Indeterminate Propositions in
Prior Analytics I.41

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In *Analytica Priora* I.41 stellt Aristoteles eine bemerkenswerte Behauptung über unbestimmte Aussagen wie ‚A kommt B zu' auf. Die Behauptung impliziert, dass solche unbestimmten Aussagen nicht gleichwertig sind zu folgender Ekthesisbedingung: ‚es gibt ein C, so dass B allem C zukommt und A allem C zukommt'. Anderseits wird oft angenommen, dass unbestimmte Aussagen gleichwertig sind zu partikulären Aussagen wie ‚A kommt einigem B zu' und dass diese wiederum gleichwertig sind zur Ekthesisbedingung. Im vorliegenden Artikel werden verschiedene Lösungsansätze für diese Schwierigkeit diskutiert.

This paper is about a puzzling statement of Aristotle's in *Prior Analytics* I.41. The statement involves indeterminate affirmative propositions such as 'A belongs to B' (formally, Υab). It also involves universal affirmative propositions such as 'A belongs to all B' (formally, $\mathrm{A}ab$). The context is a discussion of what came to be called prosleptic propositions, such as for instance $\forall z(\mathrm{A}bz \supset \Upsilon az)$ (Section 1).

The statement in question is that $\forall z(\mathrm{A}bz \supset \Upsilon az)$ does not imply $\forall z(\Upsilon bz \supset \Upsilon az)$. W. and M. Kneale argue that this is incorrect. It is incorrect if Υab is equivalent to the following condition of ecthesis: $\exists z(\mathrm{A}bz \wedge \mathrm{A}az)$. Although there is reason to accept this equivalence, I shall discuss three ways to deny it (Section 2).

Each way leads to a different semantics for the assertoric syllogistic. In each of the resulting three semantics, indeterminate affirmative propositions are treated as primitive. Their truth conditions are not defined in terms of other notions; instead, the truth conditions of the other assertoric propositions are defined in terms of them. For example, $\mathrm{A}ab$ is true if and only if $\forall z(\Upsilon bz \supset \Upsilon az)$ is true. The three semantics are in accordance with what Aristotle says about indeterminate propositions in the *Prior Analytics*, including the puzzling statement (Section 3).

Finally, I offer an explanation of why indeterminate affirmative propositions are not equivalent to the condition of ecthesis. This will allow us to reject the Kneales's argument against the puzzling statement (Section 4).

1. Prosleptic propositions in *Prior Analytics* I.41

In *Prior Analytics* I.41, Aristotle discusses constructions of the form 'whatever B belongs to, A belongs to all of it' or 'whatever B is said of all of, A is said of all of it', etc. The constructions consist of a main clause and a relative clause, introduced by the relative pronoun 'whatever' (ᾧ or καθ' οὗ). Since Theophrastus, propositions expressed by such relative clause constructions have been called prosleptic propositions (cf. Alexander *in APr.* 378.14). A good overview of prosleptic propo-

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sitions and the relevant ancient sources is given by Lejewski (1961) and Kneale & Kneale (1972). The Kneales's paper also gives an illuminating interpretation of Aristotle's discussion of prosleptic propositions in *Prior Analytics* I.41; the present section closely follows their interpretation.

In addition to the two terms A and B, the relative pronoun of prosleptic propositions introduces an implicit third term. From the perspective of modern logic, this term may be viewed as a variable bound by a universal quantifier which takes scope over a material implication. Thus the prosleptic propositions discussed by Aristotle may be formulated in terms of classical propositional and quantifier logic as follows:

$$\forall Z \text{ (B is predicated in some way of } Z \supset$$
$$A \text{ is predicated in some way of } Z)$$

Both the antecedent and the consequent of the material implication consist of categorical propositions, that is, the propositions with which Aristotle is concerned in his syllogistic in *Prior Analytics* I.1–22. Consider, for instance, the prosleptic proposition expressed by the construction 'whatever B is said of all of, A is said of all of it'. Both the antecedent and the consequent contain the quantifying expression 'all'. This indicates assertoric universal affirmative propositions, that is, the propositions which occur in the syllogistic mood Barbara. We may call them a-propositions.

Next, consider the construction 'whatever B belongs to, A belongs to all of it'. The consequent contains the quantifying expression 'all', but the antecedent contains no quantifying expression. In *Prior Analytics* I.1, propositions which contain no quantifying expression are called indeterminate (ἀδιόριστος).[1] Aristotle distinguishes between affirmative and negative indeterminate propositions, for example, 'pleasure is good' and 'pleasure is not good'. However, in chapter I.41 he only takes into account affirmative indeterminate propositions. We may call them y-propositions, the letter 'y' being derived from the verb ὑπάρχειν ('belong'). Let us say that A is y-predicated of B if and only if the y-proposition 'A belongs to B' is true. Similarly, we may say that A is a-predicated of B if and only if the a-proposition 'A belongs to all B' is true.

Y-propositions are similar in meaning to i-propositions, that is, to particular affirmative propositions such as 'A belongs to some B'. For instance, according to the *Topics*, whatever is y-predicated of a species is also y-predicated of its genus, but not vice versa: 'quadruped' is y-predicated of 'animal' but not of 'man' (*Top.* II.4 111a20–32). Also, in *Prior Analytics* I.41, Aristotle takes the i-proposition 'beautiful belongs to some white' to imply the y-proposition 'beautiful belongs to the white' (*APr.* I.41 49b18–19). We will discuss the meaning of y-propositions in more detail later. For now, it suffices to note that they are similar in meaning to i-propositions.

The prosleptic propositions discussed in *Prior Analytics* I.41 consist of only two kinds of categorical propositions, namely, a- and y-propositions. Thus chapter I.41

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[1] *APr.* I.1 24a19–22. In *Int.* 7 17b7–12, such propositions are referred to as non-universal propositions about universals.

deals with exactly four kinds of prosleptic propositions, which differ in whether the antecedent and the consequent is an a- or y-proposition. If a-propositions are represented by formulae such as A*ab* and y-propositions by formulae such as Υab, the four kinds of prosleptic propositions are:

$\forall z(\mathrm{A}bz \supset \mathrm{A}az)$	abbreviated by **AA**
$\forall z(\Upsilon bz \supset \mathrm{A}az)$	abbreviated by Υ**A**
$\forall z(\mathrm{A}bz \supset \Upsilon az)$	abbreviated by **A**Υ
$\forall z(\Upsilon bz \supset \Upsilon az)$	abbreviated by $\Upsilon\Upsilon$

Aristotle's discussion of prosleptic propositions in I.41 can be divided into three parts (49b14–20, 49b20–27, 49b27–32). The first part states that prosleptic propositions of the type ΥA are not equivalent to those of the type AA:

> It is not the same thing either to say, or for it to be the case, that whatever B belongs to, A belongs to all of that, and to say that whatever B belongs to all of, A also belongs to all of.
>
> *APr.* I.41 49b14–16

Aristotle goes on to explain why they are not equivalent (49b16–20). He does so by assuming that 'beautiful' is i-predicated, and hence also y-predicated, of 'white', but not a-predicated of it. So let *b* be the term 'beautiful', and *a* some term which is a-predicated of 'beautiful' but not of 'white' (for instance, the term 'beautiful' itself). In this case, $\forall z(\mathrm{A}bz \supset \mathrm{A}az)$ is true and $\forall z(\Upsilon bz \supset \mathrm{A}az)$ is false.

The second part of the passage contains three statements about prosleptic propositions. The first of them states that y-propositions do not imply any of the four kinds of prosleptic propositions discussed in I.41 (cf. Kneale & Kneale 1972, 204; Ebert & Nortmann 2007, 849):

> If A belongs to B, but not to everything of which B is said, then whether B belongs to all C, or merely belongs to it, then not only is it not necessary for A to belong to all C, but also it is not even necessary for it to belong at all.
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> *APr.* I.41 49b20–22

As mentioned earlier, Aristotle appears to hold that i-propositions imply y-propositions (I.41 49b18–19). Also, according to the conversions stated in *Prior Analytics* I.2, a-propositions imply i-propositions. It is therefore reasonable to assume that a-propositions imply y-propositions: if A is a-predicated of B, then it is also y-predicated of B. In this case, AΥ is the weakest of the four kinds of prosleptic propositions, and each of the other three kinds implies it. So if y-propositions do not imply AΥ, then they do not imply any of the other three kinds of prosleptic propositions. Thus, Aristotle's statement just quoted can be summarized as follows:

(1) Υab does not imply $\forall z(\mathrm{A}bz \supset \Upsilon az)$

This statement can be readily verified by an example. Let *a* be 'quadruped', *b* 'animal', and *c* 'man'; *a* is y-predicated of *b*, *b* is a-predicated of *c*, but *a* is not y-predicated of *c*.

