\[ L(y_t) = y_{t-1} \]  
\[ L^2(y_t) = y_{t-2} \]

We can thus write lag structures in terms of lag polynomials (in L) as

\[(1 + \beta_1 L + \beta_2 L^2 + \ldots) y_t = y_t + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \ldots \]

Using this notation, we can write the following (ADL for auto-distributed lag) model

\[ y_t = \rho y_{t-1} + \beta_0 x_t + \beta_1 x_{t-1} + \epsilon_t \]

in lag polynomial terms as

\[(1 - \rho L) y_t = (\beta_0 + \beta_1 L)x_t + \epsilon_t \]

or

\[ y_t = \frac{\beta_0 + L\beta_1 L}{1 - \rho L} x_t + \frac{\epsilon_t}{1 - \rho L} \]

\[ 1 + \rho L + \rho^2 L^2 + \rho^3 L^3 + \ldots = \frac{1}{1 - \rho L} \]

which shows the equivalence of the lagged dependent variable and the geometric lag model. We can also go the other way.

\[ \Phi(L)x_t = (1 + \rho L + \rho^2 L^2)(\Phi(L)x_t) \]
\[ = \Phi(L)x_t + \rho \Phi(L)x_{t-1} + \ldots \]

We can write the Box-Jenkins ARMA model as

\[ y_t = \frac{\Phi(L)}{\Theta(L)} \epsilon_t \]

Finally, consider a model with two ind vars with geometrically declining lag. We can write this as

\[ y_t = \frac{1}{1 - \rho L} x_t + \frac{1}{1 - \phi L} z_t + \epsilon_t \]
\[ (1 - \rho L)(1 - \phi L)y_t = (1 - \phi L)x_t + (1 - \rho L)z_t + (1 - \rho L)(1 - \phi L)\epsilon_t \]
\[ (1 - (\rho + \phi) L + \rho \phi L^2)y_t = 1 - \phi L)x_t + (1 - \rho L)z_t + (1 - (\rho + \phi) L + \rho \phi L^2)\epsilon_t \]

which has two lags of y, single lags of x and z and a second order moving average error.

If \( y_t = \rho y_{t-1} + \epsilon_t \), we can write this as a \( (1 - \rho L)y_t = \epsilon \), so \( \rho \) is a root of the polynomial in L. If this root is one, we have a random walk.