# The Priority Rule: A Worked Example

*Preliminaries*

Consider two research programs competing for resources. I will assume that each program has a success function—a function yielding a program's probability of achieving its goal for any given level of investment—of the form

$$s(n) = p\,\frac{n}{n + 33}$$

where $n$ is the number of worker-years devoted to the program. The coefficient $p$ represents a program's intrinsic potential; programs with higher values of $p$ have greater intrinsic potential.[1]

For the sake of the example, I will assume that the two competing programs have values for $p$ of 0.5 and 0.3. The success functions for these two values of $p$ are graphed in figure 1. The program for which $p = 0.5$ has a greater probability of success for all values of $n$ than the $p = 0.3$ program, thus it satisfies the requirement for having a higher intrinsic potential than the other program, as implied by the identification of the parameter $p$ as a measure of intrinsic potential. It can be seen that both programs offer decreasing marginal returns on investment: as $n$ increases, the probability of success increases, but at an ever-slower rate. The marginal returns are graphed explicitly in figure 3.

1. I use the same function to represent the responsiveness of both programs to investment for convenience only; it is not required for any of the mathematical results underlying my arguments.

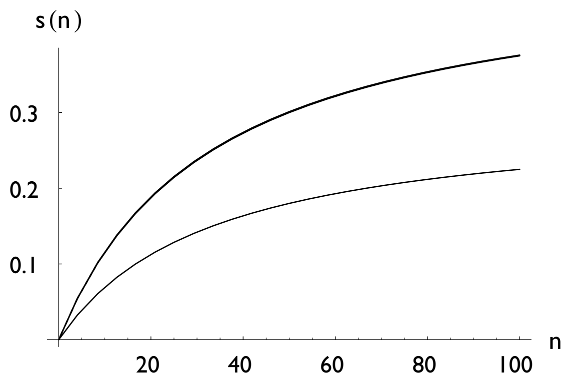


*Figure 1:* Success functions for research programs with intrinsic potentials of 0.5 and 0.3.

*The Additive Case*

Suppose that the two programs pursue independent goals, and so that the expected returns from the programs are additive. Suppose also that the goals are of equal value $v$ to society. There are, let us say, 100 worker-years to be distributed between the programs. The problem is to find the number of worker-years $n$ to allocate to the first, higher potential program so as to maximize the expected return from both programs, that is, so as to maximize

$$v(s_1(n) + s_2(N - n)).$$

The value of $v$ is unimportant to the solution of the problem; essentially the problem is to maximize the expression in parentheses, $s_1(n) + s_2(N - n)$, which is the expected number of successes.

The graph of the expected number of successes as a function of $n$ is shown in figure 2. As can be seen, the optimal distribution of labor allocates workers to both

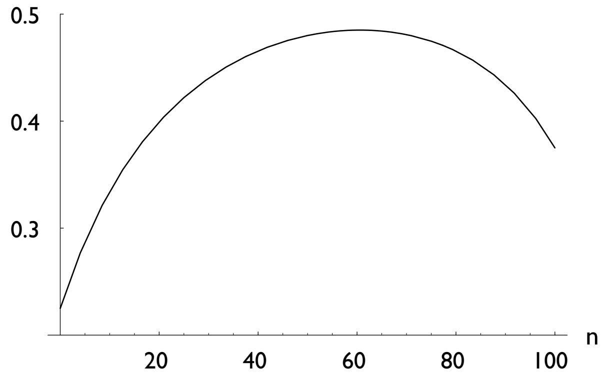


*Figure 2:* The expected number of successes when $n$ worker-years are assigned to the higher potential program and $N - n$ worker-years to the lower potential program.

programs, but more workers to the higher potential program.

The precise value of $n$ for which the expected number of successes is highest is that for which the marginal return functions are equal, that is, the $n$ for which

$$m_1(n) = m_2(N - n).$$

The marginal return functions for the two programs are shown in figure 3. One way to find the optimal value of $n$ is to graph one marginal return function against the lateral inversion of the other, as in figure 4. The point where the graphs cross is the desired value of $n$. In this case, $n$ is just over 60. Inspection of figure 2 shows that the expected number of successes is indeed maximal at this point.

An exercise for the reader: Using geometric intuition and figure 4, satisfy yourself that 100 workers, each choosing in succession the research program that offers, at the time, the higher marginal return, will indeed allocate themselves optimally.

Here is one way to think about the problem. After $n$ worker-years have been

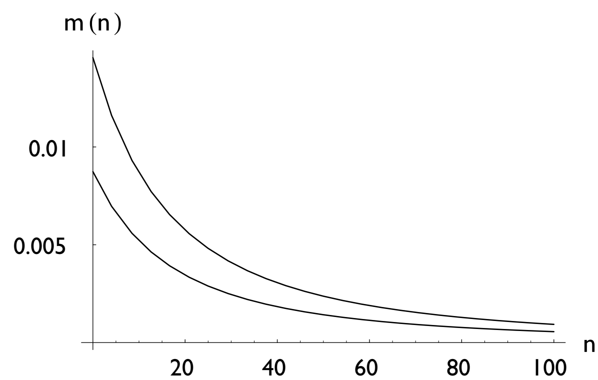


*Figure 3:* The marginal return functions for the two programs. The higher potential program has a greater marginal return on investment for any given value of $n$.

