Proof by Assumption of the Possible in *Prior Analytics* 1.15

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In Prior Analytics 1.15 Aristotle undertakes to establish certain modal syllogisms of the form XQM. Although these syllogisms are central to his modal system, the proofs he offers for them are problematic. The precise structure of these proofs is disputed, and it is often thought that they are invalid. We propose an interpretation which resolves the main difficulties with them: the proofs are valid given a small number of intrinsically plausible assumptions, although they are in tension with some claims found elsewhere in Aristotle's modal syllogistic. The proofs make use of a rule of propositional modal logic which we call the possibility rule. We investigate how this rule interacts and coheres with the core elements of the modal syllogistic.

1. Introduction

Aristotle is well known as a pioneering figure in modal logic and in philosophical thought about modality more generally. His reputation as a modal logician rests mainly on the system of modal syllogisms presented in *Prior Analytics* 1.3 and 8–22. But he also pioneered another framework for modal reasoning, which is not encoded in the body of his modal syllogisms. This second framework is based on a rule of propositional modal logic embodying the insight that anything entailed by something possible is itself possible. Its applications are distributed widely across Aristotle's works, including the *Physics*, *De Caelo*, and the *Metaphysics*. Most of these applications do not involve the framework of the *Prior Analytics*' modal syllogistic; usually the two frameworks are applied separately and in different contexts. But there is one notorious passage in which they are combined, providing a valuable opportunity to compare them and to study how they interact.

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The passage is found in *Prior Analytics* 1.15, and contains a series of proofs designed to establish four related modal syllogisms (34a34–b27, 35a35–b2). For all the difficulties presented by Aristotle's modal syllogistic, this passage has stood out as especially baffling. Commentators have attempted various reconstructions of it, but have not reached a consensus. Worse, many commentators, both ancient and modern, have come to the view that the proofs given in the passage are invalid.[1] If this is correct, it also threatens the rest of the modal syllogistic, since later on Aristotle relies on the syllogisms established by these proofs (*Pr. An.* 1.18 and 1.21).

In this paper we aim to provide an adequate reconstruction of Aristotle's proofs in the passage, and in so doing to resolve the main difficulties with them. On our reading, the proofs will be valid, if we grant Aristotle a small number of intrinsically plausible assumptions. On the other hand, although the proofs are perfectly acceptable in their own right, we will see that they are in some tension with claims Aristotle makes elsewhere in the modal syllogistic. This tension, we argue, is partly due to the fact that his proofs combine the modal syllogistic with the framework of propositional modal reasoning mentioned above, and that he did not fully think through the consequences of combining the two.

Our understanding of the latter framework is indebted to recent work by Kit Fine on an argument from *Metaphysics* Θ 4 (1047b14–26) which has important parallels with the proofs we will be discussing. Fine has shown that the argument from Θ 4 relies on the following rule of inference: given the premiss that P is possible and given a subordinate deduction from P to Q, you may infer that Q is possible.[2] Call this the *possibility rule*. We will show that this same rule is applied in Aristotle's proofs from *Prior Analytics* 1.15, and discuss how it is employed.

Before examining the proofs, we will need to discuss some preliminaries. First we will briefly explain the possibility rule as it appears

[1] Aristotle's proofs in this passage were held to be invalid by a number of commentators in antiquity, according to the reports of Alexander and Philoponus (Alexander of Aphrodisias *In Pr. An.* 191.24–192.4; Philoponus *In Pr. An.* 172.9–19). They are also thought to be invalid by von Kirchmann (1877, pp. 77–9), Becker (1933, pp. 51–4), Tredennick (1938, pp. 270–1), Mignucci (1990, pp. 332–4), Patterson (1995, pp. 158–61 and 171), and Striker (2009, pp. 146–7).

[2] See Fine 2011, pp. 1023–8. Fine calls this the rule of ◊-Introduction. It should be noted that this rule and its application at 1047b14–26 are not the main focus of his paper (which is instead on 1047b26–30). Nevertheless, Fine's appeal to the rule of ◊-Introduction in his analysis of b14–26 provides a helpful starting point for an interpretation of Aristotle's proofs in *Pr. An.* 1.15.

in Aristotle, and also introduce a formal framework for representing it (Sect. 2). We will then review some elements of Aristotle's modal syllogistic (Sects 3 and 4). This will bring us into a position to reconstruct the proofs from *Prior Analytics* 1.15 (Sects 5–8). Finally, we will explain how these proofs are in some tension with other parts of the modal syllogistic (Sect. 9).

2. Introducing the possibility rule

As mentioned above, the possibility rule licenses an inference to the effect that Q is possible from a premiss to the effect that P is possible together with a subordinate deduction of Q from P.[3] Here, 'P' and 'Q' each stand for a sentence. In the context of the *Prior Analytics*, the sentences in question are typically of the following four traditional kinds:

AaB	Every B is A
AeB	No B is A
AiB	Some B is A
AoB	Not every B is A

For expressing that a sentence is possible (by which we mean that it describes a possible state of affairs), we employ the abbreviation 'Poss(…)'. Thus, for example, Poss(AaB) means that AaB is possible. A simple argument employing the possibility rule can then be written as follows:

(1)	Poss(AeB)	(premiss)
(2)	│AeB	(assumption for the possibility rule)
(3)	│BeA	(from 2, conversion)
(4)	Poss(BeA)	(possibility rule: 1, 2–3)

This argument establishes the possibility of BeA given the possibility of AeB. It begins with the premiss that AeB is possible. In the second line, what was stated to be possible is assumed to be actually the case. We refer to this as an *assumption for the possibility rule*; it initiates an auxiliary deduction, in lines 2–3, which may be called a *modal subordinate deduction*. The modal subordinate deduction is very short, consisting in the application of one of Aristotle's conversion rules. The sentence BeA in line 3 is the *conclusion of the modal subordinate*

[3] Aristotle introduces and justifies the possibility rule in *Prior Analytics* 1.15, 34a5–33; see Rosen and Malink 2012, pp. 182–7.

deduction. From the existence of this subordinate deduction, together with the premiss in the first line, the possibility rule allows us to infer in line 4 that BeA is possible.

Now, in most cases, the modal subordinate deduction will rely on more sentences than just the assumption for the possibility rule. Further sentences may be imported into the modal subordinate deduction, but only on condition that each of them has been stated to be necessary. Using 'Nec(…)' to express necessity, this move can be described by means of the following rule of importation: you may introduce a sentence P into a modal subordinate deduction if the sentence Nec(P) has appeared outside it.[4] To give an example, here is an argument establishing the possibility of AeC given the possibility of AeB and the necessity of BaC:

(1)	Nec(BaC)	(premiss)
(2)	Poss(AeB)	(premiss)
(3)	│AeB	(assumption for the possibility rule)
(4)	│BaC	(importation: 1)
(5)	│AeC	(from 3, 4, by the syllogism Celarent)
(6)	Poss(AeC)	(possibility rule: 2, 3–5)

Aristotle's own applications of the possibility rule are more complex than this, in that they are always found embedded within a *reductio ad absurdum*. In order to familiarize ourselves with this kind of structure, let us consider one more example. The following argument is a straightforward contrapositive expansion of the previous one, establishing the impossibility of AeB given the impossibility of AeC and the necessity of BaC:

(1)	Not Poss(AeC)	(premiss)
(2)	Nec(BaC)	(premiss)
(3)	│Poss(AeB)	(assumption for *reductio*)
(4)	│Nec(BaC)	(iterated from 2)
(6)	│ │AeB	(assumption for possibility rule)
(7)	│ │BaC	(importation: 4)
(8)	│ │AeC	(from 6, 7, by the syllogism Celarent)
(9)	│Poss(AeC)	(possibility rule: 3, 6–8)
(10)	Not Poss(AeB)	(*reductio*: 1, 3–9)

[4] Fine (2011, p. 1025) calls this the rule of □-Elimination. In the event of multiple nested subordinate deductions, Nec(P) should appear within the next deduction up from the one into which P is imported.

This argument exemplifies the structure which we will encounter in Aristotle's proofs in *Prior Analytics* 1.15. Note that in line 4 of this argument, a sentence is iterated into the *reductio* subordinate deduction. This move is unproblematic: unlike modal subordinate deductions (which only admit importation of sentences whose necessity has been asserted), *reductio* subordinate deductions admit free iteration of any sentences.[5]

3. Basics of Aristotle's modal syllogistic

Aristotle's syllogistic divides into an assertoric part (*Pr. An.* 1.1–2 and 4–7) and a modal part (1.3 and 8–22). In the assertoric syllogistic he is concerned with non-modal sentences such as 'Every man is an animal'. Aristotle usually represents these sentences by means of a somewhat artificial construction using the verb 'belong to'. For example, he would say 'A belongs to all B' instead of 'Every B is A', and 'A does not belong to some B' instead of 'Not every B is A'.