The second statement of the second part is:

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> But if whatever B is truly said of, A belongs to all of that, then it will follow that
> whatever B is said of all of, A will be said of all of that.
>
> *APr.* I.41 49b22–25

Aristotle uses here the verb 'be said of' as well as 'belong to'. He gives no indication that the two verbs differ from each other in any important way; they appear to be used synonymously in I.41. Thus, the passage just quoted states (cf. Kneale & Kneale 1972, 204):

(2) $\forall z(\Upsilon bz \supset Aaz)$ implies $\forall z(Abz \supset Aaz)$

Given that a-propositions imply y-propositions, this statement is obvious. The third and last statement of the second part is less obvious:

> However, if A is said of whatever B is said of all of, then nothing prevents B from
> belonging to C while A does not belong to all C, or even does not belong to C at all.
>
> *APr.* I.41 49b25–27

This is to say, if a is y-predicated of whatever b is a-predicated, then a need not be y-predicated of whatever b is y-predicated (cf. Kneale & Kneale 1972, 204):

(3) $\forall z(Abz \supset \Upsilon az)$ does not imply $\forall z(\Upsilon bz \supset \Upsilon az)$

This statement is not readily verified by examples; it is the puzzling statement which is the main subject of this paper. We shall return to it in a moment. But for now, let us have a look at the third and last part of Aristotle's discussion of prosleptic propositions:

> it is clear that 'A is said of all of which B' means this: A is said of all those things of
> which B is said.
>
> δῆλον ὅτι τὸ καθ᾿ οὗ τὸ Β παντὸς τὸ Α λέγεσθαι τοῦτ᾿ ἔστι, καθ᾿ ὅσων τὸ Β λέγεται,
> κατὰ πάντων λέγεσθαι καὶ τὸ Α.
>
> *APr.* I.41 49b28–30

Aristotle is explaining the meaning of the relative clause construction 'A is said of all of which B' (καθ᾿ οὗ τὸ Β παντὸς τὸ Α λέγεσθαι). Such an explanation is necessary because the meaning of the construction is not clear; in particular, the syntactic position and semantic function of the quantifying expression 'all' is not clear. In the succeeding chapters of the *Prior Analytics*, such relative clause constructions are frequently used to refer to a-propositions.[2] In *Prior Analytics* I.13, Aristotle states that the relative clause construction 'of whatever B, A is possible' is equivalent to the categorical universal affirmative possibility proposition 'A possibly belongs to all B'.[3] In the same way, the relative clause construction 'A is said of all of which B' may be regarded as equivalent to the categorical

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[2] For instance, *APr.* I.46 51b41, 52a6, 52b18–19, 52b24, II.2 53b20–21, 54a15, II.21 66b40, 67a9–12, II.22 67b29–31, II.27 70b35–36. In most of these passages, Aristotle uses 'belong to' instead of 'be said of'; for instance, 'A belongs to all to which B', etc. In other passages, the quantifying expression 'all' is omitted, for instance, *APr.* I.46 52b19–20, II.2 53b21–22, II.21 67b18–19, II.22 67b39–68a1, 68a6–7.

[3] *APr.* I.13 32b29–30. Alexander (*in APr.* 166.18–19) comments on this passage: 'the so-called prosleptic proposition means the same as the categorical proposition'.

a-proposition 'A belongs to all B'.[4] In this case, the passage just quoted provides an explanation of the meaning of a-propositions.

The explanans is the construction 'A is said of all those things of which B is said' (καθ' ὅσων τὸ B λέγεται, κατὰ πάντων λέγεσθαι καὶ τὸ A). Both the relative pronoun and the quantifying expression are in the plural (ὅσων and πάντων). Such plural forms do not occur in the other constructions used to express prosleptic propositions in I.41. Instead, these constructions contain a singular relative pronoun 'whatever', which indicates the universal quantifier of the prosleptic proposition. Some of them contain also one or two singular quantifying expressions, which indicate the quantity of the categorical proposition in the antecedent or consequent of the prosleptic proposition. However, the plural quantifying expression in Aristotle's explanans (namely, πάντων) does not indicate the quantity of the categorical proposition in the consequent. Instead, this quantifying expression, together with the plural relative pronoun (namely, ὅσων), indicates the universal quantifier of the prosleptic proposition. So the categorical proposition in the antecedent and that in the consequent do not contain a quantifying expression specifying their quantity. This means that both propositions are y-propositions. Thus, Aristotle takes a-propositions to be equivalent to prosleptic propositions of the type $\Upsilon\Upsilon$:[5]

(4) $\mathbf{A}ab$ is equivalent to $\forall z(\Upsilon bz \supset \Upsilon az)$

According to classical quantifier and propositional logic, the prosleptic proposition on the right determines a binary reflexive and transitive relation between a and b. So the equivalence in (4) implies that the relation of a-predication is reflexive and transitive, in other words, that it is a preorder.

Aristotle concludes his discussion of prosleptic propositions with a further remark on a-propositions (Kneale & Kneale 1972, 203):

> And [given that A is said of all of which B] if B is said of all of something, then is A also thus; but if B is not said of all of something, then A need not be said of it all.
>
> *APr.* I.41 49b30–32

The first sentence of this passage states:

(5) $\mathbf{A}ab$ implies $\forall z(\mathbf{A}bz \supset \mathbf{A}az)$

The second sentence states:[6]

(6) $\mathbf{A}ab$ does not imply $\forall z(\Upsilon bz \supset \mathbf{A}az)$

In fact, (5) is a logical consequence of (4). For (5) is equivalent to the condition that a-predication is transitive, a condition which follows from (4). Moreover, the

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[4] Alexander *in APr.* 378.31–32 (similarly 166.29–167.2), Kneale & Kneale (1972, 202–3).

[5] Kneale & Kneale (1972, 203). This equivalence is also attributed to Aristotle by Lejewski (1961, 167). However, Lejewski justifies it not by reference to chapter I.41 but to I.13 32b27–30.

[6] (6) follows from what Aristotle said earlier. According to 49b14–16, $\Upsilon\mathbf{A}$ is not equivalent to $\mathbf{A}\mathbf{A}$. According to (2), $\Upsilon\mathbf{A}$ implies $\mathbf{A}\mathbf{A}$. So $\mathbf{A}\mathbf{A}$ does not imply $\Upsilon\mathbf{A}$. But (4) implies that $\mathbf{A}\mathbf{A}$ is equivalent to $\mathbf{A}$; see (8). Hence, $\mathbf{A}$ does not imply $\Upsilon\mathbf{A}$ – which is (6).

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converse of the implication in (5), though not explicitly stated by Aristotle, is also a logical consequence of (4):

(7) $\forall z(\mathbf{A}bz \supset \mathbf{A}az)$ implies $\mathbf{A}ab$

For (7) is equivalent to the condition that a-predication is reflexive, a condition which follows from (4). Thus, (4) implies:[7]

(8) $\mathbf{A}ab$ is equivalent to $\forall z(\mathbf{A}bz \supset \mathbf{A}az)$

Let us now summarize Aristotle's discussion of prosleptic propositions in I.41. Aristotle has stated and denied a number of implications between a-propositions, y-propositions, and the four kinds of prosleptic propositions. He has provided enough information to determine for almost every ordered pair of these six kinds of propositions whether or not an implication holds. In the diagram below, the six solid straight arrows stand for implications which are held to be valid by Aristotle or which follow from such implications.[8] The three crossed-out curved arrows stand for implications denied by Aristotle.[9]
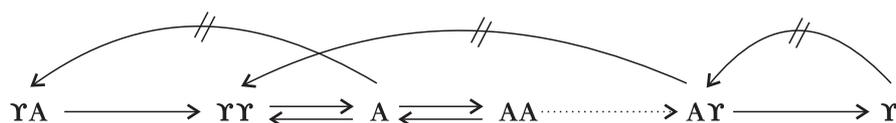
The dotted arrow indicates that $\mathbf{AA}$ implies $\mathbf{A\Upsilon}$. Aristotle does not explicitly state this implication. Given the equivalence in (4), the implication is equivalent to the condition that a-propositions imply y-propositions.[10] This condition is in turn equivalent to the condition that the relation of y-predication is reflexive.[11] So, given (4), the following three conditions are mutually equivalent:

> $\mathbf{AA}$ implies $\mathbf{A\Upsilon}$
> $\mathbf{A}$ implies $\mathbf{\Upsilon}$
> $\mathbf{\Upsilon}$ is reflexive

As noted earlier, there are strong reasons to believe that Aristotle accepts the second condition, according to which $\mathbf{A}$ implies $\mathbf{\Upsilon}$. Hence, there are also strong reasons to believe that $\mathbf{AA}$ implies $\mathbf{A\Upsilon}$, and that y-predication is reflexive.

