Philosophy and
Phenomenological Research

Philosophy and Phenomenological Research
Vol. XC No. 3, May 2015
doi: 10.1111/phpr.12185

Précis of *Aristotle's Modal Syllogistic*

MARKO MALINK
New York University

1. Introduction

Aristotle was the founder of modal logic. In his *Prior Analytics*, he developed a complex system of modal syllogistic. While influential, this system has been disputed since antiquity and is today widely regarded as incoherent or inconsistent. In view of this, *Aristotle's Modal Syllogistic* explores the prospects for understanding the modal syllogistic as a coherent and consistent system of modal logic. To this end, I introduce a model that matches all of Aristotle's claims about the validity and invalidity of modal syllogisms. This model is developed throughout the book and is summarized in Appendix B. The model shows that, contrary to what is often thought, the set of those claims is consistent. Moreover, the purpose of the book is to explain, as far as possible, why Aristotle made the claims he made in the modal syllogistic. Thus, my aim is to give an account of Aristotle's grounds for judging a given modal syllogism valid or invalid.

The book consists of three parts. The first deals with Aristotle's assertoric, or non-modal, syllogistic (*Prior Analytics* 1.1–2 and 4–7). The second part deals with the apodeictic syllogistic, in which Aristotle discusses necessity propositions (*Prior Analytics* 1.3 and 8–12). The third part deals with the problematic syllogistic, in which Aristotle discusses possibility propositions (*Prior Analytics* 1.3 and 13–22). In what follows, I give an overview of each of these three parts.

2. The Assertoric Syllogistic

In the assertoric syllogistic, Aristotle is concerned with non-modal propositions such as 'Every man is an animal' and 'Not every man is walking'. He usually represents these propositions by means of somewhat artificial phrases using the verb 'belong to'. For example, he says 'A belongs to all

B' instead of 'Every B is A'. Using the letter 'X' to indicate the lack of a modal qualifier, Aristotle's four main kinds of assertoric propositions can be represented as follows:

Aa_XB	A belongs to all B
Ae_XB	A belongs to no B
Ai_XB	A belongs to some B
Ao_XB	A does not belong to some B

Aristotle explains the semantics of a_X- and e_X-propositions in *Prior Analytics* 1.1, in a brief passage which is known as the *dictum de omni et de nullo*:

> We say "predicated of all" when none of those of the subject can be taken of which the other will not be said, and likewise for "predicated of none". (*Prior Analytics* 1.1 24b28–30)

There are at least two ways of understanding the quantification indicated by the phrase 'none of those of the subject can be taken'. On one interpretation, the quantifier ranges over individuals such as Socrates, Kallias, and Bucephalus. Accordingly, an a_X-proposition is taken to be true just in case every individual which falls under the subject term falls under the predicate term. This leads to the following semantics of Aristotle's assertoric propositions:

Aa_XB	if and only if	for every individual *z*, if *z* falls under B, *z* falls under A
Ae_XB	if and only if	for every individual *z*, if *z* falls under B, *z* does not fall under A
Ai_XB	if and only if	there is an individual *z* which falls under both B and A
Ao_XB	if and only if	there is an individual *z* which falls under B but not under A

This has been the dominant interpretation of the *dictum de omni et de nullo*, and it is often referred to as the orthodox interpretation. According to this interpretation, the semantics of Aristotle's assertoric propositions is extensional in that their truth or falsity is determined solely by the extension of the terms involved (that is, by the sets of individuals which fall under these terms).

On the other hand, some commentators have proposed another, heterodox interpretation. According to this interpretation, the quantifier in question does not range exclusively over individuals but over terms such as A and B (or over their semantic values). Thus, Michael Frede and Ben Morison take the *dictum de omni* to state that an a_X-proposition is true just in case the predicate term is a_X-predicated of everything of which the subject term is a_X-predicated.[1] This leads to the following semantics:

[1] See B. Morison, 'Aristotle, etc.', *Phronesis* 53 (2008), 209–22, at 212–15.

Aa_XB	if and only if	for every Z, if Ba_XZ then Aa_XZ
Ae_XB	if and only if	for every Z, if Ba_XZ then not Aa_XZ
Ai_XB	if and only if	there is a Z such that Ba_XZ and Aa_XZ
Ao_XB	if and only if	there is a Z such that Ba_XZ and not Aa_XZ

I discuss this heterodox semantics in detail, and argue that it should be preferred to the orthodox one as an interpretation of Aristotle's *dictum de omni et de nullo* (chapters 2–6). Unlike the orthodox semantics, the heterodox semantics does not define a_X-predication in terms of another, more primitive relation (such as that of an individual's falling under a term). Thus, it does not provide an explicit definition of what a_X-predication is, but treats a_X-predication as a primitive relation. As a result, the semantics is not extensional. The truth or falsity of assertoric propositions is not determined by the sets of individuals that fall under the terms involved.