The modal syllogistic is concerned with sentences containing the modal qualifiers 'necessarily' and 'possibly', for example 'A necessarily belongs to all B' and 'A possibly does not belong to some B'. Let us first consider sentences containing the qualifier 'necessarily'. In the literature on the modal syllogistic, this qualifier is often abbreviated by the letter 'N', and sentences which contain it are called necessity-sentences or N-sentences. There are four common kinds of N-sentences. Following the notation employed by Gisela Striker (2009) and others (Ebert and Nortmann 2007), we can write them as follows:

$\mathrm{Aa_NB}$	A necessarily belongs to all B
$\mathrm{Ae_NB}$	A necessarily belongs to no B
$\mathrm{Ai_NB}$	A necessarily belongs to some B
$\mathrm{Ao_NB}$	A necessarily does not belong to some B

Aristotle does not explain the semantics of these sentences in any detail, and it is not easy to determine what exactly he took them to mean. To begin with, one might wonder whether his N-sentences can be represented using the notation 'Nec(…)' already introduced earlier. For example, could we not write Nec(AeB) instead of $\mathrm{Ae_NB}$? There are

[5] It should be noted that the distinction between iteration and importation is not explicitly made by Aristotle. We have introduced it as an interpretative tool in order to make the logical structure of his proofs clearer.

reasons for doubt. The formula Nec(AeB) indicates a *de dicto* statement of the necessity of AeB; but in his modal syllogistic Aristotle asserts the validity of syllogisms which would be invalid on such a *de dicto* reading (e.g. that Ae_NC follows from Ae_NB and BaC). One might try instead to represent necessity-sentences by means of 'Nec(…)' in some other way. For example, Ae_NB might be taken to have a *de re* reading such as 'For every *x*, if *x* falls under B then Nec(*x* does not fall under A)'. But again, Aristotle asserts the validity of inferences which would be invalid on such a *de re* reading (e.g. that Be_NA follows from Ae_NB). Thus, neither a *de dicto* nor a *de re* reading seems to capture Aristotle's intentions. One might attempt to specify the semantics of Aristotle's necessity-sentences by means of 'Nec(…)' or sentential necessity operators in yet another, more complex way. But so far all attempts to do so have failed: none of them yields a semantics which is in accordance with all of Aristotle's claims concerning the validity and invalidity of inferences in the modal syllogistic.[6]

In view of this, it is best to formulate the modally qualified sentences of Aristotle's modal syllogistic without relying on the notation 'Nec(…)' and 'Poss(…)' used in connection with the possibility rule. To avoid begging any questions about the meaning of these sentences, we adopt the above notation employed by Striker, which is to a large extent neutral about their semantics. We will return to these issues shortly. In the meantime, the expressions 'a_N', 'e_N', 'i_N', and 'o_N' in this notation should be understood simply as abbreviations for 'necessarily belongs to all', 'necessarily belongs to no', and so on.[7]

In addition to necessity-sentences, Aristotle also considers possibility-sentences. He employs two qualifiers for possibility, corresponding to what is known as one-sided possibility (i.e. not being impossible) and two-sided possibility (i.e. being neither impossible nor necessary).

[6] The most elaborate attempt to specify the semantics of Aristotle's modal sentences by means of sentential necessity operators is offered in Nortmann 1996; but even this semantics deviates from Aristotle's claims of validity and invalidity in a number of places (see Nortmann 1996, pp. 133, 266–82, and 376).

[7] We might think of these expressions as complex copulae. The underlying idea is that the sentences of Aristotle's syllogistic have a tripartite syntax, consisting of a subject term B, a predicate term A, and a copula such as 'belongs to no' or 'necessarily belongs to all'. Thus quantifying expressions and modal qualifiers are both regarded as parts of the copula. For the view that Aristotle thinks of modal qualifiers as part of the copula, see Patterson 1995, pp. 15–22, Charles 2000, pp. 381–7, Raymond 2010, p. 195; cf. *De interpretatione* 12, 21b26–30 and 22a8–10 (on which see the discussion in Charles 2000). Similarly, Striker (2009, p. 70) writes: 'Aristotle describes the modalities — necessity and possibility — as qualifications of the belonging-relation.'

Aristotle often does not explicitly indicate whether a given possibility-sentence is of the two-sided or one-sided kind, but in the modal syllogistic it is usually clear from the context which of them he means. In the secondary literature, the qualifiers for two- and one-sided possibility are commonly indicated by the letters 'Q' and 'M' respectively, as follows:

Aa_QB	A two-sided-possibly belongs to all B
Ae_QB	A two-sided-possibly belongs to no B
Ai_QB	A two-sided-possibly belongs to some B
Ao_QB	A two-sided-possibly does not belong to some B
Aa_MB	A one-sided-possibly belongs to all B
Ae_MB	A one-sided-possibly belongs to no B
Ai_MB	A one-sided-possibly belongs to some B
Ao_MB	A one-sided-possibly does not belong to some B

Again, Aristotle does not fully explain the semantics of these sentences. One might wonder whether they can be represented using the qualifier 'Poss(…)'. Since this qualifier is to be understood in the sense of one-sided possibility, Poss(AaB) might be taken to be equivalent to Aa_MB. But as before, there are reasons for doubt. For if the two formulae were equivalent, this would require a *de dicto* reading of Aa_MB, which is incompatible with some of Aristotle's commitments in the modal syllogistic (e.g. that Aa_MC follows from Aa_MB and BaC; see n. 32 below).[8] Likewise, a *de re* reading sits uneasily with his endorsement of conversion rules for Q- and M-sentences (e.g. that Bi_MA follows from Aa_MB).

There is also a difference in the terminology used by Aristotle to express Poss(AaB) and Aa_MB. The latter sentence and the other kinds of M- and Q-sentences are always expressed by forms of the verbs 'admit' (ἐνδέχεσθαι) or 'allow' (ἐγχωρεῖν). By contrast, the qualifier 'Poss(…)' used in connection with the possibility rule is not expressed by these verbs, but by the adjective 'possible' (δυνατόν). This adjective occurs seventeen times in Aristotle's discussion of the possibility rule in *Prior Analytics* 1.15 (34a5–33); but it occurs only five

[8] Moreover, a *de dicto* reading of Q-sentences conflicts with his endorsement of syllogisms of the form QXQ and QQQ in *Pr. An.* 1.14–15.

times in the rest of chapters 1.1–22.[9] The adjective is never used to express an M- or Q-sentence in the modal syllogistic.

The overall impression, then, seems to be that in *Prior Analytics* 1.15 Aristotle combines two quite independent systems of reasoning about modality. On the one hand, there is the framework for reasoning with the modal qualifier 'Poss(…)', based on the possibility rule along with the rule of importation. On the other hand, there is the framework for reasoning with N-, Q-, and M-sentences based on a set of unproved 'perfect' syllogisms along with certain conversion rules. The latter framework forms the core of the modal syllogistic as developed in *Prior Analytics* 1.3 and 8–22, but is very rarely invoked elsewhere. By contrast, the former framework is applied in various passages throughout Aristotle's works, such as the *Physics*, *De caelo*, and *Metaphysics*.[10] Its use within the modal syllogistic is confined to the proofs in chapter 1.15 with which we are concerned in this paper.

Whereas the framework of the possibility rule is easily understood and verified in terms of contemporary modal logic, this is not the case for the framework of the modal syllogistic. The view commonly arrived at is that Aristotle's modal syllogistic is confused and incoherent: it is often suggested, for example, that the syllogistic is plagued by ambiguity and confusion between *de re* and *de dicto* readings of modal sentences.[11] However, it must be emphasized that Aristotle's system contains no formal inconsistencies. Elsewhere we have described a model for it on which all the inferences endorsed by Aristotle are valid, and all the inferences rejected by him are invalid (Malink 2006 and 2013). The model is complex and not always intuitive, but it does prove that Aristotle's system of modal syllogisms is consistent, without attributing to him any ambiguities in his use of modal sentences.[12] We therefore have the option of regarding his system as a

[9] See Mueller 1999, p. 188, n. 62. The five occurrences outside chapter 1.15 are at 1.3, 25a39; 1.5, 27a2; 1.6, 28a16; 1.11, 31b8–9; 1.13, 32b11.

[10] The application of the possibility rule in these passages is discussed in Rosen and Malink 2012.

[11] Becker 1933, p. 42; Kneale and Kneale 1962, pp. 89–91; Hintikka 1973, pp. 139–40; Sorabji 1980, p. 202; Mignucci 1998, pp. 50–2 and 61–2; Striker 2009, pp. xvi–xvii and 115. A similar ambiguity is diagnosed by Patterson (1989, pp. 15–16; 1990, pp. 155–66; 1995, pp. 41–87).

[12] The model is based on Aristotle's theory of predication and categories as developed in his *Topics*. Unavoidably, the model has some features which are in tension with an intuitive understanding of modal sentences. For example, it must accommodate the following two consequences of Aristotle's claims in the modal syllogistic: that Aa_NB is compatible with

correct elaboration of modal notions unfamiliar to us, rather than as a confused elaboration of notions with which we are familiar.

However that may be, we can largely set these issues aside for present purposes. For, fortunately, the argument of this paper does not require us to have a determinate view about the semantics of Aristotle's modal syllogistic. The proofs from *Prior Analytics* 1.15 with which we are concerned can be analysed and reconstructed without having such a determinate view.