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[7] This equivalence is attributed to Aristotle by Kneale & Kneale (1972, 204); cf. also Alexander *in APr.* 126.23–26, 375.22–23, Barnes (2007, 408–9).

[8] Two of them state that $\mathbf{\Upsilon\Upsilon}$ is equivalent to $\mathbf{A}$, which is Aristotle's statement in (4). Another two state that $\mathbf{A}$ is equivalent to $\mathbf{AA}$, which is (8) – a logical consequence of (4). Another arrow states that $\mathbf{\Upsilon A}$ implies $\mathbf{\Upsilon\Upsilon}$. This follows from Aristotle's statement in (2) and the equivalence of $\mathbf{\Upsilon\Upsilon}$ and $\mathbf{AA}$ (which is a logical consequence of (4)). Another arrow states that $\mathbf{A\Upsilon}$ implies $\mathbf{\Upsilon}$. This follows from the reflexivity of $\mathbf{A}$, which is a logical consequence of (4).

[9] These correspond to the statements in (1), (3), and (6), respectively.

[10] Consider the condition that $\mathbf{AA}$ implies $\mathbf{A\Upsilon}$. By virtue of (8) (which follows from (4)), this is equivalent to the condition that $\mathbf{A}$ implies $\mathbf{A\Upsilon}$, i.e., that for any a and b, $\mathbf{A}ab$ implies $\forall z(\mathbf{A}bz \supset \mathbf{\Upsilon}az)$. By virtue of the reflexivity and transitivity of $\mathbf{A}$ (which follows from (4)), this is equivalent to the condition that for any a and b, $\mathbf{A}ab$ implies $\mathbf{\Upsilon}ab$.

[11] Consider the condition that $\mathbf{A}ab$ implies $\mathbf{\Upsilon}ab$. Given (4), this is equivalent to the condition that $\forall z(\mathbf{\Upsilon}bz \supset \mathbf{\Upsilon}az)$ implies $\mathbf{\Upsilon}ab$. This is, according to classical logic, equivalent to the reflexivity of $\mathbf{\Upsilon}$.

A natural question at this point is whether Aristotle's statements in his discussion of prosleptic propositions are true. There is no simple answer to this question. If y-predication is reflexive, the implication from ΥA to $\Upsilon\Upsilon$ is valid. The other five implications whose validity is indicated by solid straight arrows in the diagram are logical consequences of (4), as explained in note 8. So given (4) and the reflexivity of y-predication, the validity of all implications held to be valid by Aristotle in his discussion of prosleptic propositions in I.41 is verified.

What about Aristotle's statements of invalidity, indicated by the three crossed-out curved arrows? The next section suggests a way to verify them, especially the puzzling statement in (3).

2. The condition of ecthesis

The puzzling statement is:

(3) $\forall z(\mathrm{A}bz \supset \Upsilon az)$ does not imply $\forall z(\Upsilon bz \supset \Upsilon az)$

The Kneales (1972, 203–4) argue that this is incorrect. Their argument for why it is incorrect is complex; we shall discuss it later. In any case, given the transitivity of a-predication, (3) is incorrect if we accept that y-propositions Υab are equivalent to the following condition:[12]

$\exists z(\mathrm{A}bz \wedge \mathrm{A}az)$

This may be called the condition of ecthesis, because, according to a certain interpretation of Aristotle's proofs by ecthesis, it plays an important role in them.

The syllogism Barbara, a cornerstone of Aristotle's syllogistic, implies that a-predication is transitive. So (3) requires us to deny that y-propositions are equivalent to the condition of ecthesis. Aristotle does not say that they are equivalent, neither in chapter I.41 nor elsewhere in the *Prior Analytics*. Nevertheless, it would not be unreasonable to accept that equivalence. For instance, one might argue that the y-propositions discussed in I.41 are equivalent to those discussed in the syllogistic in I.1–22, that the latter are equivalent to i-propositions, and that these are equivalent to the condition of ecthesis:

              indeterminate affirmative propositions in *APr.* I.41: Υab

$\Leftrightarrow$ indeterminate affirmative propositions in *APr.* I.1–22

$\Leftrightarrow$ particular affirmative propositions (i-propositions)

$\Leftrightarrow$ condition of ecthesis: $\exists z(\mathrm{A}bz \wedge \mathrm{A}az)$

Aristotle's statement in (3) requires us to deny at least one of these three equivalences. Let us consider each of them in turn.

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[12] Assume $\forall z(\mathrm{A}bz \supset \Upsilon az)$ and $\neg\forall z(\Upsilon bz \supset \Upsilon az)$. Due to the equivalence in question, the latter formula implies $\mathrm{A}zs \wedge \mathrm{A}bs$ and $\neg\exists v(\mathrm{A}zv \wedge \mathrm{A}av)$. Now, $\forall z(\mathrm{A}bz \supset \Upsilon az)$ and $\mathrm{A}bs$ imply Υas. Due to the equivalence in question, Υas implies $\mathrm{A}sv \wedge \mathrm{A}av$. Since a-predication is transitive, $\mathrm{A}zs$ and $\mathrm{A}sv$ imply $\mathrm{A}zv$. So we have $\mathrm{A}zv \wedge \mathrm{A}av$, which contradicts $\neg\exists v(\mathrm{A}zv \wedge \mathrm{A}av)$.

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If we deny the first equivalence, the indeterminate affirmative propositions discussed in I.41 are not equivalent to those discussed in I.1–22. The latter may be equivalent to i-propositions and to the condition of ecthesis, the former not. In what follows, we shall use the symbol Υ and the terms 'y-proposition' and 'y-predicated' exclusively for the indeterminate affirmative propositions in I.41, not for those in I.1–22.

The term 'indeterminate' (ἀδιόριστος), frequently used in I.1–22, does not occur in I.41. In chapter I.1, indeterminate propositions are defined as propositions which do not contain quantifying expressions such as 'all', 'some', 'no', etc.[13] The y-propositions discussed in chapter I.41 meet this criterion, and Aristotle gives no indication that they are of a different kind than those discussed in I.1–22. It is therefore natural to assume that they are equivalent to those discussed in I.1–22.

If we accept the first equivalence, we may deny the second equivalence. In this case, the indeterminate affirmative propositions discussed in I.1–22 are not equivalent to i-propositions.[14] It is often assumed that indeterminate affirmative propositions are equivalent to i-propositions.[15] But there is no clear evidence for this in *Prior Analytics* I.1–22. When Aristotle discusses indeterminate propositions in I.1–22, he usually focusses on inconcludence of premiss pairs rather than on validity of syllogistic moods. He often states that a given inconcludent premiss pair remains inconcludent when the particular premiss(es) is (or are) replaced by the corresponding indeterminate proposition(s) of the same quality and modality.[16]

There are only two statements of validity involving indeterminate propositions in *Prior Analytics* I.1–22. Both of them deal with assertoric (that is, non-modalized) valid moods one of whose premisses is an i-proposition. Aristotle states that when that i-proposition is replaced by the corresponding indeterminate affirmative proposition, there will be the same *syllogismos* (I.4 26a28–30, I.7 29a27–29). This can be understood in two ways. Either the conclusion of the resulting valid mood remains an i- or o-proposition, or it is replaced by an affirmative or negative indeterminate proposition as well as the particular premiss. Alexander prefers the latter option, others the former.[17] In either case, everything Aristotle says about

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[13] *APr.* I.1 24a19–22. The term 'indeterminate' is also used in a wider sense in the *Prior Analytics*. In this sense, particular propositions (that is, i- and o-propositions) are also indeterminate because they may be true regardless of whether the corresponding universal proposition (that is, a- or e-proposition) of the same quality (and modality) is true or false (*APr.* I.4 26b14–16, I.5 27b20–22, 27b28, I.6 28b28–30, 29a6, I.15 35b11); cf. Alexander *in APr.* 66.2–18, 67.3–7, 88.6–8, 88.31–33, 105.22–26, Maier (1896, 162–3), Crivelli (2004, 245 note 21). This paper is exclusively concerned with the narrow sense of 'indeterminate', defined by the lack of quantifying expressions.

