Précis of Aristotle’s Modal Syllogistic

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1. Introduction

Aristotle was the founder of modal logic. In his Prior Analytics, he developed a complex system of modal syllogistic. While influential, this system has been disputed since antiquity and is today widely regarded as incoherent or inconsistent. In view of this, Aristotle’s Modal Syllogistic explores the prospects for understanding the modal syllogistic as a coherent and consistent system of modal logic. To this end, I introduce a model that matches all of Aristotle’s claims about the validity and invalidity of modal syllogisms. This model is developed throughout the book and is summarized in Appendix B. The model shows that, contrary to what is often thought, the set of those claims is consistent. Moreover, the purpose of the book is to explain, as far as possible, why Aristotle made the claims he made in the modal syllogistic. Thus, my aim is to give an account of Aristotle’s grounds for judging a given modal syllogism valid or invalid.

The book consists of three parts. The first deals with Aristotle’s assertoric, or non-modal, syllogistic (Prior Analytics 1.1–2 and 4–7). The second part deals with the apodeictic syllogistic, in which Aristotle discusses necessity propositions (Prior Analytics 1.3 and 8–12). The third part deals with the problematic syllogistic, in which Aristotle discusses possibility propositions (Prior Analytics 1.3 and 13–22). In what follows, I give an overview of each of these three parts.

2. The Assertoric Syllogistic

In the assertoric syllogistic, Aristotle is concerned with non-modal propositions such as ‘Every man is an animal’ and ‘Not every man is walking’. He usually represents these propositions by means of somewhat artificial phrases using the verb ‘belong to’. For example, he says ‘A belongs to all
B’ instead of ‘Every B is A’. Using the letter ‘X’ to indicate the lack of a modal qualifier, Aristotle’s four main kinds of assertoric propositions can be represented as follows:

- **AaXB**: A belongs to all B
- **AeXB**: A belongs to no B
- **AiXB**: A belongs to some B
- **AoXB**: A does not belong to some B

Aristotle explains the semantics of aX- and eX-propositions in *Prior Analytics* 1.1, in a brief passage which is known as the *dictum de omni et de nullo*:

> We say “predicated of all” when none of those of the subject can be taken of which the other will not be said, and likewise for “predicated of none”.

(*Prior Analytics* 1.1 24b28–30)

There are at least two ways of understanding the quantification indicated by the phrase ‘none of those of the subject can be taken’. On one interpretation, the quantifier ranges over individuals such as Socrates, Kallias, and Bucephalus. Accordingly, an aX-proposition is taken to be true just in case every individual which falls under the subject term falls under the predicate term. This leads to the following semantics of Aristotle’s assertoric propositions:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Statement</th>
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<tbody>
<tr>
<td><strong>AaXB</strong></td>
<td>if and only if for every individual ( z ), if ( z ) falls under B, ( z ) falls under A</td>
</tr>
<tr>
<td><strong>AeXB</strong></td>
<td>if and only if for every individual ( z ), if ( z ) falls under B, ( z ) does not fall under A</td>
</tr>
<tr>
<td><strong>AiXB</strong></td>
<td>if and only if there is an individual ( z ) which falls under both B and A</td>
</tr>
<tr>
<td><strong>AoXB</strong></td>
<td>if and only if there is an individual ( z ) which falls under B but not under A</td>
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This has been the dominant interpretation of the *dictum de omni et de nullo*, and it is often referred to as the orthodox interpretation. According to this interpretation, the semantics of Aristotle’s assertoric propositions is extensional in that their truth or falsity is determined solely by the extension of the terms involved (that is, by the sets of individuals which fall under these terms).

On the other hand, some commentators have proposed another, heterodox interpretation. According to this interpretation, the quantifier in question does not range exclusively over individuals but over terms such as A and B (or over their semantic values). Thus, Michael Frede and Ben Morison take the *dictum de omni* to state that an aX-proposition is true just in case the predicate term is aX-predicated of everything of which the subject term is aX-predicated.¹ This leads to the following semantics:

AaXB if and only if for every Z, if BaXZ then AaXZ
AeXB if and only if for every Z, if BaXZ then not AaXZ
AiXB if and only if there is a Z such that BaXZ and AaXZ
AoXB if and only if there is a Z such that BaXZ and not AaXZ

I discuss this heterodox semantics in detail, and argue that it should be preferred to the orthodox one as an interpretation of Aristotle’s *dictum de omni et de nullo* (chapters 2–6). Unlike the orthodox semantics, the heterodox semantics does not define aX-predication in terms of another, more primitive relation (such as that of an individual’s falling under a term). Thus, it does not provide an explicit definition of what aX-predication is, but treats aX-predication as a primitive relation. As a result, the semantics is not extensional. The truth or falsity of assertoric propositions is not determined by the sets of individuals that fall under the terms involved.