4. Barbara XQM

As we have seen, Aristotle's modal syllogistic is concerned with sixteen types of sentences, twelve of them modally qualified and four assertoric. When assertoric sentences occur in the context of the modal syllogistic, the standard convention is to indicate them by the letter 'X'. Thus, for example, instead of AaB and AeB, we will write $\mathrm{Aa_XB}$ and $\mathrm{Ae_XB}$. Aristotle investigates syllogisms built out of sentences of these sixteen types, each syllogism consisting of two premisses and one conclusion. In *Prior Analytics* 1.15 he endorses the following syllogism, known as Barbara XQM:

Major premiss:	$\mathrm{Aa_XB}$	A belongs to all B
Minor premiss:	$\mathrm{Ba_QC}$	B two-sided-possibly belongs to all C
Conclusion:	$\mathrm{Aa_MC}$	A one-sided-possibly belongs to all C

(The term 'Barbara' encodes the fact that all three sentences are a-sentences, and 'XQM' specifies the modality of each of them.) Aristotle also endorses three other XQM-syllogisms, namely Celarent, Darii, and Ferio XQM.[13] There is a question whether Aristotle is right to endorse these syllogisms; he himself discusses, and rejects, an apparent counterexample to Barbara XQM (*Pr. An.* 1.15, 34b11–14):

Major premiss:	Man belongs to all moving (supposed to be TRUE)
Minor premiss:	Moving two-sided-possibly belongs to all horse (TRUE)
Conclusion:	Man one-sided-possibly belongs to all horse (FALSE)

$\mathrm{Ae_QB}$; that $\mathrm{Aa_QB}$ implies $\mathrm{Ai_XB}$ if B is a substance term. See Malink 2013, pp. 195–210 and 240–7.

[13] In Celarent XQM, $\mathrm{Ae_MC}$ is inferred from $\mathrm{Ae_XB}$ and $\mathrm{Ba_QC}$. In Darii XQM, $\mathrm{Ai_MC}$ is inferred from $\mathrm{Aa_XB}$ and $\mathrm{Bi_QC}$. In Ferio XQM, $\mathrm{Ao_MC}$ is inferred from $\mathrm{Ae_XB}$ and $\mathrm{Bi_QC}$.

Is this a genuine counterexample, proving the invalidity of Barbara XQM? If we had a determinate semantics for Q- and M-sentences, we could use it as a standard to decide the question and to determine whether Barbara XQM is valid. But in the absence of such a semantics, we cannot settle the matter in such a direct way. There might be various strategies for rejecting the putative counterexample.[14] Aristotle's own response is that its major premiss should be rejected as not being an instance of 'predication *simpliciter*', but rather of 'predication limited in respect of time'. His response is standardly interpreted as requiring that the major premiss be omnitemporally true.[15] However, there are several problems with this interpretation, for example that elsewhere in the modal syllogistic Aristotle frequently admits non-omnitemporal premisses such as 'Moving belongs to all animal' or 'Being awake belongs to all animal'.[16] But whatever exactly his reasons were, it is clear that in so far as Aristotle endorsed Barbara XQM, he must have regarded himself as justified in rejecting the counterexample.

Given that we are not to decide the validity of the four XQM-syllogisms by means of an antecedently given semantics, the natural thing to do is to examine the proofs which Aristotle gives for them.

5. First proof of Barbara XQM (34a34–b2)

Aristotle's proof of Barbara XQM begins his discussion of XQM-syllogisms in *Prior Analytics* 1.15. In fact, he even offers two proofs for it. In what follows we will discuss the first proof, leaving the second for the next section. Aristotle's first step is to state the premisses and conclusion of the syllogism to be proved:

> [i] Let A belong to all B and let it be possible for B to belong to all C. Then it is necessary for it to be possible for A to belong to all C. (*Pr. An.* 1.15, 34a34–6)

[14] For instance, on the semantics described in n. 12, the counterexample can be rejected as follows. Since 'horse' is a substance term, the minor premiss 'Moving two-sided-possibly belongs to all horse' implies the assertoric sentence 'Moving belongs to some horse'. This conflicts with the major premiss, so that the two premisses cannot be true together.

[15] Philoponus *In Pr. An.* 175.21–4; Hintikka 1973, pp. 136–8; Nortmann 1990, p. 64 and 1996, pp. 45–6; Barnes 2007, p. 18.

[16] Cf. Patterson 1995, pp. 167–9. It is also worth noting that Patterson (1995) and Becker (1933, pp. 58–9) raise doubts about the authenticity of (parts of) the passage at 34b7–18.

Although he formulates the minor premiss and the conclusion using the same expression for possibility, it is generally agreed that he understands the minor premiss as a two-sided possibility-sentence ($\mathrm{Ba_QC}$) and the conclusion as a one-sided possibility-sentence ($\mathrm{Aa_MC}$). Aristotle's first proof of this syllogism then proceeds as follows:

> [ii] For let it not be possible, [iii] and posit that B belongs to all C; [iv] this is false but not impossible. [v] Therefore, if it is not possible for A to belong to all C and B belongs to all C, then it is not possible for A to belong to all B; for a deduction comes about through the third figure. [vi] But it was assumed that it is possible for A to belong to all B. [vii] Therefore, it is necessary for it to be possible for A to belong to all C; [viii] for while something false but not impossible was posited, the result is impossible. (*Pr. An.* 1.15, 34a36–b2)

The proof proceeds by *reductio ad absurdum*, with the assumption for *reductio* being introduced in point [ii]. Given that the intended conclusion of Barbara XQM is $\mathrm{Aa_MC}$, the assumption for *reductio* must be its contradictory. We will label this contradictory 'Not $\mathrm{Aa_MC}$'.[17]

Next, Aristotle sets up an application of the possibility rule. He does so, in point [iii], by introducing an assumption for the possibility rule, namely the assertoric sentence $\mathrm{Ba_XC}$. It should be noted that this sentence is not supposed to follow from the premisses of Barbara XQM, but is merely introduced into the proof as an assumption initiating a modal subordinate deduction. Thus, Aristotle states in point [iv] that $\mathrm{Ba_XC}$ is 'false but not impossible'. Presumably, he does not intend positively to assert the falsehood of this sentence; as explained by Alexander of Aphrodisias, when Aristotle says 'false but not impossible' in this context, we should understand him as meaning something like 'at worst false, perhaps true, but not impossible'.[18] This last phrase is equivalent to 'possible' (understood in the one-sided sense of possibility). Thus, in point [iv], Aristotle is asserting that $\mathrm{Ba_XC}$ is possible, that is, he is asserting $\mathrm{Poss(Ba_XC)}$. Aristotle appears to take this assertion to be justified

[17] Prima facie, it is plausible that this contradictory is identical with $\mathrm{Ao_NC}$. On the other hand, we will shortly encounter some reason to doubt this identification. The label 'Not $\mathrm{Aa_MC}$' is meant to be neutral on this issue.

[18] Alexander of Aphrodisias *In Pr. An.* 185.16–20. See also Nortmann 1996, pp. 196–7, Mueller 1999, p. 39, and Striker 2009, p. 145.

by the two-sided possibility-sentence Ba_QC, which is the minor premiss of Barbara XQM. Aristotle appears, then, to hold that Ba_QC implies Poss(Ba_XC).[19]

Aristotle's proof up to point [iv] contains two assumptions, an assumption for *reductio* and an assumption for the possibility rule, and can be written as follows:

(1)	Aa_XB	(major premiss)
(2)	Ba_QC	(minor premiss)
(3)	$\mid$ Not Aa_MC	(assumption for *reductio*)
(4)	$\mid$ Ba_QC	(iterated from 2)
(5)	$\mid$ Poss(Ba_XC)	(from 4)
(6)	$\mid\quad\mid$ Ba_XC	(assumption for possibility rule)

We now have to consider how Aristotle proceeds within the modal subordinate deduction initiated by the assumption of Ba_XC. As we know from point [v] of the passage quoted above, what he does there is to apply a syllogism in his third figure.[20] One of the premisses of this syllogism is the assumption for the possibility rule, Ba_XC, and the other premiss is the assumption for *reductio*, Not Aa_MC. Thus, Aristotle seems to think that this latter premiss is available not only within the *reductio* subordinate deduction, but also within the modal subordinate deduction. If he is to be justified in this, we should take him to apply the rule of importation to make it available there. The rule, however, requires the presence outside the modal subordinate deduction of the sentence Nec(Not Aa_MC). It is not immediately obvious where this sentence could come from; but we may take Aristotle to infer it from the assumption for *reductio*, Not Aa_MC. We will discuss shortly whether and how this inference can be justified.

The conclusion of the third-figure syllogism applied in point [v] is expressed by means of the same kind of phrase used to express the

[19] See Alexander of Aphrodisias *In Pr. An.* 186.10–14 and Patterson 1995, p. 160. Although the implication from Ba_QC to Poss(Ba_XC) is prima facie plausible, it does place substantive constraints on the semantics of a_Q-sentences. The implication would be invalid, for example, on a *de re* reading of these sentences: if B is 'being my favourite number' and C is 'number' then Ba_QC is true on a *de re* reading, since each number is possibly my favourite number and possibly not my favourite number; but Poss(Ba_XC) is false since it is not possible that every number is my favourite number.