[14] Authors who deny this equivalence include Mulhern (1974, 145–6), Whitaker (1996, 86), Hafemann (1999, 114–16).

[15] Alexander *in APr.* 30.29–31, 49.15, 62.24, 111.30–112.2, 267.2, Alexander *in Top.* 288.27–289.4, Philoponus *in APr.* 79.4–5, 252.35, Philoponus *in APost.* 296.10–11, Waitz (1844, 369), Kneale & Kneale (1962, 55; 1972, 203), Ackrill (1963, 129), Owen (1965, 86–87), Thom (1981, 19), Barnes (2007, 141); cf. also Crivelli (2004, 244 note 19).

[16] *APr.* I.4 26a32, 26a39, 26b23–24, I.5 27b38, I.6 29a8, I.14 33a37, I.15 35b15, I.16 36b12, I.17 37b14, I.18 38a10–11, I.19 38b36, I.20 39b2, I.21 40a1.

[17] Alexander *in APr.* 51.24–30. For the former option, see Whitaker (1996, 86), Drechsler (2005, 374–6 and 543–5).

indeterminate propositions in I.1–22 can be explained by assuming that these are equivalent to the corresponding particular propositions of the same quality and modality. Perhaps this is even the best explanation of Aristotle's statements about indeterminate propositions in I.1–22. Nevertheless, his statements do not entail that equivalence. For instance, they can also be explained by the weaker assumption that every valid (or invalid) mood and conversion remains valid (or invalid) when every particular proposition in it is replaced by the corresponding indeterminate proposition of the same quality and modality.

There is no conclusive evidence in the *Prior Analytics* that indeterminate propositions are equivalent to particular propositions. But there may be some evidence for it in the *Topics*. In *Topics* III.6, Aristotle mentions the two indeterminate propositions 'pleasure is good' and 'pleasure is not good'.[18] He goes on to explain how these propositions can be established and rejected. In doing so, however, he uses the examples 'some pleasure is good' and 'some pleasure is not good' instead of the original ones.[19] He gives no indication of any difference between these two particular propositions on the one hand and the original indeterminate ones on the other. As Alexander points out, this suggests that Aristotle regards them as equivalent (Alexander *in Top.* 288.27–289.4).

If we accept the first two equivalences, we may deny the third equivalence. In this case, i-propositions are not equivalent to the condition of ecthesis. Aristotle does not say that they are equivalent. Nevertheless, it is not unreasonable to accept this equivalence. It is often thought that Aristotle's proofs by ecthesis are based on the equivalence.[20] Moreover, the equivalence is valid in most semantic interpretations of the assertoric syllogistic.

Consider, for instance, what may be called the standard non-empty set semantics of the assertoric syllogistic. In it, the semantic value of argument terms of categorical propositions is a non-empty subset of a given primitive non-empty set of individuals. Thus, the domain of semantic values of terms is the powerset of the primitive set of individuals with the empty set removed. An a-proposition (or i-proposition) is true if and only if the semantic value of the subject term is a subset of (or has an individual in common with) the semantic value of the predicate term. Now, the condition of ecthesis contains an existential quantifica-

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[18] *Top.* III.6 120a7–8. The latter proposition is also used in *APr.* I.1 24a21–22. Some commentators read εἴ τινα ἔφησεν in 120a7; Brunschwig (1967, 77), Crivelli (2004, 245 note 21). In this case, the passage would concern o- and i-propositions, not indeterminate propositions in the narrow sense defined by the lack of quantifying expressions. However, there is little evidence for that reading in the manuscripts (Brunschwig 1967, 77 and 163).

[19] *Top.* III.6 120a8–20. These two examples are indeterminate in the wide sense described in note 13 above. The term 'indeterminate' (ἀδιορίστου) in 120a6 is therefore probably used in the wide sense. This is confirmed by the phrase διωρισμένης δὲ in 120a20–21, which corresponds to the phrase ἀδιορίστου μὲν in 120a6. For that phrase appears to refer to propositions which are not indeterminate in the wide sense. Nevertheless, in 120a7–8 Aristotle mentions propositions which are indeterminate in the narrow (and hence also wide) sense.

[20] Łukasiewicz (1957, 61–4), Patzig (1968, 161–4), Rescher & Parks (1971, 685), Rescher (1974, 11), Smith (1983, 226; 1989, xxiii), Detel (1993, 164), Lagerlund (2000, 8). Some of these authors mention only the implication from i-propositions to the condition of ecthesis. This is the substantive part of the equivalence, as the converse follows by means of Darapti.

tion, applied to a variable of the syntactic type of argument terms of categorical propositions. A common way to interpret quantifications is what is known as the objectual interpretation. According to it, an existential quantification $\exists z$ requires the formula to which it is applied to be true for some assignment of a semantic value to the variable z. Given this objectual interpretation, i-propositions are equivalent to the condition of ecthesis in the non-empty set semantics (cf. Smith 1983, 228).

In other kinds of set-theoretic semantics, empty sets are admitted as semantic values of terms. For instance, the domain of semantic values of terms can be taken to be the powerset of a given primitive non-empty set of individuals, including the empty set. An a-proposition is true if and only if the semantic value of the subject term is (1) not the empty set, and (2) a subset of the semantic value of the predicate term.[21] The truth conditions of i-propositions are the same as in the non-empty set semantics. Given the objectual interpretation of quantification, i-propositions are equivalent to the condition of ecthesis in this kind of set-theoretic semantics. The equivalence is also valid in some non-set-theoretic semantics of the assertoric syllogistic (for instance, Martin 1997, 5; Malink 2006, 115–16).

To sum up, each of the three equivalences is plausible, and for each of them there are reasons to accept it. Nevertheless, Aristotle's statement in (3) requires us to deny at least one of them. It is not my intention here to decide which of them should be denied and which not. Instead, I want to explore the consequences of denying each of them individually. This is the subject of the next section. In the remainder of the present section, I want to suggest a semantics which is in accordance with the statement in (3).

The statement implies that y-propositions (that is, the indeterminate affirmative propositions discussed in *Prior Analytics* I.41) are not equivalent to the condition of ecthesis. This suggests that y-predication cannot be defined in terms of a-predication; for the most natural definiens in terms of a-predication would be the condition of ecthesis. On the other hand, a-predication can be defined in terms of y-predication; for as noted in (4) above, Aristotle regards a-propositions as equivalent to prosleptic propositions of the type ϒϒ. Thus we may say that y-predication is more primitive than a-predication, the latter being definable in terms of the former, but not vice versa. Following this idea, we may regard y-predication as a primitive relation not defined in terms of another relation. This allows us to verify Aristotle's puzzling statement in (3) by means of suitable models. Consider, for instance, a model which consists of four items:

$$\circ\!\cdots\!\cdots\!\circ\!\cdots\!\cdots\!\circ\!\cdots\!\cdots\!\circ$$
$$a \qquad b \qquad c \qquad d$$
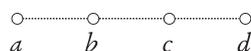
The primitive relation of y-predication holds exactly between those items which are connected by a line not interrupted by another item, with every item being understood to be connected to itself. For instance, y-predication holds between a and a, a and b, b and a, but not between a and c, etc. Thus, y-predication is a

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[21] Prior (1962, 169), Wedin (1990, 135), Bäck (2000, 241–3), Ebert & Nortmann (2007, 333).

reflexive and symmetric relation in the model; it is the reflexive and symmetric closure of $\{ab, bc, cd\}$ on the domain $\{a, b, c, d\}$.