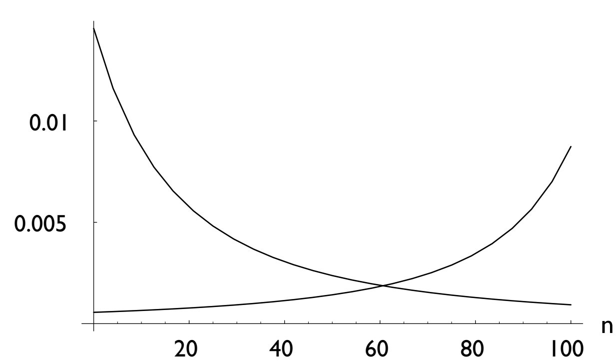


*Figure 4:* The optimal value of $n$ occurs at the point where the marginal return functions cross.

invested in a program, the reward for investing a further year is $m(n)$. Imagine a ball sitting on the marginal return function, marking the current return for an investment of a further year. When $i$ worker-years have been invested in the program, then, the ball will sit at the point on the marginal return function directly above $i$. As years are invested, the ball in effect rolls down the graph. Since there are two programs and two marginal return functions, there are two balls rolling towards one another. When they meet, all 100 worker-years have been invested. You need to find this meeting point. If workers always the select the program offering the higher marginal return, it is at any given moment the ball that is currently higher which descends the graph. What you need to show is that, under this condition, the balls meet where the graphs meet.

Hint: The point where the marginal return functions cross is the low point of the "valley" marked out by their graphs. Satisfy yourself that neither ball will ever roll beyond this low point. The rest follows easily.

### *The Winner-Confers-All Case*

In a winner-confers-all scenario, society receives the highest possible payoff by maximizing not the expected number of successes, as in an additive scenario, but the probability of at least one success. Assuming that the successes of two competing programs are stochastically independent, the probability of at least one success, given an allocation of $n$ out of a total of 100 worker-years to the higher potential program, is

$$s_1(n) + s_2(N - n) - s_1(n)s_2(N - n).$$

This function is shown in figure 5. The optimal value of $n$ is just over 65, com-

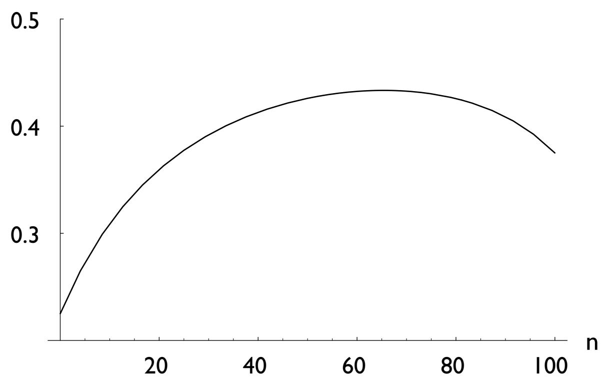


*Figure 5:* The probability of at least one success, as a function of the number of workers allocated to the higher potential program

pared to 60 in the additive case. In a winner-confers-all case, then, relatively more workers should be assigned to the higher potential research program.

The optimal value of $n$ is that for which the adjusted marginal return functions are equal, that is, for which

$$m_1(n)(1 - s_2(N - n)) = m_2(N - n)(1 - s_1(n)).$$

Figure 6 shows the adjusted marginal return function for the higher potential program (dashed line) compared to its regular marginal return function (solid line).

As in the additive case, the optimal allocation can be determined by superimposing the functions to be equalized, with one laterally inverted. Figure 7 shows this done for both the adjusted marginal return functions (dashed lines) and the regular marginal return functions (solid lines). The optimal allocation for the winner-confers-all case occurs where the dashed lines cross; the optimal al-

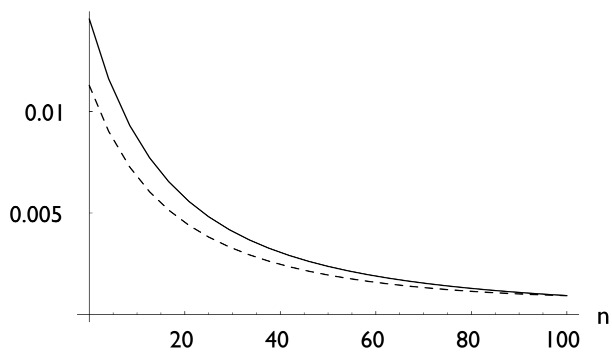


*Figure 6:* The adjusted marginal return function for the higher potential program (dashed line) compared to the regular marginal return function for the same program (solid line)

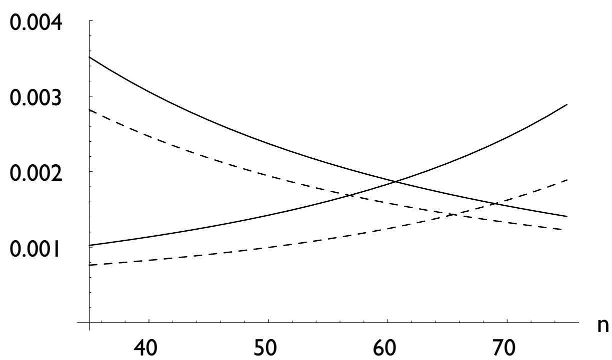


*Figure 7:* The optimal value of $n$ for a winner-confers-all case is marked by the point where the adjusted marginal return functions (dashed lines) cross. The optimal value for an additive case (the crossing point of the marginal return functions, represented by solid lines) is shown by way of comparison.

location for the additive case where the solid lines cross. I leave it to the reader to satisfy themselves that the optimal value of $n$ for the winner-confers-all case is always higher than for the additive case.