[20] Third-figure syllogisms are those in which the two terms of the conclusion appear as the predicate terms of the two premisses. By contrast, Barbara XQM is a first-figure syllogism, meaning that the subject term of the conclusion is the subject term of one premiss, and the predicate term of the conclusion is the predicate term of the other premiss.

premiss Not Aa_MC, and so it can be written as follows: Not Aa_MB. Thus, Aristotle's proof up through the application of the syllogism in point [v] is as follows:

(1)	Aa_XB	(major premiss)
(2)	Ba_QC	(minor premiss)
(3)	Not Aa_MC	(assumption for *reductio*)
(4)	Nec(Not Aa_MC)	(from 3, to be discussed below)
(5)	Ba_QC	(iterated from 2)
(6)	Poss(Ba_XC)	(from 5)
(7)	Ba_XC	(assumption for possibility rule)
(8)	Not Aa_MC	(importation: 4)
(9)	Not Aa_MB	(from 7, 8, by third-figure syllogism)

The sentence Not Aa_MB in line 9 is the conclusion of the modal subordinate deduction. An application of the possibility rule then allows us to leave the modal subordinate deduction by inferring the sentence Poss(Not Aa_MB) within the *reductio* subordinate deduction. Although Aristotle does not explicitly express this sentence, we will include it in our reconstruction as the conclusion of the *reductio* subordinate deduction in order to make the logical structure of his proof clearer.

In point [vi], Aristotle says 'but it was assumed that it is possible for A to belong to all B'. Now, in fact, what Aristotle has laid down is that A *does* belong to all B, that is, Aa_XB; this is the major premiss of Barbara XQM, which was stated in point [i]. Aristotle appears in point [vi], then, to be deriving either Aa_QB or Aa_MB from the original assertoric premiss Aa_XB.[21] It is not permissible to infer an a_Q-sentence from an a_X-sentence; for example, the a_X-sentence 'Animal belongs to all man' is true but the corresponding a_Q-sentence is false. But it is plausible that an a_X-sentence implies the corresponding a_M-sentence. So we should assume that in point [vi] Aristotle infers Aa_MB from Aa_XB.[22]

At this stage, then, Aristotle has on the one hand asserted Aa_MB, and on the other hand constructed a *reductio* subordinate deduction from Not Aa_MC to the conclusion Poss(Not Aa_MB). He takes this to

[21] For this view, see Maier 1900, Vol. 2, p. 140, Tredennick 1938, p. 270, Ross 1949, p. 339, Angelelli 1979, pp. 196–7, Striker 2009, p. 146. We disagree with those commentators who neglect or excise the term ἐνδέχεσθαι in point [vi], and who take Aristotle there to express the assertoric sentence Aa_XB. These include Alexander of Aphrodisias (*In Pr. An.* 186.23–5), Philoponus (*In Pr. An.* 173.29–31), Becker (1933, p. 56), and Flannery (1993, p. 202).

[22] Angelelli 1979, pp. 196–7; Striker 2009, p. 146.

put him in a position to complete his *reductio* proof by inferring the desired conclusion of Barbara XQM, namely Aa_MC. He does this in point [vii]. Now, it must be noted that Aa_MB does not immediately contradict Poss(Not Aa_MB). But Aristotle apparently thinks that the two sentences are incompatible. This is confirmed by what he says in point [viii], 'while something false but not impossible was posited, the result is impossible'. The thing which 'was posited' is the assumption for the possibility rule, namely Ba_XC, and the 'result' is the conclusion of the modal subordinate deduction, namely Not Aa_MB. By calling this result impossible, Aristotle in effect asserts the sentence Not Poss(Not Aa_MB). Although no derivation of this sentence is explicitly given by Aristotle, we may take him to infer it from Aa_MB; we will discuss shortly whether and how this inference can be justified. Before that, however, we can now reconstruct the entirety of Aristotle's proof in points [i]–[viii], as follows:

(1)	Aa_XB	(major premiss)
(2)	Ba_QC	(minor premiss)
(3)	│ Not Aa_MC	(assumption for *reductio*)
(4)	│ Nec(Not Aa_MC)	(from 3, by principle of necessitation)
(5)	│ Ba_QC	(iterated from 2)
(6)	│ Poss(Ba_XC)	(from 5)
(7)	│ │ Ba_XC	(assumption for possibility rule)
(8)	│ │ Not Aa_MC	(importation: 4)
(9)	│ │ Not Aa_MB	(from 7, 8)
(10)	│ Poss(Not Aa_MB)	(possibility rule: 6, 7–9)
(11)	Aa_MB	(from 1)
(12)	Not Poss(Not Aa_MB)	(from 11, by principle of necessitation)
(13)	Aa_MC	(*reductio*: 3–10, 12)

Two issues require explanation in connection with this reconstruction. First, it remains to consider the inferences drawn in line 4 and line 12, which we have provisionally labelled as relying on a 'principle of necessitation'. Second, there are questions as to the nature of the syllogism employed in line 9 within the modal subordinate deduction.

Beginning with the first issue, let us consider the inference drawn in line 12 from Aa_MB to Not Poss(Not Aa_MB). As we saw above, this inference appears to be performed by Aristotle, but it is not immediately clear whether and how it is justified. A reasonable proposal, put forward by Paolo Fait, is that Aristotle is assuming a principle to the effect that modally qualified sentences are necessary if true, and impossible if false.[23] Thus modally qualified sentences are either necessary or impossible; unlike assertoric sentences, they are never contingently true or false. For example, if the N-sentence 'Animal necessarily belongs to all horse' is true, then this sentence is necessary (i.e. it is necessary that animal necessarily belongs to all horse). If the N-sentence 'Being beautiful necessarily belongs to all horse' is false, then this sentence is impossible (i.e. it is impossible that being beautiful necessarily belongs to all horse). Likewise for Q- and M-sentences (but not for X-sentences). The principle in question may be stated as follows:

Principle of necessitation:
Where 'Y' stands for any N-, Q-, or M-sentence:
(i) If Y then Nec(Y)
(ii) If Not Y then Nec(Not Y)

Given that 'Nec(…)' is equivalent to 'Not Poss(Not …)', the principle also implies that if Y then Not Poss(Not Y), and that if Not Y then Not Poss(Y). Although this principle is never explicitly discussed by Aristotle, it is in itself plausible and can reasonably be attributed to him.[24] Supposing that Aristotle did indeed hold the principle of necessitation, this justifies both the inference in line 12 and the inference in line 4.

The inference from Not Aa_MC to Nec(Not Aa_MC) in line 4 is needed to make the former statement available within the modal subordinate deduction (line 8). If this statement were not available there, Aristotle would not be in a position to apply his third-figure syllogism (line 9). Accordingly, commentators who do not acknowledge that he performs the inference in line 4 tend to think that his proof is flawed. For example, Patterson argues that Aristotle's application of the third-figure syllogism rests on a confusion between the possibility of Ba_XC (stated in line 6) and the joint possibility of Ba_XC with Not Aa_MC.

[23] Fait 1999, p. 145. Similarly, Angelelli (1979, p. 196) suggests that the argument relies on a principle to the effect that if an N-sentence can be false, it is false.

[24] It may be noted that the principle of necessitation has affinities with characteristic theorems of the systems S4 and S5 in modern modal logic. This does not mean that the principle is innocent. For example, it would be invalid if we adopted a *de re* reading of Aristotle's N-, Q-, and M-sentences. Thus if A is 'hopping' and B is 'green', then Aa_MB is true on a *de re* reading if it is assumed that all green things are frogs. But Nec(Aa_MB) is false, since it is possible that some green things are stones and hence that Aa_MB is false, on a *de re* reading.

Only their joint possibility can justify Aristotle's assertion of the possibility of Not Aa_MB (line 10), but according to Patterson Aristotle has not established this joint possibility.[25] On our interpretation, by contrast, Aristotle is not guilty of such a confusion, and does in effect establish joint possibility. If we are correct, he infers the necessity of Not Aa_MC by means of the principle of necessitation (line 4). Combining this with the possibility of Ba_XC (line 6), it follows that Ba_XC is jointly possible with Not Aa_MC, since everything that is possible is jointly possible with everything that is necessary.

The second issue we want to discuss concerns the nature of the syllogism which Aristotle applies within the modal subordinate deduction. In lines 7–9 of the above reconstruction, this syllogism is represented as follows:

Major premiss:	Not Aa_MC	A does not one-sided-possibly belong to all C
Minor premiss:	Ba_XC	B belongs to all C
Conclusion:	Not Aa_MB	A does not one-sided-possibly belong to all B

The major premiss and the conclusion of this syllogism is each the contradictory of an a_M-sentence. Prima facie, it seems plausible to suppose that these contradictories are o_N-sentences. Thus, one might suppose that Not Aa_MC simply is Ao_NC. Hence commentators often take Aristotle to be applying a syllogism of the form Bocardo NXN, which may be represented as follows:[26]

Major premiss:	Ao_NC	A necessarily does not belong to some C
Minor premiss:	Ba_XC	B belongs to all C
Conclusion:	Ao_NB	A necessarily does not belong to some B

This leads to a problem, however, because Aristotle has denied the validity of Bocardo NXN earlier in the *Prior Analytics* (1.11, 32a4–5). In response to this, some commentators take Aristotle to be applying not

[25] Patterson 1995, pp. 158–61 and 171. A similar fallacious move from separate to joint possibility is diagnosed in Aristotle's proof by Becker (1933, pp. 51–4) and Mignucci (1990, pp. 332–4).

[26] Tredennick 1938, p. 270; Ross 1949, pp. 338–9; Angelelli 1979, p. 195; Sainati 1988, p. 45; Patterson 1995, p. 162; Striker 2009, p. 146; Rini 2011, p. 150; Smith 2011, pp. 421–2.