A-predication is defined in terms of y-predication by means of the equivalence in (4). There are exactly two items of which b is a-predicated, namely, b and a. Since a is y-predicated of both of them, the formula $\forall z(Abz \supset \Upsilon az)$ is true in the model. On the other hand, b is y-predicated of c, but a is not y-predicated of c; so $\forall z(\Upsilon bz \supset \Upsilon az)$ is false. Thus Aristotle's statement in (3) is verified. Y-predication is not equivalent to the condition of ecthesis in the model; for b is y-predicated of c, but the condition of ecthesis does not hold between these two items.

The above model can also be used to verify the other two statements of invalidity in Aristotle's discussion of prosleptic propositions in I.41, namely, the statements in (1) and (6). The former states that Υ does not imply $A\Upsilon$; in the above model, c is y-predicated of b, while b is a-predicated of a but c is not y-predicated of a. The latter states that A does not imply ΥA; in the above model, a is a-predicated of a, while a is y-predicated but not a-predicated of b.

In addition, the above model satisfies the equivalence in (4) and the reflexivity of y-predication. As mentioned earlier, all implications held to be valid by Aristotle in his discussion of prosleptic propositions in I.41 follow from (4) and the reflexivity of y-predication. Thus, all of Aristotle's statements of validity and invalidity in his discussion of prosleptic propositions in I.41 can be verified if y-predication is taken as a primitive relation.

However, we should like to verify not only what Aristotle says about indeterminate propositions in I.41, but also what he says about them in the syllogistic in *Prior Analytics* I.1–22. I cannot discuss the modal syllogistic (I.3 and 8–22) here, and consider only the assertoric syllogistic (I.1–2, 4–7). The next section suggests a semantics for the assertoric syllogistic which is in accordance with what Aristotle says about indeterminate propositions in the assertoric syllogistic and in I.41. More precisely, we will consider three such semantics, depending on which of the three equivalences mentioned above is denied. Each of the three semantics will be based on the primitive relation of y-predication.

3. Syllogistic based on y-predication

The assertoric syllogistic deals with six kinds of propositions: universal, particular, and indeterminate propositions, each of them affirmative and negative. The purpose of this section is to show that all of them can be interpreted (that is, that their truth conditions can be defined) in terms of y-predication. The interpretation of a-propositions is determined by the equivalence in (4). The interpretation of i-propositions and indeterminate propositions depends on which of the three equivalences discussed in the previous section is denied. Given an interpretation of i- and a-propositions, that of e- and o-propositions is determined by the assertoric square of opposition; for according to it, a-propositions are contradictory to o-propositions, and i- to e-propositions (*APr.* II.8 59b8–11, II.15 63b23–30).

As to the interpretation of i-propositions, let us first assume that the first equivalence is denied while the other two are accepted. In this case, the indeterminate

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affirmative propositions discussed in I.41 (that is, y-propositions) are not equiv-
alent to those discussed in I.1–22. At the same time, the latter are equivalent
to i-propositions and to the condition of ecthesis. Thus, it is natural to assume
that the indeterminate negative propositions discussed in I.1–22 are equivalent
to o-propositions. This leads to the following interpretation of the six kinds of
assertoric propositions in I.1–22:

a-propositions $\mathbf{A}ab$:	$\forall z(\Upsilon bz \supset \Upsilon az)$
i-propositions $\mathbf{I}ab$ and indeterminate affirmative propositions:	$\exists z(\mathbf{A}bz \wedge \mathbf{A}az)$
e-propositions $\mathbf{E}ab$:	$\neg \exists z(\mathbf{A}bz \wedge \mathbf{A}az)$
o-propositions $\mathbf{O}ab$ and indeterminate negative propositions:	$\neg \mathbf{A}ab$

A-propositions are interpreted in terms of y-predication. The other kinds of
propositions are interpreted in terms of a-predication, and hence also in terms of
y-predication. There are no axioms governing the primitive relation of y-predi-
cation.

Consider the class of standard first-order models for a first-order language
whose only predicate symbol is Υ. Each of these models can also be regarded
as a model for the language of categorical propositions: any of the six kinds of
categorical propositions is true in such a model if and only if the formula assigned
to it by the above interpretation is true in the model. Let us call this class of models
the y1-semantics of the assertoric syllogistic. A syllogistic mood or conversion is
valid in the y1-semantics if and only if its conclusion is true in every model of the
y1-semantics in which the premiss(es) is (or are) true.

Is the y1-semantics adequate for Aristotle's assertoric syllogistic? In other
words, are all assertoric moods and conversions held to be valid (or invalid)
by Aristotle valid (or invalid) in the y1-semantics? The answer is affirmative. To
see this, we only need to consider universal and particular propositions. For all of
what Aristotle says about indeterminate propositions in I.1–22 can be explained
by assuming that these are equivalent to the corresponding particular proposi-
tions.

As to validity, we only need to consider four inferences: the moods Barbara
and Celarent, and the conversions of i- and a-propositions. Given the assertoric
square of opposition, these four inferences imply all other purely universal and
particular inferences held to be valid by Aristotle (Smiley 1973, 141–2). As noted
above, the interpretation of a-propositions in terms of y-propositions implies
that a-predication is reflexive and transitive. Transitivity of a-predication implies
the validity in the y1-semantics of Barbara and Celarent. The conversion of
i-propositions is valid in the y1-semantics because the condition of ecthesis is
symmetric. The conversion of a-propositions is valid in the y1-semantics by virtue
of the reflexivity of a-predication: assume that a is a-predicated of b. Owing to
the reflexivity of a-predication, there is something, namely b, of which both a and
b are a-predicated. Hence, b is i-predicated of a in the y1-semantics.

At the same time, every mood and conversion held to be invalid by Aristotle in the assertoric syllogistic is invalid in the y1-semantics. This can be proved by suitable models, which are given at the end of the paper. These models can be constructed in such a way that y-predication is reflexive and symmetric. Thus, y-predication can be assumed to be reflexive in the y1-semantics. In this case, all implications held to be valid by Aristotle in his discussion of prosleptic propositions in I.41 are valid in the y1-semantics. As a result, the y1-semantics is in accordance with what Aristotle says about indeterminate propositions in the assertoric syllogistic and in I.41.

In the set-theoretic semantics, the conversion of a-propositions implies that empty terms (that is, terms whose semantic value is the empty set) cannot serve as the subject of true a-propositions. This is known as the problem of existential import. On the other hand, there is no such problem in the y1-semantics. In fact, the notion of an empty term does not play a role in the y1-semantics; for the semantic value of terms is not taken to be a set of individuals. Rather, the semantic value of terms is a primitive zero-order individual, or at least it is considered as such. The y1-semantics does not specify what kind of item that semantic value is. Thus, the distinction between a term and its semantic value is not as important in the y1-semantics as in the set-theoretic semantics.

In the y1-semantics, we deny the first of the three equivalences discussed in the previous section. We can also consider y2- and y3-semantics, in which we deny the second and third equivalence, respectively. Since the y2-semantics is more complicated than the y3-semantics, let us start with the latter. So assume that the third equivalence is denied while the other two are accepted. In this case, i-propositions are not equivalent to the condition of ecthesis, but the indeterminate affirmative propositions discussed in I.41 (that is, y-propositions) are equivalent to those discussed in I.1–22 and to i-propositions. Thus, it is natural to assume that the indeterminate negative propositions discussed in I.1–22 are equivalent to o-propositions.

I-propositions are convertible: if A is i-predicated of B, then B is i-predicated of A. Since y-propositions are equivalent to i-propositions in the y3-semantics, y-propositions should also be convertible. In other words, the primitive relation of y-predication should be symmetric in the y3-semantics. Moreover, since a-propositions imply i-propositions, they should also imply y-propositions. In other words, a-predication should imply y-predication in the y3-semantics. As noted above, this implication is equivalent to the condition that y-predication is reflexive.[22] Thus, y-predication should be reflexive in the y3-semantics.