The heterodox *dictum de omni* entails that the relation of aX-predication is a preorder (i.e., that it is both reflexive and transitive). The semantics of the other three kinds of assertoric propositions is defined in terms of this primitive preorder. An advantage of this semantics is that, unlike the orthodox one, it validates all of Aristotle’s assertoric syllogisms and conversion rules—including the notorious rule of accidental conversion from AaXB to BiXA (which is not valid in the orthodox semantics). Thus the heterodox semantics, unlike the orthodox one, provides a solution to the problem of existential import (see chapters 3 and 4).

3. The Apodeictic Syllogistic

In the apodeictic syllogistic, Aristotle is concerned with necessity propositions, that is, with propositions which contain modal qualifiers such as ‘necessarily’. He focuses on four kinds of these propositions:

- **AaXB**: A necessarily belongs to all B
- **AeXB**: A necessarily belongs to no B
- **AiXB**: A necessarily belongs to some B
- **AoXB**: A necessarily does not belong to some B

Aristotle discusses inferences from mixed premise pairs consisting of an assertoric proposition and a necessity proposition (*Prior Analytics* 1.9–11). For example, he takes the following syllogism, known as Barbara NXN, to be valid:

- **Major premise**: AaXB
- **Minor premise**: BaXC
- **Conclusion**: AaXC

<table>
<thead>
<tr>
<th>Major premise</th>
<th>Minor premise</th>
<th>Conclusion</th>
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<tbody>
<tr>
<td>AaXB</td>
<td>BaXC</td>
<td>AaXC</td>
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A necessarily belongs to all B
B belongs to all C
A necessarily belongs to all C
This syllogism has been the subject of controversy since antiquity. It was rejected by Aristotle’s pupil Theophrastus, who gave the following counter-example to it:\(^2\)

| Major premise: | Animal necessarily belongs to all man | (TRUE) |
| Minor premise: | Man belongs to all moving | (supposed to be TRUE) |
| Conclusion: | Animal necessarily belongs to all moving | (FALSE) |

While Aristotle does not explain in detail why Barbara NXN is valid, its validity is fundamental to the modal syllogistic. It is crucial, therefore, to understand why he took Barbara NXN to be valid and how he would respond to Theophrastus’s counterexample.

To answer these questions, I argue that Aristotle’s endorsement of Barbara NXN is motivated by the theory of predication that Aristotle sets forth in the *Topics*. In particular, I focus on the *Topics*’ theory of predicables and categories (chapters 8–10). According to the theory of the predicables, A is predicated of B just in case A is a definition, genus, differentia, proprium, or accident of B (*Topics* 1.4 and 1.9). If A is a definition, genus, or differentia of B, then the predication is essential. Otherwise the predication is accidental.

In addition, the *Topics*’ theory of categories introduces a distinction between two kinds of terms. The first group contains substance terms like ‘animal’ and ‘man’, and non-substance terms like ‘color’, ‘redness’, and ‘motion’. Call these essence terms. The second group contains non-substance terms like ‘colored’, ‘red’, and ‘moving’. Call these non-essence terms.

\[
\begin{align*}
\text{Essence terms:} & \quad \text{‘animal’, ‘man’, ‘color’, ‘redness’, ‘motion’, \ldots} \\
\text{Non-essence terms:} & \quad \text{‘colored’, ‘red’, ‘walking’, \ldots}
\end{align*}
\]

In the *Topics*, Aristotle imposes various restrictions on the kinds of predications in which essence terms can occur. For example, essence terms can be predicated only of terms of which they are predicated essentially. For example, ‘animal’ is predicated only of terms like ‘man’, ‘horse’, and ‘Socrates’, and it is predicated essentially of all of them. It is not predicated of terms like ‘walking’, on the grounds that ‘animal’ is not a definition, genus, differentia, proprium, or accident of ‘walking’. Likewise, ‘color’ is predicated only of terms like ‘redness’, ‘scarlet redness’, and ‘crimson redness’, and it is predicated essentially of all of them. Thus, essence terms are predicated essentially of everything of which they are predicated. On the other hand, non-essence terms can be predicated non-essentially of subjects; for example, both ‘red’ and ‘walking’ may be predicated as accidents of ‘man’.