Bocardo NXN but Bocardo NXX, yielding the assertoric conclusion Ao_XB instead of Ao_NB.[27] This too, however, is problematic, for two reasons. First, when Aristotle expresses the conclusion of his syllogism in point [v] of the passage above, he uses the phrase 'it is not possible for A to belong to all B', and this cannot plausibly be read as a statement of the assertoric sentence Ao_XB.[28] Second, if the conclusion of the syllogism is taken to be the assertoric sentence Ao_XB, Aristotle's proof, as we have reconstructed it, would fail. For in this case, one would need the sentence Not Poss(Ao_XB) instead of Not Poss(Not Aa_MB) in line 12; but since the principle of necessitation is not applicable to assertoric sentences, there is no way to derive this sentence from the major premiss Aa_XB in line 1. It is vital to the success of the proof that the conclusion of the syllogism applied within the modal subordinate deduction is not an assertoric but a modally qualified sentence, or indeed the contradictory of a modally qualified sentence. Hence, the view that this syllogism is Bocardo NXX should be rejected.

Therefore, if the only options are Bocardo NXN and NXX, we should prefer the first, even though this would conflict with Aristotle's earlier rejection of this syllogism. Indeed, some commentators hold that the earlier rejection of Bocardo NXN was a mistake on Aristotle's part, which he corrects here in *Prior Analytics* 1.15.[29] But there is also a third option available, albeit one with its own costs, on which we do not need to attribute to Aristotle an earlier mistake. This option is to deny that the contradictory of an a_M-sentence is an o_N-sentence.[30] As a result, the syllogism in question is not identified with Bocardo NXN, although it does yield as a conclusion the

[27] Alexander of Aphrodisias *In Pr. An.* 186.19–23; Philoponus *In Pr. An.* 171.29–172.3; Pacius 1597, p. 188; Becker 1933, p. 56; McCall 1963, p. 84; Mignucci 1972, p. 60; Flannery 1993, pp. 202–3; Mueller 1999, p. 39. Others, surprisingly, take the syllogism to be Felapton NXN (Maier 1900, Vol. 1, p. 159; Wieland 1972, pp. 149–50). However, this requires that the assumption for *reductio* in line 3 be Ae_NC, which is implausible because Ae_NC is certainly not the contradictory of Aa_MC, the conclusion of Barbara XQM.

[28] Such a reading would require that the phrase 'it is not possible' in 'it is not possible for A to belong to all B' (τὸ A οὐ παντί τῷ B ἐνδέχεται, 34a39) be taken as an expression of *necessitas consequentiae*. But this is implausible: see Ross 1949, p. 338, and Patterson 1995, p. 162.

[29] Sainati 1988, p. 45, Patterson 1995, pp. 162–3; similarly Angelelli 1979, p. 195.

[30] For an argument that Aristotle denies some such relations of contradiction between N-sentences and M-sentences, see n. 42 below and Malink 2013, pp. 201–10.

contradictory of a modally qualified sentence, namely Not Aa_MB. It was in order to leave this option open that we have been writing 'Not Aa_MB' instead of 'Ao_NB'. We think this option is worthy of consideration, but remain neutral as to whether this or the first option should in the end be preferred.

Regardless of whether the first or the third option is ultimately preferred, the syllogism in question can be regarded as a contraposed version of the following syllogism, called Barbara MXM:[31]

Major premiss:	Aa_MB	A one-sided-possibly belongs to all B
Minor premiss:	Ba_XC	B belongs to all C
Conclusion:	Aa_MC	A one-sided-possibly belongs to all C

Now, Aristotle does not explicitly discuss Barbara MXM in the modal syllogistic, but there is reason to think that he would accept it as valid. At least, he regards Barbara NXN and Barbara QXQ as 'perfect' syllogisms, by which he means that their validity is obvious and not in need of proof. In light of this, it is plausible that he would also accept Barbara MXM as valid and perfect.[32] Thus, we take it that the syllogism applied by Aristotle within the modal subordinate deduction is a contraposed version of Barbara MXM, whether or not this is to be identified with Bocardo NXN.

This completes our reconstruction of Aristotle's first proof of Barbara XQM. We have seen that, in addition to the possibility rule and the rule of *reductio*, the proof relies on the validity of four items: the inference from Ba_QC to Poss(Ba_XC) (line 6), the principle of necessitation for modally qualified sentences (lines 4 and 12), Barbara MXM (line 9), and the principle that any X-sentence implies the corresponding M-sentence (line 11). While there may be legitimate questions about each of them, none of these four items is implausible. Given that they are valid, then Aristotle's proof of Barbara XQM is valid, and hence Barbara XQM is a valid syllogism.

[31] For the notion of a contraposed, or, as Aristotle calls it, 'converted' (ἀντιστραφέν), version of a syllogism, see *Pr. An.* 2.8–10.

[32] The considerations which Aristotle offers for the validity of Barbara NXN and QXQ (1.9, 30a21–3; 1.15, 33b33–6) appear as if they can be adapted to cover Barbara MXM. It is understandable that Aristotle did not explicitly discuss the case of Barbara MXM. For in general, although he discusses syllogisms with an M-sentence as conclusion, he does not discuss any syllogisms with an M-sentence among their premisses.

Before we move on, let us make one final remark on Aristotle's proof. When Aristotle infers Poss(Ba_XC) from Ba_QC, it does not seem to be essential to the inference that the latter sentence is a two-sided as opposed to one-sided possibility sentence. Since 'Poss(…)' is to be understood in the sense of one-sided possibility, it is natural to suppose that the inference would be equally acceptable if Ba_QC is replaced by Ba_MC. Given that Aristotle accepts the latter inference, his proof of Barbara XQM can easily be turned into a proof of Barbara XMM. Moreover, the only way the major premiss of Barbara XQM, Aa_XB, is exploited in Aristotle's proof is by inferring the one-sided possibility sentence Aa_MB in line 11. Hence, the proof remains intact if the major premiss is replaced by this latter sentence, yielding a proof of Barbara MMM.[33] Thus, Aristotle's proof in effect allows us to establish the validity of Barbara MMM by means of Barbara MXM. On the face of it, the former syllogism is significantly stronger than the latter, so that one might not see how the first can be proved valid by means of the second. Aristotle's deployment of the possibility rule allows him to accomplish this feat.

6. Second proof of Barbara XQM (34b2–6)

The above proof of Barbara XQM is followed by a complicated and disputed passage in which Aristotle seems to give a second, alternative proof of the same syllogism, as follows:

> [i] It is also possible to produce the impossibility through the first figure, [ii] positing that B belongs to C. [iii] For if B belongs to all C, [iv] and it is possible for A to belong to all B, [v] then it would be possible for A to belong to all C. [vi] But it was assumed not to be possible for it to belong to all C. (*Pr. An.* 1.15, 34b2–6)

Commentators have taken divergent views about this passage. Becker and others, including Ross, despair of extracting a relevant and sensible argument from the passage, and so they excise it as a gloss, despite the fact that it is found in all manuscripts.[34] Others think that the passage is genuine and that it contains a proof of the syllogism

[33] Although Aristotle does not explicitly discuss Barbara MMM, it is plausible that he would accept it as valid, given that he also accepts as valid Barbara NNN and Barbara QQQ.

[34] Becker 1933, p. 57; Ross 1949, p. 339; Mignucci 1969, p. 326; Barnes 1984, p. 55; Smith 1989, pp. 23 and 132.

Bocardo NXN, which, as we saw, Aristotle can be taken to apply in the preceding proof discussed above.[35] While in principle the passage can be interpreted as giving a proof by *reductio* of Bocardo NXN, the way in which the passage is introduced in point [i] does not support this kind of interpretation. For point [i] seems to announce an alternative way of proving the same result that has been proved before, namely the validity of Barbara XQM.[36] We will show that the passage can indeed be interpreted as giving a sensible alternative proof of Barbara XQM, so that there is no reason to doubt its authenticity.

In his first proof of Barbara XQM, Aristotle stated that the syllogism employed within the modal subordinate deduction is in the third figure. In point [i] of the present passage, Aristotle seems to state that Barbara XQM can also be proved in such a way that the syllogism employed there is in the first figure. On this interpretation, the phrase 'the impossibility' in point [i] refers to the conclusion of the modal subordinate deduction, Not $\mathrm{Aa_MB}$.[37] Aristotle is indicating that this conclusion can be reached in a different way than before. Thus, points [i]–[vi] can be understood as describing a modification within the modal subordinate deduction, with the rest of the proof remaining the same.

In point [ii], Aristotle reminds us of the assumption for the possibility rule which initiates the modal subordinate deduction, namely $\mathrm{Ba_XC}$. In points [iii]–[vi], he explains the modified structure of the modal subordinate deduction. In the first proof he applied a third-figure syllogism, which we took to be a contraposed version of the first-figure syllogism Barbara MXM. In the present passage, Aristotle explains how to replace this contraposed version with an application

[35] Colli 1955, pp. 857–8; Thom 1996, p. 79; Fait 1999, p. 143. Colli and Fait hold that the passage contains a proof of both Bocardo NXN and Disamis NXN.

[36] Authors who hold that the passage gives an alternative proof of Barbara XQM include Gohlke (1936, pp. 82–3), Angelelli (1979, p. 197), Patterson (1995, pp. 164–6), and Ebert and Nortmann (2007, pp. 570–1).