Unlike the y1-semantics, the y3-semantics requires the primitive relation of y-predication to have certain logical properties, namely, symmetry and reflexivity. These properties can be guaranteed by axioms governing the primitive relation of y-predication. So the y3-semantics is constituted by two axioms and by the following interpretation of categorical propositions:

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[22] Given the equivalence in (4), cf. note 11 above.

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two axioms:	Υaa and $\Upsilon ab \leftrightarrow \Upsilon ba$
a-propositions $\mathbf{A}ab$:	$\forall z(\Upsilon bz \supset \Upsilon az)$
i-propositions $\mathbf{I}ab$ and indeterminate affirmative propositions:	Υab
e-propositions $\mathbf{E}ab$:	$\neg\Upsilon ab$
o-propositions $\mathbf{O}ab$ and indeterminate negative propositions:	$\neg\mathbf{A}ab$

The y3-semantics is adequate for Aristotle's assertoric syllogistic. This can be seen as follows. Barbara is valid in the y3-semantics for the same reason as in the y1-semantics. Celarent and the conversion of i-propositions are valid in the y3-semantics by virtue of the symmetry of y-predication. The conversion of a-propositions is valid in the y3-semantics by virtue of the symmetry and reflexivity of y-predication. The models showing that every mood and conversion held to be invalid by Aristotle is invalid in the y3-semantics are given at the end of the paper.

The y3-semantics is also in accordance with the statements of invalidity in Aristotle's discussion of prosleptic propositions in I.41, including the puzzling statement in (3). For the model given above to verify these statements is admissible in the y3-semantics, as y-predication is reflexive and symmetric in it. Thus, the y3-semantics is in accordance with Aristotle's statements about indeterminate propositions in the assertoric syllogistic and in I.41.

Let us now consider the y2-semantics. In it, the second equivalence is denied, and the other two equivalences are accepted. This is to say, the indeterminate affirmative propositions discussed in I.1–22 are not equivalent to i-propositions, but equivalent to the indeterminate affirmative propositions discussed in I.41 (that is, to y-propositions); and i-propositions are equivalent to the condition of ecthesis. Consequently, what Aristotle says about indeterminate propositions in the assertoric syllogistic cannot be explained by assuming that these are equivalent to the corresponding particular propositions of the same quality. Instead, it can be explained by assuming that every valid (or invalid) mood and conversion remains valid (or invalid) when every particular proposition in it is replaced by the corresponding indeterminate proposition of the same quality. We want the y2-semantics to satisfy this assumption. The primitive relation of y-predication should therefore be reflexive and symmetric in the y2-semantics for the same reasons as in the y3-semantics.

Given that y-predication is reflexive and symmetric, and given the usual definition of a-predication in terms of y-predication, y-predication follows from the condition of ecthesis: Υab follows from $\exists z(\mathbf{A}bz \wedge \mathbf{A}az)$.[23] This means that y-propositions follow from i-propositions in the y2-semantics. This is in accordance with a passage from I.41 mentioned above (49b18–19) where Aristotle

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[23] Assume $\mathbf{A}bz \wedge \mathbf{A}az$. Since y-predication is reflexive, $\mathbf{A}bz$ implies Υbz (cf. note 11 above). Since y-predication is symmetric, Υbz implies Υzb. Given the definition of a-predication in terms of y-predication, $\mathbf{A}az$ implies $\forall y(\Upsilon zy \supset \Upsilon ay)$. This and Υzb imply Υab.

appears to infer an y-proposition from the corresponding i-proposition. On the other hand, i-propositions do not follow from y-propositions in the y2-semantics (as shown by the model given in the previous section[24]). So y-propositions follow from i-propositions in the y2-semantics, but not vice versa.

Accordingly, we may also want indeterminate negative propositions to follow from o-propositions in the y2-semantics, but not vice versa. To this end, indeterminate negative propositions can be interpreted as follows (with a being the predicate term and b the subject term):

(9) $\exists z(\Upsilon bz \wedge \neg\exists u(Azu \wedge Aau))$

Thus, indeterminate negative propositions are true in the y2-semantics if and only if the subject term is y-predicated of something of which the predicate term is e-predicated. Similarly, y-propositions are true in the y2-semantics if and only if the subject term is y-predicated of something of which the predicate term is a-predicated; for given the symmetry of y-predication and the usual definition of a-predication in terms of y-predication, Υab is equivalent to $\exists z(\Upsilon bz \wedge Aaz)$.[25] In view of this equivalence, (9) seems to be a natural interpretation of indeterminate negative propositions. Given this interpretation, indeterminate negative propositions follow from o-propositions in the y2-semantics,[26] but not vice versa.[27]

The y2-semantics is constituted by two axioms and by the following interpretation of categorical propositions:

two axioms:	Υaa and $\Upsilon ab \leftrightarrow \Upsilon ba$
a-propositions Aab:	$\forall z(\Upsilon bz \supset \Upsilon az)$
i-propositions Iab:	$\exists z(Abz \wedge Aaz)$
indeterminate affirmative propositions:	Υab
e-propositions Eab:	$\neg\exists z(Abz \wedge Aaz)$
o-propositions Oab:	$\neg Aab$
indeterminate negative propositions:	$\exists z(\Upsilon bz \wedge \neg\exists u(Azu \wedge Aau))$

Is the y2-semantics adequate for the assertoric syllogistic? As far as purely universal and particular moods and conversions are concerned, the y2-semantics is identical with the y1-semantics, and hence adequate. What about indeterminate propositions? It suffices to prove that every valid (or invalid) mood and

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[24] In this model, Υbc is true and $\exists z(Acz \wedge Abz)$ is false.

[25] First, assume Υab. Since y-predication is symmetric, Υab implies Υba. Given the definition of a-predication, a-predication is reflexive. So we have $\Upsilon ba \wedge Aaa$, and hence $\exists z(\Upsilon bz \wedge Aaz)$. Second, assume $\Upsilon bz \wedge Aaz$. According to the definition of a-predication, Aaz implies $\forall u(\Upsilon zu \supset \Upsilon au)$. This and Υbz imply Υab, since y-predication is symmetric.

[26] The interpretation of o-propositions in the y2-semantics is $\neg Aab$. Due to the definition of a-predication, this implies $\Upsilon bz \wedge \neg\Upsilon az$. $\neg\Upsilon az$ implies $\neg\exists u(Azu \wedge Aau)$ (cf. note 23 above). So we have (9).

[27] Consider the model given in the previous section: there are exactly two items of which b is a-predicated, namely, b and a. Similarly, c is only a-predicated of c and of d. So $\neg\exists u(Acu \wedge Abu)$ is true. Since b is y-predicated of c, $\exists z(\Upsilon bz \wedge \neg\exists u(Azu \wedge Abu))$ is also true. But b is not o-predicated of b.

conversion remains valid (or invalid) when every particular proposition in it is replaced by the corresponding indeterminate proposition of the same quality. For, as noted above, this suffices to explain what Aristotle says about indeterminate propositions in the assertoric syllogistic.

The y2-semantics can be shown to satisfy that condition. As to invalidity, the appropriate models of the y2-semantics are given at the end of the paper. As to validity, it suffices to consider six inferences: the conversion of a-propositions, the conversion of i-propositions, and the four moods Darii, Ferio, Baroco, and Bocardo. These six inferences imply the validity of all other moods which contain particular propositions and are held to be valid by Aristotle in the assertoric syllogistic. As far as the two conversions and Darii are concerned, the truth in the y2-semantics of the condition has already been proved by the y3-semantics. As a matter of fact, the condition is also true in the y2-semantics for Ferio, Baroco, and Bocardo.[28]

It is worth pointing out that Baroco and Bocardo remain valid in the y2-semantics when o-propositions are replaced by indeterminate negative propositions. Now, Aristotle's indirect proofs for Baroco and Bocardo are based on the assumption that o-propositions are contradictory to a-propositions. But indeterminate negative propositions are not contradictory to a-propositions in the y2-semantics. Thus, the y2-semantics shows that a semantics can be adequate for the assertoric syllogistic although o-propositions are not contradictory to a-propositions. In such a semantics, Baroco and Bocardo are valid although Aristotle's indirect proofs for them are not sound.

To sum up, all three y-semantics are in accordance with what Aristotle says about indeterminate propositions in the assertoric syllogistic and in I.41.