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I argue that the relations of aX- and aN-predication employed in the modal syllogistic are closely connected with the *Topics*’ relations of predication and essential predication (chapters 8–10). Thus, essence terms are aX-predicated only of terms of which they are aN-predicated. For example, ‘animal’ is aX-predicated only of terms like ‘man’ and ‘horse’. It is not aX-predicated of terms like ‘walking’, on the grounds that it is not *predicated* of them. Although every individual that falls under ‘walking’ also falls under ‘animal’, the latter term is not aX-predicated of the former; for aX-predication is not extensional and is not determined by the sets of individuals that fall under the terms involved. Likewise, ‘man’ is not aX-predicated of ‘moving’ even if it happens to be the case that every individual that is moving is a man. Thus, Theophrastus’s counterexample to Barbara NXN can be rejected since its minor premise is false.

On this account, essence terms are aN-predicated of everything of which they are aX-predicated:

**Thesis 1:** For any B, if B is an essence term then B is aN-predicated of everything of which it is aX-predicated.

In addition, I argue that Aristotle imposes a restriction to the effect that only essence terms can serve as the subjects of aN-predications (that is, of *true* aN-propositions). Non-essence terms cannot serve as subjects of aN-predications.\(^3\) This is because aN-predications need to be grounded in the definable essence of their subjects and non-essence terms lack such an essence (pp. 125–7). Thus, every subject of an aN-predication is an essence term. For example, ‘color’ is aN-predicated of ‘redness’, but ‘colored’ is not aN-predicated of ‘red’.

**Thesis 2:** For any A and B, if A is aN-predicated of B then B is an essence term.

Given Theses 1 and 2, the premise pair AaNB and BaXC entails BaNC. In other words, the premise pair of Barbara NXN entails the premise pair of Barbara NNN.\(^4\) Since the validity of the latter syllogism is generally accepted, the above two theses suffice to justify the validity of Barbara NXN: the premise pair of Barbara NXN implies that of Barbara NNN, and therefore yields an aN-conclusion. Although Aristotle does not explicitly state the two theses in the modal syllogistic, I argue that he endorsed them, based on the *Topics*’ theory of predication, and that he took them to justify the validity of Barbara NXN (chapters 8–10).

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3 This is to say that non-essence terms cannot serve as subjects of *true* aN-propositions, although they can serve as subjects of *false* aN-propositions.

4 This does not mean that the two premise pairs are identical; they are distinct because their minor premises differ in the presence or absence of the qualifier ‘necessarily’.

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A similar explanation can be given for Celarent NXN, another syllogism endorsed by Aristotle in the apodeictic syllogistic:

- **Major premise:** $\text{AeNB}$: A necessarily belongs to no B
- **Minor premise:** $\text{BaN}$: B belongs to all C
- **Conclusion:** $\text{AeNC}$: A necessarily belongs to no C

It is often thought that Aristotle’s commitment to Celarent NXN conflicts with his commitment to the following rule of $e_N$-conversion:

- **Premise:** $\text{AeNB}$: A necessarily belongs to no B
- **Conclusion:** $\text{BeNA}$: B necessarily belongs to no A

The prevailing view is that $e_N$-conversion requires a *de dicto* reading of $e_N$-propositions, whereas Celarent NXN requires a *de re* reading. If this is correct, Aristotle’s use of $e_N$-propositions in the modal syllogistic is ambiguous and incoherent. By contrast, I argue that the alleged ambiguity of $e_N$-propositions can be avoided by adopting an alternative semantics of $e_N$-propositions which is neither purely *de dicto* nor purely *de re*, as follows:

- $\text{AeNB}$ if and only if (i) both A and B are essence terms, and (ii) A is $e_X$-predicated of B

Clearly, this interpretation of $e_N$-propositions is symmetric, and hence validates $e_N$-conversion. At the same time, it validates Celarent NXN (because essence terms are $a_X$-predicated only of essence terms but not of non-essence terms). Thus, both Celarent NXN and $e_N$-conversion are validated by a single interpretation of $e_N$-propositions, and there is no need to attribute to Aristotle an ambiguity in his use of $e_N$-propositions (chapter 11).

### 4. The Problematic Syllogistic

In the problematic syllogistic, Aristotle is concerned with possibility propositions, that is, with propositions which contain modal qualifiers such as ‘possibly’. Aristotle distinguishes two kinds of these propositions, usually referred to as one- and two-sided possibility propositions. Being two-sided possible means being neither impossible nor necessary, and being one-sided possible simply means being not impossible. For example, ‘Possibly no man is a horse’ is true if understood as a one-sided possibility proposition, but not if understood as a two-sided possibility proposition.

Two-sided possibility propositions prevail in Aristotle’s modal syllogistic. Using the letter ‘Q’ to indicate two-sided possibility, the four kinds of these propositions can be represented as follows:

- $\text{AaQB}$: A two-sided-possibly belongs to all B
- $\text{AeQB}$: A two-sided-possibly belongs to no B

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Similarly, one-sided possibility propositions are indicated by the letter ‘M’ and represented by formulae such as AaMB.