[37] In principle, this phrase might also be taken to refer to the conclusion of the *reductio* subordinate deduction (i.e. Poss(Not $\mathrm{Aa_MB}$)). However, this is unlikely given the immediately preceding sentence in Aristotle's text: 'for while something false but not impossible was posited, the result is impossible' (34b1–2). As discussed above, that which is called impossible in that sentence is the conclusion of the modal subordinate deduction. Hence, 'the impossibility' in point [i] of the present passage should also refer to the conclusion of the modal subordinate deduction.

of plain Barbara MXM embedded within an additional *reductio*. The resulting proof can be reconstructed as follows:

(1)	Aa_XB	(major premiss)
(2)	Ba_QC	(minor premiss)
(3)	$\mid$ Not Aa_MC	(assumption for *reductio*)
(4)	$\mid$ Nec(Not Aa_MC)	(from 3, by principle of necessitation)
(5)	$\mid$ Ba_QC	(iterated from 2)
(6)	$\mid$ Poss(Ba_XC)	(from 5)
(7)	$\mid\ \mid$ Ba_XC	(assumption for possibility rule)
(8)	$\mid\ \mid$ Not Aa_MC	(importation: 4)
(9)	$\mid\ \mid\ \mid$ Aa_MB	(assumption for *reductio*)
(10)	$\mid\ \mid\ \mid$ Ba_XC	(iterated from 7)
(11)	$\mid\ \mid\ \mid$ Aa_MC	(from 9, 10, by Barbara MXM)
(12)	$\mid\ \mid$ Not Aa_MB	(*reductio*: 8, 9–11)
(13)	$\mid$ Poss(Not Aa_MB)	(possibility rule: 6, 7–12)
(14)	Aa_MB	(from 1)
(15)	Not Poss(Not Aa_MB)	(from 14, by principle of necessitation)
(16)	Aa_MC	(*reductio*: 3–13, 15)

The application of Barbara MXM in lines 9–11 corresponds to points [iii]–[v] in Aristotle's text. Specifically, the statement 'it is possible for A to belong to all B' in point [iv] specifies the major premiss for this application of Barbara MXM. In the above reconstruction, this statement is treated as the assumption for *reductio* in line 9, even though Aristotle does not give an explicit indication of an assumption for *reductio* at this point. In point [vi] of the text, we take Aristotle to refer to the statement in line 8, which allows him to perform the step of *reductio* in line 12.

In sum, Aristotle's second proof of Barbara XQM on this interpretation is exactly like his first proof, except that the contraposed version of Barbara MXM is replaced by a combination of plain Barbara MXM and *reductio*.[38]

7. Proof of Celarent XQM (34b19–27)

Shortly after the two proofs of Barbara XQM, Aristotle undertakes a proof of Celarent XQM. As we will see, this proof closely follows the

[38] As Aristotle points out at *Pr. An.* 2.11, 61a17–27, there is a close connection between proofs by *reductio* and contraposed versions of syllogisms. His second proof of Barbara XQM can be seen as taking advantage of this connection.

pattern of the first proof of Barbara XQM. Aristotle begins by stating the premisses and the conclusion of the syllogism he wishes to establish:

> [i] Let A be taken to belong to no B, and let it be possible for B to belong to all C. With these having been posited as premisses, then, it is necessary for it to be possible for A to belong to no C. (*Pr. An.* 1.15, 34b20–2)

As before, the minor premiss is a two-sided possibility-sentence, whereas, as Aristotle will emphasize at 34b27–37, the conclusion is a one-sided possibility-sentence. In our notation, the syllogism described by Aristotle reads as follows:

Major premiss:	$\mathrm{Ae_XB}$	A belongs to no B
Minor premiss:	$\mathrm{Ba_QC}$	B two-sided-possibly belongs to all C
Conclusion:	$\mathrm{Ae_MC}$	A one-sided-possibly belongs to no C

Aristotle goes on to prove Celarent XQM as follows:

> [ii] For let it not be possible, [iii] and let it be posited that B belongs to C, just as before. [iv] Then it is necessary for A to belong to some B; for a deduction comes about in the third figure. [v] But this is impossible. [vi] Consequently, it would be possible for A to belong to no C; [vii] for while something false was posited, the result is impossible. (*Pr. An.* 1.15, 34b22–7)

As before, the proof proceeds by *reductio*. In point [ii], Aristotle introduces the assumption for *reductio*, namely Not $\mathrm{Ae_MC}$. In point [iii] he introduces the assumption for the possibility rule, $\mathrm{Ba_XC}$, thus initiating a modal subordinate deduction. In point [iv], Aristotle applies a third-figure syllogism within the modal subordinate deduction. As in the first proof of Barbara XQM, the minor premiss of this syllogism is presumably the assumption for the possibility rule, $\mathrm{Ba_XC}$. Also as before, its major premiss is presumably the assumption for *reductio*, which in this proof is Not $\mathrm{Ae_MC}$. Thus Aristotle may be taken to import Not $\mathrm{Ae_MC}$ into the modal subordinate deduction, relying as before on the principle of necessitation.

Aristotle states the conclusion of the third-figure syllogism applied within the modal subordinate deduction by saying ‘it is necessary for A to belong to some B’ (point [iv]). Commentators are divided on the interpretation of this phrase. Some read it as an assertion of $\mathrm{Ai_NB}$, and accordingly take the third-figure syllogism applied by Aristotle in point [iv] to be Disamis NXN.[39] The problem with this interpretation is that earlier in the *Prior Analytics*, Aristotle has denied the validity of Disamis NXN (1.11, 31b20–33).

Because of this, Alexander and others take Aristotle in point [iv] to state the assertoric conclusion Ai_XB, and to be applying the syllogism Disamis NXX.[40] On this interpretation, the qualification 'necessary' is not the modal qualifier of an i_N-sentence, but instead expresses *necessitas consequentiae*, indicating that the assertoric conclusion necessarily follows from the premisses.[41] Although this would be a natural reading of the Greek, it leads to problems in the reconstruction of Aristotle's proof. In particular, if the conclusion of the modal subordinate deduction were an assertoric sentence, Aristotle's proof would fail, for the same reason we saw above in connection with his first proof of Barbara XQM. There, we explained, the proof fails if Aristotle is taken to apply Bocardo NXX in the modal subordinate deduction.

If the only options are Disamis NXN and NXX, we should prefer the first, despite Aristotle's earlier rejection of this syllogism. As before, however, there is a third option available, if we are willing to deny that the contradictory of an e_M-sentence is an i_N-sentence.[42] In this case, Aristotle's phrase in point [iv] can be taken to express not the i_N-sentence Ai_NB, but the contradictory of an e_M-sentence, that is, to express Not Ae_MB. The third-figure syllogism applied

[39] Patterson 1995, p. 182; Thom 1996, p. 80; Striker 2009, p. 148; Rini 2011, p. 159, n. 3. In Disamis NXN, the conclusion Ai_NB is inferred from the premisses Ai_NC and Ba_XC.

[40] Alexander of Aphrodisias *In Pr. An.* 193.30–3; Philoponus *In Pr. An.* 177.4–7; Pacius 1597, p. 190; Maier 1900, Vol. 1, p. 164; Tredennick 1938, p. 274; Ross 1949, p. 341; McCall 1963, p. 90; Angelelli 1979, p. 196; Mignucci 1990, p. 333; Mueller 1999, p. 194, n. 149. In Disamis NXX, the conclusion Ai_XB is inferred from the premisses Ai_NC and Ba_XC.

[41] The phrase 'it is necessary' in point [iii] translates the Greek word 'ἀνάγκη'. Aristotle often uses this word to express *necessitas consequentiae* in his syllogistic, whereas he more commonly uses 'ἐξ ἀνάγκης' as the modal qualifier in N-sentences; see Patzig 1968 (pp. 16–21). However, he does sometimes use 'ἀνάγκη' as the modal qualifier in N-sentences (*Pr. An.* 1.3, 25a29–33, b12; 1.10, 30b29; 1.11, 31b7, b16–17, b29, b36, 32a2; 1.15, 34b41; 1.20, 39b4; 1.22, 40a14, a36).

[42] An independent reason for such a denial is given by Aristotle's statement that Datisi QNX is invalid (*Pr. An.* 1.22, 40a39–b2). If i_N-sentences were contradictory to e_M-sentences, Datisi QNX could be proved to be valid as follows. Begin with the premisses Aa_QB and Ci_NB. For *reductio*, assume the contradictory of Ai_XC; this is Ae_XC, which can be converted to Ce_XA. Ce_XA and Aa_QB yield Ce_MB by Celarent XQM. If i_N-sentences were contradictory to e_M-sentences, Ce_MB would contradict Ci_NB, which would complete the *reductio* proof of Datisi QNX.

within the modal subordinate deduction can then be represented as follows:

Major premiss:	Not Ae_MC	A does not one-sided-possibly belong to no C
Minor premiss:	Ba_XC	B belongs to all C
Conclusion:	Not Ae_MB	A does not one-sided-possibly belong to no B

If the contradictories of e_M-sentences are identified with i_N-sentences, then this syllogism is simply Disamis NXN. But if they are not so identified, then it is neither Disamis NXN nor Disamis NXX, but a third syllogism. We wish to remain neutral between this third option and the Disamis NXN option. On either of these two options, the syllogism in question is a contraposed version of Celarent MXM. Now, Aristotle does not explicitly discuss Celarent MXM in the modal syllogistic, but given that he regards Celarent NXN and Celarent QXQ as valid perfect syllogisms, it is reasonable to think that he would also accept Celarent MXM as valid.[43] Thus, Aristotle can be taken in point [iv] to apply the above contraposed version of Celarent MXM, and to state Not Ae_MB.