The heterodox *dictum de omni* entails that the relation of a_X-predication is a preorder (i.e., that it is both reflexive and transitive). The semantics of the other three kinds of assertoric propositions is defined in terms of this primitive preorder. An advantage of this semantics is that, unlike the orthodox one, it validates all of Aristotle's assertoric syllogisms and conversion rules—including the notorious rule of accidental conversion from Aa_XB to Bi_XA (which is not valid in the orthodox semantics). Thus the heterodox semantics, unlike the orthodox one, provides a solution to the problem of existential import (see chapters 3 and 4).

3. The Apodeictic Syllogistic

In the apodeictic syllogistic, Aristotle is concerned with necessity propositions, that is, with propositions which contain modal qualifiers such as 'necessarily'. He focuses on four kinds of these propositions:

Aa_NB	A necessarily belongs to all B
Ae_NB	A necessarily belongs to no B
Ai_NB	A necessarily belongs to some B
Ao_NB	A necessarily does not belong to some B

Aristotle discusses inferences from mixed premise pairs consisting of an assertoric proposition and a necessity proposition (*Prior Analytics* 1.9–11). For example, he takes the following syllogism, known as Barbara NXN, to be valid:

Major premise:	Aa_NB	A necessarily belongs to all B
Minor premise:	Ba_XC	B belongs to all C
Conclusion:	Aa_NC	A necessarily belongs to all C

This syllogism has been the subject of controversy since antiquity. It was rejected by Aristotle's pupil Theophrastus, who gave the following counterexample to it:[2]

Major premise:	Animal necessarily belongs to all man	(TRUE)
Minor premise:	Man belongs to all moving	(supposed to be TRUE)
Conclusion:	Animal necessarily belongs to all moving	(FALSE)

While Aristotle does not explain in detail why Barbara NXN is valid, its validity is fundamental to the modal syllogistic. It is crucial, therefore, to understand why he took Barbara NXN to be valid and how he would respond to Theophrastus's counterexample.

To answer these questions, I argue that Aristotle's endorsement of Barbara NXN is motivated by the theory of predication that Aristotle sets forth in the *Topics*. In particular, I focus on the *Topics*' theory of predicables and categories (chapters 8–10). According to the theory of the predicables, A is predicated of B just in case A is a definition, genus, differentia, proprium, or accident of B (*Topics* 1.4 and 1.9). If A is a definition, genus, or differentia of B, then the predication is essential. Otherwise the predication is accidental.

In addition, the *Topics*' theory of categories introduces a distinction between two kinds of terms. The first group contains substance terms like 'animal' and 'man', and non-substance terms like 'color', 'redness', and 'motion'. Call these essence terms. The second group contains non-substance terms like 'colored', 'red', and 'moving'. Call these non-essence terms.

Essence terms:	'animal', 'man', 'color', 'redness', 'motion', . . .
Non-essence terms:	'colored', 'red', 'walking', . . .

In the *Topics*, Aristotle imposes various restrictions on the kinds of predications in which essence terms can occur. For example, essence terms can be predicated only of terms of which they are predicated essentially. For example, 'animal' is predicated only of terms like 'man', 'horse', and 'Socrates', and it is predicated essentially of all of them. It is not predicated of terms like 'walking', on the grounds that 'animal' is not a definition, genus, differentia, proprium, or accident of 'walking'. Likewise, 'color' is predicated only of terms like 'redness', 'scarlet redness', and 'crimson redness', and it is predicated essentially of all of them. Thus, essence terms are predicated essentially of everything of which they are predicated. On the other hand, non-essence terms can be predicated non-essentially of subjects; for example, both 'red' and 'walking' may be predicated as accidents of 'man'.

---

[2] See Alexander of Aphrodisias, *Commentary on Prior Analytics*, 124.24–5.

I argue that the relations of a_X- and a_N-predication employed in the modal syllogistic are closely connected with the *Topics*' relations of predication and essential predication (chapters 8–10). Thus, essence terms are a_X-predicated only of terms of which they are a_N-predicated. For example, 'animal' is a_X-predicated only of terms like 'man' and 'horse'. It is not a_X-predicated of terms like 'walking', on the grounds that it is not *predicated* of them. Although every individual that falls under 'walking' also falls under 'animal', the latter term is not a_X-predicated of the former; for a_X-predication is not extensional and is not determined by the sets of individuals that fall under the terms involved. Likewise, 'man' is not a_X-predicated of 'moving' even if it happens to be the case that every individual that is moving is a man. Thus, Theophrastus's counterexample to Barbara NXN can be rejected since its minor premise is false.