4. Y-predication vs. the condition of ecthesis

Y-predication, I have argued, can be viewed as a primitive, reflexive and symmetric relation.[29] As a result, y-predication is strictly weaker than the condition of ecthesis: y-predication follows from the condition of ecthesis, but not vice versa.[30]

In this section, we shall focus on two questions. Firstly, is there an intuitive explanation of why y-predication is not equivalent to the condition of ecthesis? Secondly, what does y-predication mean if it is not equivalent to the condition of ecthesis? I want to approach these questions by considering modern systems of what is sometimes called mereotopology. I have in mind systems which are based

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[28] Ferio: since y-predication is symmetric, $\neg\exists z(\mathbf{A}bz \wedge \mathbf{A}az)$ and Υbc imply $\exists z(\Upsilon cz \wedge \neg\exists u(\mathbf{A}zu \wedge \mathbf{A}au))$. Baroco: since a-predication is transitive, $\mathbf{A}ba$ and $\exists z(\Upsilon cz \wedge \neg\exists u(\mathbf{A}zu \wedge \mathbf{A}bu))$ imply $\exists z(\Upsilon cz \wedge \neg\exists u(\mathbf{A}zu \wedge \mathbf{A}au))$. Bocardo: $\exists z(\Upsilon bz \wedge \neg\exists u(\mathbf{A}zu \wedge \mathbf{A}au))$ and $\forall z(\Upsilon bz \supset \Upsilon cz)$ imply $\exists z(\Upsilon cz \wedge \neg\exists u(\mathbf{A}zu \wedge \mathbf{A}au))$.

[29] In the y2- and y3-semantics y-predication is required to be reflexive and symmetric; in the y1-semantics it may, but need not, be reflexive and symmetric.

[30] Cf. note 23 and note 24 above.

on a primitive, reflexive and symmetric relation of topological connection: *a* is connected with *b*. According to a tradition which goes back to de Laguna (1922, 452) and Whitehead (1929, 418), this primitive relation can be used to define a mereological part-whole relation as follows: *b* is a part of *a* if and only if *a* is connected with everything with which *b* is connected.[31]

This is similar to the definition of a-predication in terms of y-predication by means of Aristotle's equivalence in (4): *a* is a-predicated of *b* if and only if *a* is y-predicated of everything of which *b* is y-predicated. If a-predication is regarded as a part-whole relation, the condition of ecthesis amounts to mereological overlap, that is, to the relation of sharing a common part. Y-predication, on the other hand, may be regarded as a topological relation of connection.

In many mereotopological systems, the domain of objects under consideration consists of extended magnitudes. Two such magnitudes overlap if and only if they have a common extended magnitude as their part. Topological connection, on the other hand, indicates a weaker kind of contact. It may be understood in such a way that the two magnitudes have certain items in common, for instance, common boundaries or common points. Crucially, however, these items are not extended magnitudes and therefore do not belong to the domain of objects under consideration; at least they are not extended in the same number of dimensions as the members of that domain. Two magnitudes may be connected by sharing common boundaries or points without having a common extended magnitude as their part.

The difference between y-predication and the condition of ecthesis can be explained in a similar way. The three y-semantics do not specify what kind of items the semantic values of terms are. For instance, we may assume that they are beings (ὄντα) such as described in *Prior Analytics* I.27. According to I.27 43a25–43, there are three kinds of beings. First, there are beings such as Kallias which are not predicated of anything else, but of which other beings are predicated. Such beings may be called individuals. Second, there are beings of which nothing else is predicated, but which are themselves predicated of other beings. Finally, there are beings such as man of which other beings are predicated, and which are themselves predicated of other beings. For instance, man is predicated of Kallias, and animal of man. Beings of the second and third group may be called universals.

We may consider models of the three y-semantics in which the domain of semantic values of terms is the class of beings described in I.27. But we may also consider models in which that domain is only a subclass of the whole class of beings. For instance, we may consider models whose domain includes only universals and no individuals. Or we may consider models whose domain includes only those beings for which the language under consideration has a term to name

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[31] For instance, Clarke (1981, 206; 1985, 62), Biacino & Gerla (1991, 242–3), Gotts et al. (1996, 59), Varzi (1997, 31–32), Cohn & Varzi (2003, 361).

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it.[32] In this case, we may also assume that every term is its own semantic value.[33] The condition of ecthesis states that there is a member of the domain of semantic values of terms of which both argument terms are a-predicated. More precisely, it states that the open formula $\mathrm{A}bz \wedge \mathrm{A}az$ is true for some assignment of a semantic value to the variable z.

Y-predication, on the other hand, can be regarded as a weaker kind of particular affirmative predication. For instance, y-predication may be taken to hold between two terms if and only if there is an individual of which both terms are truly said, regardless of whether or not that individual is a member of the domain of semantic values of terms. In this case, the requirement that y-predication be reflexive amounts to the requirement that every term be truly said of some individual. Now, assume that Kallias is a man who is walking. In this case, both 'man' and 'walking' are truly said of Kallias, so that 'walking' is y-predicated of 'man'. If Kallias is the semantic value of the variable z, then both 'walking' and 'man' are a-predicated of z; for there is only one individual of which z is truly said, namely, Kallias, so that both 'walking' and 'man' are y-predicated of everything of which z is y-predicated. If the individual Kallias is a member of the domain of semantic values of terms, the condition of ecthesis holds between 'walking' and 'man'. However, if Kallias is not a member of that domain, then the condition of ecthesis need not hold between 'walking' and 'man'. It does not hold if the domain includes no item x such that 'walking' and 'man' are a-predicated of the variable z when x is the semantic value of z.

This is one way to explain why y-predication is not equivalent to the condition of ecthesis. Let me suggest another one. Y-predication may be taken to obtain between two terms if and only if it is possible for there to be an individual of which both terms are truly said. In this case, y-predication is an implicitly modal notion, and may be understood as a relation of compatibility between terms. The requirement that y-predication be reflexive amounts to the requirement that it be possible for any term to be truly said of some individual. In other words, self-incompatible terms such as 'round square' are ruled out. If y-predication is a modal notion, a-predication is also a modal notion. A-predication can then be viewed as a relation of necessary entailment; for we may assume that b necessarily entails a if and only if a is compatible with everything with which b is compatible.[34]

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[32] Aristotle holds that the number of names (ὀνόματα) and phrases (λόγοι) is finite while things (πράγματα) are infinite in number (*Soph. el.* 1 165a10–12). If homonymy is excluded, two things cannot have the same name. As a result, there are things which do not have a name.

[33] For an example of such a semantics, cf. Smith (1982, 123–4).

[34] Similarly, Brandom (2008, 121–4) defines a notion of necessary entailment in terms of incompatibility: p entails q if and only if everything incompatible with q is incompatible with p. In other words, if and only if everything compatible with p is compatible with q. Brandom takes incompatibility, and hence also compatibility, to be symmetric. However, he allows self-incompatibility, so that compatibility need not be reflexive. He regards incompatibility as a semantically primitive, implicitly modal relation which obtains between 'Aristotelian contraries' (2008, 125–6). For instance, being made of pure copper is incompatible with being an electrical insulator because 'it is impossible for one and the same object simultaneously to have both properties.' (2008, 191)

Now, 'walking' is compatible with 'man' and is therefore y-predicated of it. Since Kallias is essentially a man, 'Kallias' necessarily entails 'man'. So 'man' is a-predicated of 'Kallias'. On the other hand, 'walking' is not a-predicated of 'Kallias', given that 'sitting' is compatible with 'Kallias' but not with 'walking'. However, both 'walking' and 'man' are a-predicated of the compound term 'walking man'; for everything compatible with 'walking man' is also compatible with 'walking' and with 'man'. It is not immediately clear what the semantic value of the compound term 'walking man' is in the three y-semantics. But if the domain of semantic values of terms includes such a value, the condition of ecthesis holds between 'walking' and 'man'. On the other hand, if that domain does not include such a value or another suitable item, the condition of ecthesis need not hold between 'walking' and 'man'.

This is another way to explain why y-predication is not equivalent to the condition of ecthesis. Both explanations are based on the assumption that the domain of semantic values of terms may fail to include certain items. In the first explanation, the domain does not include individuals such as Kallias. In the second explanation, the domain does not include the semantic values of compound terms such as 'walking man'. Thus, the assumption is that the domain of semantic values of terms may be rather poor.