The sentence Not Ae_MB is the conclusion of the modal subordinate deduction. In point [v], Aristotle states that this conclusion is impossible, that is, he asserts Not Poss(Not Ae_MB). This statement can be derived from Ae_XB, the major premiss of Celarent XQM, by first inferring Ae_MB and then applying the principle of necessitation. Finally, in point [vi] Aristotle completes his proof by stating the conclusion of Celarent XQM, which he is now in a position to infer by *reductio*. The whole proof can then be reconstructed in close parallel to the first proof of Barbara XQM, as follows:

(1)	Ae_XB	(major premiss)
(2)	Ba_QC	(minor premiss)
(3)	$\quad\vert$ Not Ae_MC	(assumption for *reductio*)
(4)	$\quad\vert$ Nec(Not Ae_MC)	(from 3, by principle of necessitation)
(5)	$\quad\vert$ Ba_QC	(iterated from 2)
(6)	$\quad\vert$ Poss(Ba_XC)	(from 5)
(7)	$\quad\vert\quad\vert$ Ba_XC	(assumption for possibility rule)
(8)	$\quad\vert\quad\vert$ Not Ae_MC	(importation: 4)

[43] Cf. van Rijen 1989, p. 196.

(9)	Not $Ae_M B$	(from 7, 8, by contraposed version of Celarent MXM)
(10)	Poss(Not $Ae_M B$)	(possibility rule: 6, 7–9)
(11)	$Ae_M B$	(from 1)
(12)	Not Poss(Not $Ae_M B$)	(from 11, by principle of necessitation)
(13)	$Ae_M C$	(*reductio*: 3–10, 12)

As in the case of Barbara XQM, the present proof relies on four items: the inference from $Ba_Q C$ to Poss($Ba_X C$) (line 6), the principle of necessitation (lines 4 and 12), Celarent MXM (line 9, as opposed to Barbara MXM in the earlier proofs), and the principle that any X-sentence implies the corresponding M-sentence (line 11). Given the validity of these four items, Aristotle's proof of Celarent XQM is valid.

It remains to consider Aristotle's concluding remark in point [vii]: 'for while something false was posited, the result is impossible'. This remark has a close parallel in point [viii] of the first proof of Barbara XQM: 'for while something false but not impossible was posited, the result is impossible' (34b1–2). Both remarks can be understood as an explanation of the way in which the possibility rule is applied in the proofs. 'Something false (but not impossible)' refers to the assumption for the possibility rule, $Ba_X C$, and 'the result' refers to the conclusion of the modal subordinate deduction, which in the present proof is Not $Ae_M B$. Thus Aristotle's statement that 'the result is impossible' corresponds to Not Poss(Not $Ae_M B$) in line 12 of our reconstruction. The word 'false' in point [vii] can be understood as an abbreviation for the phrase 'false but not impossible', which appeared in its entirety in 34b1–2. As explained above, this phrase should be understood to mean 'at worst false, perhaps true, but not impossible', which is equivalent to 'possible' (see n. 18). Thus, Aristotle in effect states that the assumption for the possibility rule is possible, which corresponds to line 6 in our reconstruction.

In the standard translations of the *Prior Analytics*, point [vii] is rendered in a way that is incompatible with the interpretation just given. Barnes's revised Oxford translation of point [vii] reads: 'for if that is supposed false, the consequence is impossible'; Smith's translation is similar.[44] In this translation, 'that' refers to the conclusion of Celarent XQM stated in point [vi], and 'if that is supposed false' refers to the assumption for *reductio*, Not $Ae_M C$. Point [vii] would then

[44] Barnes 1984, p. 56. Smith's translation (1989, p. 23) reads: 'for when that was put as false, the result was impossible'. On the other hand, the translation in Striker 2009 (p. 24) is in line with our interpretation: 'for assuming a falsehood has led to an impossible result'.

explain Aristotle's application of the rule of *reductio*, rather than the use of the possibility rule within the *reductio*. However, such an interpretation makes little sense because the method of *reductio* is familiar to readers of the *Prior Analytics*, and does not call for explanation at this stage. The possibility rule, by contrast, has just been introduced earlier in chapter 1.15, and so it is reasonable for Aristotle to give some additional explanation of its application here. This speaks in favour of our interpretation of point [vii] over the one implied by Barnes's and Smith's translations.[45]

8. Proofs of Darii XQM and Ferio XQM (35a35–b2)

Aristotle also asserts the validity of Darii XQM and Ferio XQM (*Pr. An.* 1.15, 35a35–b2). He does not carry out a proof of these syllogisms, but it seems clear that he thinks their proofs would proceed in the same way as the ones already given. The required proofs can easily be constructed, in close parallel to Aristotle's first proof of Barbara XQM and his proof of Celarent XQM. There are only two modifications to take note of. First, where the earlier proofs relied on an inference from $\mathrm{Ba_QC}$ to $\mathrm{Poss(Ba_XC)}$, the new proofs will involve an inference from $\mathrm{Bi_QC}$ to $\mathrm{Poss(Bi_XC)}$. Second, different syllogisms will be applied within the modal subordinate deduction: where the earlier proofs relied on Barbara MXM and Celarent MXM respectively, the new proofs will rely on Darii MXM and Ferio MXM. Apart from that, these proofs will have exactly the same structure as the ones already given.

9. Two problems

Now that we have reconstructed Aristotle's proofs of the four first-figure XQM-syllogisms, we would like to close by discussing two problems to which these proofs give rise. Both problems concern the coherence of the proofs with the remainder of Aristotle's modal syllogistic. The first of them concerns only the proof of Celarent XQM, whereas the second is a broader issue affecting the proofs of all four syllogisms.

[45] Moreover, the similarity of phrasing and placement between 34b1–2 and 34b26–7 creates a presumption that both remarks have the same function. The phrase 'false but not impossible' in the first of these remarks makes it clear that Aristotle is describing an assumption for the possibility rule, not an assumption for *reductio*. For Aristotle used this phrase shortly beforehand in his justification of the possibility rule (34a25–9).

9.1 Is Celarent MXM valid?

Aristotle's proof of Celarent XQM, as reconstructed above, relies on the validity of Celarent MXM. However, some of Aristotle's claims elsewhere in the modal syllogistic suggest that he should deny the validity of Celarent MXM. In particular, Aristotle holds that the premiss pair $\mathrm{Be_QA}$, $\mathrm{Ba_XC}$ does not yield a valid syllogism (1.18, 37b19–23). However, if Celarent MXM were valid, there would be reason to think that this premiss pair yields the conclusion $\mathrm{Ae_MC}$. For, first of all, it is reasonable to think that any Q-sentence implies the corresponding M-sentence, and therefore that $\mathrm{Be_QA}$ implies $\mathrm{Be_MA}$.[46] Secondly, Aristotle holds that $\mathrm{e_M}$-sentences are convertible, so that $\mathrm{Be_MA}$ implies $\mathrm{Ae_MB}$.[47] Hence, the premiss pair $\mathrm{Be_QA}$, $\mathrm{Ba_XC}$ entails the premiss pair $\mathrm{Ae_MB}$, $\mathrm{Ba_XC}$. So, if Celarent MXM were valid, then the former premiss pair would yield the conclusion $\mathrm{Ae_MC}$.

This problem suggests that Aristotle's proof of Celarent XQM, with its reliance on Celarent MXM, does not fully cohere with the body of claims in the rest of his modal syllogistic. This does not mean that the syllogism Celarent XQM itself fails to cohere with the rest of the modal syllogistic; rather, the trouble arises from the specific way in which Aristotle chooses to prove this syllogism.

9.2 Alexander's problem

Whereas the problem just discussed was confined to the proof of Celarent XQM, there is a second problem of a more general nature. In particular, there is reason to fear that the method which Aristotle employs in his proofs of XQM-syllogisms is too strong, in that it could be used to establish inferences which Aristotle in fact rejects. In order to lead into the problem we have in mind, it will be helpful to start with a worry raised by Alexander of Aphrodisias in his commentary on the *Prior Analytics*. Alexander notes that Aristotle's method of proving Celarent XQM might appear as if it could also yield a proof of Celarent XQX.[48] Yet Aristotle would certainly deny the validity of

[46] Although Aristotle does not explicitly state this principle, it is very natural to think that he would accept it. He seems to assume that an $\mathrm{o_Q}$-sentence is incompatible with the denial of the corresponding $\mathrm{o_M}$-sentence in his proof of Bocardo QXM (*Pr. An.* 1.21, 39b31–9). This amounts to assuming that an $\mathrm{o_Q}$-sentence implies the corresponding $\mathrm{o_M}$-sentence.

[47] *Pr. An.* 1.3, 25b3–13. See also 1.18, 37b29, where the conversion of $\mathrm{e_M}$-sentences is used in the proof of Camestres QXM.