Aristotle’s treatment of possibility propositions raises questions concerning their relation to necessity propositions (chapter 13). It is clear from some of Aristotle’s proofs in the problematic syllogistic that he endorses the following principles of incompatibility:

\[
\begin{align*}
&AaQB \text{ is incompatible with } AeNB \\
&AiQB \text{ is incompatible with } AeNB
\end{align*}
\]

At the same time, however, Aristotle commits himself to denying some other such principles. Most strikingly, he is committed to denying that eQ-propositions are incompatible with the corresponding aN-propositions (pp. 201–10). This follows from his claim that the premise pair BeQA, BaNC does not yield any conclusion (Prior Analytics 1.19 38a26–b4); for if eQ-propositions were incompatible with aN-propositions, it would be easy to prove that this premise pair yields the conclusion AoXC.\(^5\) Aristotle is therefore committed to the view that AeQB is compatible with AaNB. If we are to verify Aristotle’s claims of invalidity in the modal syllogistic, we have to accept that some eQ-predications coincide with aN-predications.

This consequence is counterintuitive and constitutes a major difficulty in interpreting the problematic syllogistic. It is often taken to show that Aristotle’s modal syllogistic is inconsistent. By contrast, I argue that the modal syllogistic can be viewed as consistent if we accept that in some cases AeQB and AaNB are both true at the same time (chapters 13 and 14).

This approach puts us in a position to specify a deductive system for Aristotle’s modal syllogistic (chapter 15). The deductive system is adequate with respect to the modal syllogistic in the sense that every schema held to be valid by Aristotle in the modal syllogistic is deducible in this system but no schema held to be invalid by him is deducible in it.

I proceed to specify a semantic interpretation of Aristotle’s possibility propositions (chapters 17 and 18). This interpretation is based on three primitive relations:

\(^5\) Suppose BeQA and BaNC. In order to establish the conclusion AoXC, assume for reductio its contradictory, AaXC. From BeQA and AaXC we infer BeC by means of Celarent QXQ (which Aristotle accepts as valid). So, if BeC were incompatible with BaNC, the proof by reductio would be successful.
(i) $a_X$-predication
(ii) $a_N$-predication
(iii) strong $a_N$-predication

The third relation, that of strong $a_N$-predication, picks out those $a_N$-predications whose subject is a substance term and which do not coincide with an $a_M$-predication (and hence also not with an $a_Q$-predication). The three primitive relations are governed by the *Topics*’ theory of predication as outlined above. They suffice to specify an interpretation of Aristotle’s assertoric and modal propositions. Thus, all of Aristotle’s $N$-, $Q$-, $M$-, and $X$-propositions are interpreted solely by means of the three primitive relations. The resulting semantics is called the predicable semantics of the modal syllogistic.

The predicable semantics verifies all of Aristotle’s claims of validity and invalidity: every syllogism held to be valid by Aristotle in the modal syllogistic is valid in the predicable semantics, and every syllogism held to be invalid by him is invalid in it. Thus, the predicable semantics establishes the consistency of the modal syllogistic (that is, the consistency of all of Aristotle’s claims concerning the validity and invalidity of modal syllogisms).

In addition, the predicable semantics helps explain why Aristotle made the claims he made in the modal syllogistic. The predicable semantics provides such an explanation for large parts of the apodeictic syllogistic, but it cannot provide it for the whole modal syllogistic (see pp. 268–71). This is so for two reasons. First, while the predicable semantics can account for all of Aristotle’s claims about the validity and invalidity of modal syllogisms, it cannot account for all of his *proofs* of these claims. Consequently, it cannot establish that these proofs and the assumptions on which they rely are consistent with the rest of the modal syllogistic.

Second, in some cases the interpretation in the predicable semantics of Aristotle’s modal propositions is rather complex and technical. In particular, this is true of $o_N$-propositions and some kinds of $Q$- and $M$-propositions. When such interpretations are involved, the predicable semantics does not reflect the reasons Aristotle had for his claims of validity and invalidity. Thus, the predicable semantics cannot properly explain some of Aristotle’s claims concerning $o_N$-propositions. For example, it cannot explain why he took Baroco $XNN$ and Bocardo $NXN$ to be invalid. Nor is the predicable semantics intended to provide such an explanation. Instead, I argue that Aristotle’s treatment of those two syllogisms is motivated, not by any specific semantic interpretation of $o_N$-propositions, but rather by general considerations about certain monotonicity properties of $N$-propositions (pp. 186–90). Thus, the explanatory goal of the book is achieved in part by the predicable semantics, in part by more general considerations extending beyond the predicable semantics, and in some cases it is left for future research.