[3]

[3]This part of the proof generalizes the proof of Brams, Kilgour, and Sanver [2004, Proposition 4].

The second statement follows directly from the result of Brams and Kilgour [2000, Theorem 4] that the FB_w procedure determines the set of all alternatives that minimize the maximum distance. ■

We will refer to MV_w and FB_w committees as weighted minisum and weighted minimax committees, respectively. Theorem 1 and its proof provide a simple procedure for finding all such committees.

Begin with compressed ballot data (Q, m) and a weight, w_j, for each voted-for subset, q^j. Compute a $2^k \times \ell$ matrix in which the rows represent subsets and the columns voted-for subsets. The (p, j) entry is $w_j d(p, q^j)$. Then find the sum and the maximum of the entries in each row. The rows with minimum sum correspond to the weighted minisum committees, and the rows with the minimum maximum to the weighted minimax committees.

We illustrate first with Example 2 using count weights. The columns of the table correspond to the voted-for committees, which are listed across the top. Below each voted-for committee is its weight. (Here, the weights are the components of the count vector.) The rows of the table correspond to the possible winning committees, which in this illustration are all subsets of $C = \{1, 2, 3\}$. Each entry of the table is the weighted distance between the column subset and the row subset. The two right-most columns record the row sum and row maximum.

Voted-for Committee	100	110	101		
Weight	1	1	2	$\sum$	max
000	1	2	4	7	4
100	0	1	2	3*	2*
010	2	1	6	9	6
001	2	3	2	7	3
110	1	0	4	5	4
101	1	2	0	3*	2*
011	3	2	4	9	4
111	2	1	2	5	2*

The winning subsets are indicated with asterisks. The weighted minisum committees are 100 and 101, and the weighted minimax committees are 100, 101, and 111. These results accord exactly with those given earlier.

Note also that the table simplifies an extended procedure we recommend elsewhere [Brams, Kilgour, and Sanver, 2005]: Choose either minisum and minimax as the primary criterion; use the other as a secondary criterion to break ties. Doing so leads us to discard 111 as a minimax committee, leaving 100 and 101 as the two committees selected according to both procedures.[4]

We next analyze the same example using proximity weights. As determined by (9), these work out to be $w_1 = \frac{1}{3}$, $w_2 = \frac{1}{5}$, and $w_3 = \frac{2}{3}$. (For instance, from $q^1 = 100$ the Hamming distance to $q^2 = 110$ is 1 and to $q^3 = 101$ is also 1; but q^1, q^2, and q^3 are selected by $m_1 = 1$, $m_2 = 1$, and $m_3 = 2$ voters respectively, so $w_1 = \frac{1}{1\times1+2\times1} = \frac{1}{3}$.) To make the table a little neater, we have cleared the denominators by multiplying the weights by 15, producing $15 \times \frac{1}{3} = 5$, $15 \times \frac{1}{5} = 3$, and $15 \times \frac{2}{3} = 10$ shown in the table.

Voted-for Committee	100	110	101		
Weight	5	3	10	$\sum$	max
000	5	6	20	31	20
100	0	3	10	13	10
010	10	3	30	43	30
001	10	9	10	29	10
110	5	0	20	25	20
101	5	6	0	11*	6*
011	15	6	20	41	20
111	10	3	10	23	10

Now the unique winning committee is 101 by both the minisum and minimax criteria.

Of course, there may be restrictions on the size or composition of the committee to be elected. Restrictions might be imposed, for example, to fix the exact size of the committee, to establish bounds (upper, lower, or both) on its size, or to ensure that the committee contains representatives of certain subgroups of the candidate set. One important feature of our procedures is that they can be adapted to satisfy restrictions of this sort.

In fact, changing the procedure to incorporate such restrictions is easy. Simply delete from the table all rows that correspond to subsets that fail to meet the restrictions. For example, in the table above (Example 2, using proximity weights), a plausible restriction might be that the committee to be elected is to have exactly one member. If so, only the second, third, and fourth rows of the table correspond to eligible subsets; all other rows must be removed. Clearly, the committee selected would be 100 by the minisum criterion, and either 100 or 001 by the minimax criterion.[5]

[4] A more sophisticated approach would be to order the rows of the table using the leximin ordering—see Fishburn [1974] for details.

[5] Application of the two criteria in sequence, as discussed above, would break the tie in favor of 100.

5 Conclusions

We have analyzed how minisum and minimax criteria can be used to elect a representative committee from approval ballots. As we suggested, approval balloting has a natural connection to committee elections, because both the ballot and the outcome are subsets of the candidates. The methodologies discussed here have several desirable properties—they are anonymous (treat all voters equally), neutral (treat all candidates equally), and symmetric (voting for or against a candidate influences the outcome equally).

Based on the construction and analysis of the table, it is possible to draw some inferences about the complexity of the minisum and minimax criteria. From compressed ballot data, (Q, m), we know the number of voters, n, the number of candidates, k, and the number of voted-for committees, ℓ. If weights are given, and there are no restrictions on the winning subset, the minisum winner(s) can be calculated from the weighted votes of the candidates, making the complexity of the minisum procedure roughly ℓk.

For the minimax procedure, we note that the table has $\ell 2^k$ entries, each of which is a (weighted) distance determined by ℓk comparisons. Once the table is constructed, the determination of each row maximum, and the comparison of the maxima, takes relatively few steps. We conclude that the complexity of the minimax procedure is roughly $\ell^2 k 2^k$.

Clearly, the minisum procedure has a lower order of complexity than the minimax procedure. Nonetheless, even when $\ell = n$ (the worst case), the minimax procedure is exponential in the number of candidates, but it is polynomial in the number of voters.[6]

[6] Of course, we have not included the calculation of weights. For example, proximity weights require the determination of all ℓ^2 distances between voted-for subsets.

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