[48] Alexander of Aphrodisias *In Pr. An.* 217.8–18. This kind of problem is also discussed in Nortmann 2000 (p. 302) and in Ebert and Nortmann 2007 (p. 566); see also Hintikka 1973 (p. 186, n. 16).

this inference.[49] Alexander thinks he can explain why the method would not in fact yield such a proof (*In Pr. An.* 217.19–24), but it is difficult to see how exactly his explanation is supposed to work. On the other hand, we can see why the method cannot succeed in proving Celarent XQX if we write out an attempt at such a proof within the framework used above:

(1)	$\mathrm{Ae_XB}$		(major premiss)
(2)	$\mathrm{Ba_QC}$		(minor premiss)
(3)	$\quad\mathrm{Ai_XC}$		(assumption for *reductio*)
(4)	$\quad\mathrm{Nec(Ai_XC)}$	**	(from 3, by principle of necessitation)
(5)	$\quad\mathrm{Ba_QC}$		(iterated from 2)
(6)	$\quad\mathrm{Poss(Ba_XC)}$		(from 5)
(7)	$\quad\quad\mathrm{Ba_XC}$		(assumption for possibility rule)
(8)	$\quad\quad\mathrm{Ai_XC}$		(importation: 4)
(9)	$\quad\quad\mathrm{Ai_XB}$		(from 7, 8, by Disamis)
(10)	$\quad\mathrm{Poss(Ai_XB)}$		(possibility rule: 6, 7–9)
(11)	$\mathrm{Not\ Ai_XB}$		(from 1)
(12)	$\mathrm{Not\ Poss(Ai_XB)}$	**	(from 11, by principle of necessitation)
(13)	$\mathrm{Ae_XC}$		(*reductio*: 3–10, 12)

This attempted proof has two faulty steps, which are marked by double asterisks. In both of these steps, the principle of necessitation is applied to assertoric sentences, whereas, as we noted above, this principle is only applicable to modally qualified sentences. Because of this, there is no way validly to introduce the sentence $\mathrm{Ai_XC}$ into the modal subordinate deduction, nor to derive the required statement Not $\mathrm{Poss(Ai_XB)}$ in line 12. Thus, Aristotle's method of proving XQM-syllogisms does not lead to the undesired result that Celarent XQX, or other XQX-inferences, are valid.

However, Alexander's worry can be adapted to yield a more pressing difficulty, concerning Celarent NQN and other NQN-inferences. Aristotle states that Celarent NQN is invalid.[50] Yet a proof for it can be

[49] Aristotle denies the validity of Darapti, Disamis, Felapton, and Ferison XQX (*Pr. An.* 1.21, 39b7–9, 39b30–1), thus implying the invalidity of Darii and Ferio XQX. Also, he denies the validity of Barbara NQX (*Pr. An.* 1.16, 35b26–8), which strongly suggests that he would also deny the validity of Barbara XQX. All this suggests that Aristotle would deny the validity of Celarent XQX.

[50] *Pr. An.* 1.16, 35b34–6; cf. Ebert and Nortmann 2007, p. 591.

constructed which does not suffer from the faults of the above attempted proof of Celarent XQX, as follows:

(1)	Ae$_N$B	(major premiss)
(2)	Ba$_Q$C	(minor premiss)
(3)	│ Not Ae$_N$C	(assumption for *reductio*)
(4)	│ Nec(Not Ae$_N$C)	(from 3, by principle of necessitation)
(5)	│ Ba$_Q$C	(iterated from 2)
(6)	│ Poss(Ba$_X$C)	(from 5)
(7)	│ │ Ba$_X$C	(assumption for possibility rule)
(8)	│ │ Not Ae$_N$C	(importation: 4)
(9)	│ │ Not Ae$_N$B	(from 7, 8, by contraposed version of Celarent NXN)
(10)	│ Poss(Not Ae$_N$B)	(possibility rule: 6, 7–9)
(11)	Not Poss(Not Ae$_N$B)	(from 1, by principle of necessitation)
(12)	Ae$_N$C	(*reductio*: 3–10, 11)

This proof relies on the validity of three items. First, it employs in line 9 a contraposed version of Celarent NXN, a syllogism which Aristotle accepts as valid and perfect in *Prior Analytics* 1.9. Second, it contains an inference from Ba$_Q$C to Poss(Ba$_X$C) in line 6. Finally, it relies on the principle of necessitation applied to N-sentences in lines 4 and 11. So if our proposed proof of Celarent NQN is to be blocked, one of these three items must be rejected.

Celarent NXN is unassailable, given that it has Aristotle's explicit endorsement. The inference from Ba$_Q$C to Poss(Ba$_X$C) cannot be rejected because it is used in Aristotle's own proofs of XQM-syllogisms. This leaves the principle of necessitation for N-sentences. When we introduced the principle of necessitation, we took it to apply to all modally qualified sentences, although we only needed to use it in application to M-sentences. One might attempt to restrict the principle to M-sentences, and deny that it applies to N-sentences. However, this seems to be an ad hoc solution with little intrinsic plausibility. Thus, Aristotle's method of proving XQM-syllogisms leads to genuine problems with Celarent NQN. The same problems also arise in connection with Barbara, Darii, and Ferio NQN, whose validity Aristotle denies as well.[51] They too could be proved valid, in exactly the same way as Celarent NQN.

[51] Aristotle denies their validity at *Pr. An.* 1.16, 35b34–8; see Ebert and Nortmann 2007, pp. 591–2, Striker 2009, p. 152.

These problems show that it is not easy to interpret Aristotle's proofs of XQM-syllogisms in such a way that they cohere with the remainder of his modal syllogistic. It seems to us that this lack of coherence is largely due to the fact that, as we noted above, Aristotle's proofs combine two independent frameworks of modal reasoning. They employ both the framework of the possibility rule, centred around the modal qualifier 'Poss(…)', and the framework of the modal syllogistic, dealing with N-, Q-, and M-sentences based on perfect syllogisms and conversion rules. The interaction of the two frameworks in *Prior Analytics* 1.15 is effected by means of two bridging principles. First is the principle that an affirmative Q-sentence entails the possibility of the corresponding X-sentence; thus Aa_QB is taken to entail Poss(Aa_XB). Second is the principle of necessitation, according to which Not Poss(Not Y) follows from Y, where 'Y' stands for any N-, Q-, or M-sentence. These are the two principles which connect the modal syllogistic with the framework of the possibility rule.

Bridging the two frameworks in this way is not straightforward, because they seem to reflect quite different conceptions of modality. The qualifier 'Poss(…)' attributes possibility to whole sentences or states of affairs, in the way with which we are familiar from contemporary modal logic. This is not the case for the modal qualifiers occurring in N-, Q-, and M-sentences. One difference is that these modal qualifiers do not seem to specify the modal status of whole sentences or states of affairs (otherwise they would require a *de dicto* reading of Aristotle's N-, Q-, and M-sentences). Rather they seem to signify special kinds of modal relations between parts of these sentences or states of affairs, that is, between terms (see n. 7). Moreover, it is not clear how the modal relations in question connect to our familiar notions of possibility and necessity (see Sect. 3 above). Despite the differences between the frameworks of the possibility rule on the one hand and of the modal syllogistic on the other, we should expect that there is some connection between them. But it is also clear that Aristotle did not fully think through the consequences of connecting them in the way he did. The two bridging principles mentioned above embody substantive assumptions about the semantics of N-, Q-, and M-sentences (see nn. 19 and 24). If Aristotle wants to uphold all his claims of validity and invalidity in the modal syllogistic, and given that the possibility rule and the rule of importation are in themselves unobjectionable, he cannot endorse both of the bridging principles. In particular, he must deny at least one of them if he is to uphold his commitment to the validity of Celarent NXN and the invalidity of Celarent NQN.

In sum, Aristotle's proofs of XQM-syllogisms are in some tension with the rest of his modal syllogistic. The tension might be relieved in different ways, perhaps by revising some of his claims elsewhere in the modal syllogistic, or perhaps by imposing restrictions on the two bridging principles. In any case, if we set aside these problems arising from the broader context of the modal syllogistic, the proofs in themselves turn out to be perfectly valid under a small number of intrinsically plausible assumptions.

The reconstruction of Aristotle's proofs presented in this paper enjoys, we think, some advantages over previous reconstructions put forward in the secondary literature. First, unlike the authors cited in note 1 above, we do not need to attribute to Aristotle a fallacious and invalid argument, but are in a position to understand his proofs of the first-figure XQM-syllogisms as valid. Second, this is achieved without relying on over-strong and complicated assumptions regarding the semantics of Aristotle's modal sentences.[52] Finally, the reconstruction makes it clear how Aristotle's use of the possibility rule in the present proofs instantiates a general pattern of proof found throughout his works — a pattern which has been elaborated by us elsewhere (2012) and by Kit Fine (2011) in his treatment of *Metaphysics* Θ 4. All in all we hope to have shown that, considered in their own right, Aristotle's proofs are, despite their complexity, well-reasoned and elegant.[53]

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[52] An example of a reconstruction relying on such over-strong assumptions is, we think, the one given by Ebert and Nortmann (2007, pp. 552–76). They claim that Aristotle's proofs cannot be reconstructed without appealing to the framework of modern modal predicate logic and possible world semantics (see esp. pp. 560–5). Another reconstruction which incorporates unattractively strong assumptions is found in Rini 2011 (pp. 146–68). Her reconstruction builds into Aristotle's proofs a hidden premiss about the nature of the middle term in the XQM syllogisms to be proved: namely, that this term is predicated necessarily of everything of which it is predicated (pp. 153–5, 160, 164).

[53] This paper grew out of presentations given in a seminar led by Jonathan Beere at the Humboldt University of Berlin. For their valuable comments we would like to thank the participants of that seminar, as well as the editor of *Mind* and two anonymous referees.

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