On this account, essence terms are a_N-predicated of everything of which they are a_X-predicated:

> Thesis 1: For any B, if B is an essence term then B is a_N-predicated of everything of which it is a_X-predicated.

In addition, I argue that Aristotle imposes a restriction to the effect that only essence terms can serve as the subjects of a_N-predications (that is, of *true* a_N-propositions). Non-essence terms cannot serve as subjects of a_N-predications.[3] This is because a_N-predications need to be grounded in the definable essence of their subjects and non-essence terms lack such an essence (pp. 125–7). Thus, every subject of an a_N-predication is an essence term. For example, 'color' is a_N-predicated of 'redness', but 'colored' is not a_N-predicated of 'red'.

> Thesis 2: For any A and B, if A is a_N-predicated of B then B is an essence term.

Given Theses 1 and 2, the premise pair Aa_NB and Ba_XC entails Ba_NC. In other words, the premise pair of Barbara NXN entails the premise pair of Barbara NNN.[4] Since the validity of the latter syllogism is generally accepted, the above two theses suffice to justify the validity of Barbara NXN: the premise pair of Barbara NXN implies that of Barbara NNN, and therefore yields an a_N-conclusion. Although Aristotle does not explicitly state the two theses in the modal syllogistic, I argue that he endorsed them, based on the *Topics*' theory of predication, and that he took them to justify the validity of Barbara NXN (chapters 8–10).

---

[3] This is to say that non-essence terms cannot serve as subjects of *true* a_N-propositions, although they can serve as subjects of *false* a_N-propositions.

[4] This does not mean that the two premise pairs are identical; they are distinct because their minor premises differ in the presence or absence of the qualifier 'necessarily'.

A similar explanation can be given for Celarent NXN, another syllogism endorsed by Aristotle in the apodeictic syllogistic:

Major premise:	Ae_NB	A necessarily belongs to no B
Minor premise:	Ba_XC	B belongs to all C
Conclusion:	Ae_NC	A necessarily belongs to no C

It is often thought that Aristotle's commitment to Celarent NXN conflicts with his commitment to the following rule of e_N-conversion:

Premise:	Ae_NB	A necessarily belongs to no B
Conclusion:	Be_NA	B necessarily belongs to no A

The prevailing view is that e_N-conversion requires a *de dicto* reading of e_N-propositions, whereas Celarent NXN requires a *de re* reading. If this is correct, Aristotle's use of e_N-propositions in the modal syllogistic is ambiguous and incoherent. By contrast, I argue that the alleged ambiguity of e_N-propositions can be avoided by adopting an alternative semantics of e_N-propositions which is neither purely *de dicto* nor purely *de re*, as follows:

Ae_NB	if and only if	(i) both A and B are essence terms, and (ii) A is e_X-predicated of B

Clearly, this interpretation of e_N-propositions is symmetric, and hence validates e_N-conversion. At the same time, it validates Celarent NXN (because essence terms are a_X-predicated only of essence terms but not of non-essence terms). Thus, both Celarent NXN and e_N-conversion are validated by a single interpretation of e_N-propositions, and there is no need to attribute to Aristotle an ambiguity in his use of e_N-propositions (chapter 11).

4. The Problematic Syllogistic

In the problematic syllogistic, Aristotle is concerned with possibility propositions, that is, with propositions which contain modal qualifiers such as 'possibly'. Aristotle distinguishes two kinds of these propositions, usually referred to as one- and two-sided possibility propositions. Being two-sided possible means being neither impossible nor necessary, and being one-sided possible simply means being not impossible. For example, 'Possibly no man is a horse' is true if understood as a one-sided possibility proposition, but not if understood as a two-sided possibility proposition.

Two-sided possibility propositions prevail in Aristotle's modal syllogistic. Using the letter 'Q' to indicate two-sided possibility, the four kinds of these propositions can be represented as follows:

Aa_QB	A two-sided-possibly belongs to all B
Ae_QB	A two-sided-possibly belongs to no B

Ai_QB A two-sided-possibly belongs to some B
Ao_QB A two-sided-possibly does not belong to some B

Similarly, one-sided possibility propositions are indicated by the letter 'M' and represented by formulae such as Aa_MB.

