$$L(y_t) = y_{t-1} \tag{1}$$
$$L^2(y_t) = y_{t-2} \tag{2}$$

We can thus write lag structures in terms of lag polynomials (in L) as

$$(1 + \beta_1 L + \beta_2 L^2 + \dots)y_t = y_t + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \dots \tag{3}$$

Using this notation, we can write the following (ADL for auto-distributed lag) model

$$y_t = \rho y_{t-1} + \beta_0 x_t + \beta_1 x_{t-1} + \epsilon_t \tag{4}$$

in lag polynomial terms as

$$(1 - \rho L)y_t = (\beta_0 + \beta_1 L)x_t + \epsilon_t \tag{5}$$

or

$$y_t = \frac{\beta_0 + L\beta_1 L}{1 - \rho L} x_t + \frac{\epsilon_t}{1 - \rho L} \tag{6}$$

$$1 + \rho L + \rho^2 L^2 + \rho^3 L^3 + \dots = \frac{1}{1 - \rho L} \tag{7}$$

which shows the equivalence of the lagged dependent variable and the geometric lag model. We can also go the other way.

$$\frac{\Phi(L)}{1 - \rho L} x_t = (1 + \rho L + \rho^2 L^2)(\Phi(L) x_t \tag{8}$$
$$= \Phi(L)x_t + \rho\Phi(L)x_{t-1} \dots \tag{9}$$

We can write the Box-Jenkins ARMA model as

$$y_t = \frac{\Phi(L)}{\Theta(L)} \epsilon_t \tag{10}$$

Finally, consider a model with two ind vars with geometrically declining lag. We can write this as

$$\begin{aligned}
y_t = &\quad \tfrac{1}{1-\rho L} x_t + \tfrac{1}{1-\phi L} z_t + \epsilon_t \\
(1 - \rho L)(1 - \phi L) y_t = &\quad (1 - \phi L)x_t + (1 - \rho L)z_t \\
&\quad +(1 - \rho L)(1 - \phi L)\epsilon_t \\
(1 - (\rho + \phi)L + \rho\phi L^2)y_t = &\quad 1 - \phi L)x_t + (1 - \rho L)z_t \\
&\quad +(1 - (\rho + \phi)L + \rho\phi L^2)\epsilon_t
\end{aligned}$$

which has two lags of y, single lags of x and z and a second order moving average error.

If $y_t = \rho y_{t-1} + \epsilon_t$, we can write this as a $(1 - \rho L)y_t = \epsilon$, so ρ is a root of the polynomial in L. If this root is one, we have a random walk.