Aristotle's treatment of possibility propositions raises questions concerning their relation to necessity propositions (chapter 13). It is clear from some of Aristotle's proofs in the problematic syllogistic that he endorses the following principles of incompatibility:

Aa_QB is incompatible with Ae_NB
Ai_QB is incompatible with Ae_NB

At the same time, however, Aristotle commits himself to denying some other such principles. Most strikingly, he is committed to denying that e_Q-propositions are incompatible with the corresponding a_N-propositions (pp. 201–10). This follows from his claim that the premise pair Be_QA, Ba_NC does not yield any conclusion (*Prior Analytics* 1.19 38a26–b4); for if e_Q-propositions were incompatible with a_N-propositions, it would be easy to prove that this premise pair yields the conclusion Ao_XC.[5] Aristotle is therefore committed to the view that Ae_QB is compatible with Aa_NB. If we are to verify Aristotle's claims of invalidity in the modal syllogistic, we have to accept that some e_Q-predications coincide with a_N-predications.

This consequence is counterintuitive and constitutes a major difficulty in interpreting the problematic syllogistic. It is often taken to show that Aristotle's modal syllogistic is inconsistent. By contrast, I argue that the modal syllogistic can be viewed as consistent if we accept that in some cases Ae_QB and Aa_NB are both true at the same time (chapters 13 and 14).

This approach puts us in a position to specify a deductive system for Aristotle's modal syllogistic (chapter 15). The deductive system is adequate with respect to the modal syllogistic in the sense that every schema held to be valid by Aristotle in the modal syllogistic is deducible in this system but no schema held to be invalid by him is deducible in it.

I proceed to specify a semantic interpretation of Aristotle's possibility propositions (chapters 17 and 18). This interpretation is based on three primitive relations:

---

[5] Suppose Be_QA and Ba_NC. In order to establish the conclusion Ao_XC, assume for *reductio* its contradictory, Aa_XC. From Be_QA and Aa_XC we infer Be_QC by means of Celarent QXQ (which Aristotle accepts as valid). So, if Be_QC were incompatible with Ba_NC, the proof by *reductio* would be successful.

(i) a_X-predication

(ii) a_N-predication

(iii) strong a_N-predication

The third relation, that of strong a_N-predication, picks out those a_N-predications whose subject is a substance term and which do not coincide with an o_M-predication (and hence also not with an e_Q-predication). The three primitive relations are governed by the *Topics*' theory of predication as outlined above. They suffice to specify an interpretation of Aristotle's assertoric and modal propositions. Thus, all of Aristotle's N-, Q-, M-, and X-propositions are interpreted solely by means of the three primitive relations. The resulting semantics is called the predicable semantics of the modal syllogistic.

The predicable semantics verifies all of Aristotle's claims of validity and invalidity: every syllogism held to be valid by Aristotle in the modal syllogistic is valid in the predicable semantics, and every syllogism held to be invalid by him is invalid in it. Thus, the predicable semantics establishes the consistency of the modal syllogistic (that is, the consistency of all of Aristotle's claims concerning the validity and invalidity of modal syllogisms).

In addition, the predicable semantics helps explain why Aristotle made the claims he made in the modal syllogistic. The predicable semantics provides such an explanation for large parts of the apodeictic syllogistic, but it cannot provide it for the whole modal syllogistic (see pp. 268–71). This is so for two reasons. First, while the predicable semantics can account for all of Aristotle's claims about the validity and invalidity of modal syllogisms, it cannot account for all of his *proofs* of these claims. Consequently, it cannot establish that these proofs and the assumptions on which they rely are consistent with the rest of the modal syllogistic.

Second, in some cases the interpretation in the predicable semantics of Aristotle's modal propositions is rather complex and technical. In particular, this is true of o_N-propositions and some kinds of Q- and M-propositions. When such interpretations are involved, the predicable semantics does not reflect the reasons Aristotle had for his claims of validity and invalidity. Thus, the predicable semantics cannot properly explain some of Aristotle's claims concerning o_N-propositions. For example, it cannot explain why he took Baroco XNN and Bocardo NXN to be invalid. Nor is the predicable semantics intended to provide such an explanation. Instead, I argue that Aristotle's treatment of those two syllogisms is motivated, not by any specific semantic interpretation of o_N-propositions, but rather by general considerations about certain monotonicity properties of N-propositions (pp. 186–90). Thus, the explanatory goal of the book is achieved in part by the predicable semantics, in part by more general considerations extending beyond the predicable semantics, and in some cases it is left for future research.