In the set-theoretic semantics, on the other hand, the domain of semantic values of terms is rather rich. More precisely, it is the powerset of a non-empty set of individuals, possibly with the empty set removed. Given an individual such as Kallias, there is a semantic value corresponding to the term 'Kallias', namely, the singleton set whose only member is Kallias. And given two terms 'walking' and 'man', there is a semantic value corresponding to the compound term 'walking man', namely, the intersection of the semantic value of 'walking' and that of 'man' (at least, if the empty set is removed, when 'walking man' is truly said of at least one individual).

Let me conclude this paper by considering the Kneales's argument for why Aristotle's puzzling statement in (3) is incorrect – the statement which implies that y-predication is not equivalent to the condition of ecthesis. First of all, the Kneales assume that y-propositions are equivalent to i-propositions (formally, $\mathbf{I}ab$). As a result, (3) is equivalent to:

(10) $\forall z(\mathbf{A}bz \supset \mathbf{I}az)$ does not imply $\forall z(\mathbf{I}bz \supset \mathbf{I}az)$

The Kneales's (1972, 204) objection to (10) is that

(11) $\forall z(\mathbf{A}bz \supset \mathbf{I}az)$ implies $\mathbf{A}ab$

Given the validity of Darii, $\mathbf{A}ab$ implies $\forall z(\mathbf{I}bz \supset \mathbf{I}az)$. So (11) is sufficient to reject (10).

The Kneales justify (11) by a complex argument which appeals to compound terms such as $b \cap \bar{a}$.[35] They argue that the a-proposition $\mathbf{A}b(b \cap \bar{a})$ is always true, while the i-proposition $\mathbf{I}a(b \cap \bar{a})$ is always false. So if the formula $\forall z(\mathbf{A}bz \supset \mathbf{I}az)$ is

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[35] Kneale & Kneale (1972, 198–9). Cf. also Lejewski's (1976, 4) summary of their argument.

true, the compound term $b \cap \bar{a}$ must not be admissible; for otherwise it could serve as an instance of the variable z and could thereby falsify the formula. According to the Kneales, any term which is not admissible is either empty or universal (that is, the complement of an empty term). The Kneales argue that $b \cap \bar{a}$ cannot be universal, so that it is empty; for if it was universal, the term a would be empty and therefore not admissible. Finally, they argue that the fact that $b \cap \bar{a}$ is empty implies $\mathbf{E}\bar{a}b$ and $\mathbf{A}ab$.

This argument involves some substantive assumptions. One of them is that empty terms are not admissible in Aristotle's syllogistic. From this the Kneales (1972, 204) infer that Aristotle's puzzling statement in (3) 'is a sign that he did not fully recognize what was involved in his commitment to existential import. It is in line with his failure to recognize *Barbari*.'

This conclusion is, I think, not justified. The Kneales's argument involves many more assumptions, some of which Aristotle might have rejected. For instance, two such assumptions are that for any term a there is a complement $\bar{a}$, and that for any two terms b and $\bar{a}$ there is a compound term $b \cap \bar{a}$.[36]

As to complements, Aristotle does not use negative terms such as 'not-man' in *Prior Analytics* I.1–22. In *de Interpretatione*, he says that such terms are not names in the proper sense, but merely indefinite names (*Int.* 2 16a29–32, 10 19b8–10, similarly 3 16b11–14). This does not imply that such terms are not admitted in Aristotle's language of categorical propositions. Aristotle may consider languages in which every term has a complement. However, he may equally well consider languages in which not every term has a complement. Accordingly, we may consider domains of semantic values of terms in which not every item has a complement.

As to compounds, according to *de Interpretatione*, the two terms 'walking' and 'man' do not make up a unity.[37] Thus the term 'walking man' does not stand for a single unity. On the other hand, Aristotle holds that the subject and predicate term of simple apophantic sentences must stand for a single unity. Even if the terms 'horse' and 'man' were grouped together under a single name 'cloak', the sentence 'cloak is white' would not be a simple apophantic sentence because 'cloak' does not stand for a single unity.[38] Similarly, the sentence 'walking man is white' would not be a simple apophantic sentence because 'walking man' does not stand for a single unity. The categorical propositions discussed in *Prior Analytics* I.1–22 are simple apophantic sentences.[39] So terms such as 'walking man' should not be able to serve as argument terms of categorical propositions.

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[36] Lejewski (1976, 6–7) proves (11) without appealing to compound terms. Instead, he assumes that every term has a complement which meets the traditional principles of obversion, and that i-propositions are equivalent to the condition of ecthesis. His proof can be divided into two parts. First, the equivalence of i-propositions and the condition of ecthesis is used to prove that $\forall z(\mathbf{A}bz \supset \mathbf{I}az)$ implies $\forall z(\mathbf{I}bz \supset \mathbf{I}az)$, as indicated in note 12 above. Then obversion is used to prove that $\forall z(\mathbf{I}bz \supset \mathbf{I}az)$ implies $\mathbf{A}ab$: suppose $\neg \mathbf{A}ab$; this implies $\mathbf{O}ab$ (square of opposition), $\mathbf{I}\bar{a}b$ (obversion), and $\mathbf{I}b\bar{a}$ (conversion); $\mathbf{I}b\bar{a}$ and $\forall z(\mathbf{I}bz \supset \mathbf{I}az)$ imply $\mathbf{I}a\bar{a}$, which is a contradiction.

[37] *Int.* 11 20b15–22, 21a7–14, cf. Whitaker (1996, 150–1), Crivelli (2004, 156–8 and 177).

[38] *Int.* 8 18a18–21, cf. Whitaker (1997, 95–98), Crivelli (2004, 155–180).

[39] Cf. *APr.* I.1 24a16–17 and *Int.* 5 17a20–22; cf. also Alexander *in APr.* 11.6–8.

Nevertheless, Aristotle uses compound terms such as 'sleeping horse' as argument terms of categorical propositions in *Prior Analytics* I.1–22. However, such compounds are only used twice in I.1–22, and do not play an important role in the syllogistic.[40] In any case, even if Aristotle considers languages which include compound terms, he may equally well consider languages which do not include compound terms for any two given terms. Accordingly, we may consider domains of semantic values of terms which do not include compound items for any two given items.

Given that complements and compounds are not always available, the Kneales's argument for why (3) is incorrect is not applicable.

Appendix

The purpose of this appendix is to prove that the three y-semantics are adequate for Aristotle's assertoric syllogistic. In other words, that all assertoric moods and conversions held to be valid (or invalid) by Aristotle are valid (or invalid) in the three y-semantics. The claim about validity has already been proved. It remains to prove that every assertoric mood and conversion held to be invalid by Aristotle is invalid in the three y-semantics.

Let us start with the y1- and y3-semantics. In both of them, indeterminate propositions are equivalent to particular propositions. So we need not consider indeterminate propositions, but can focus on purely universal and particular moods and conversions. There is only one such conversion held to be invalid by Aristotle in the assertoric syllogistic, namely, that of o-propositions. This can be shown to be invalid in the y1- and y3-semantics.[41] There are many assertoric moods held to be invalid by Aristotle. Aristotle holds that the premiss pair of all these moods is inconcludent – that is, that no assertoric conclusion follows from the premiss pair in the figure under consideration.

Now, the two principles of subalternation are valid in the y1- and y3-semantics: a- and e-propositions imply i- and o-propositions, respectively.[42] So in order to prove that a premiss pair is inconcludent in the y1- and y3-semantics, it suffices to show that no i- and no o-conclusion follows from it. This can be done by two models in which the premiss pair is true while the major term is not i- or o-predicated of the minor term, respectively.

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[40] Compound terms such as 'sleeping horse' are used in two counterexamples to prove the inconcludence of the modal premiss pairs ae-3-QN and ie-3-QN (I.22 40a37–8, 40b10–12). Aristotle has already established the inconcludence of ae-1-QN by counterexamples which do not involve compound terms (I.16 36a30). These counterexamples are also used to prove the inconcludence of ae-1-QX and ao-1-QX (I.15 35a24, 35b10). The same counterexamples can be used to prove the inconcludence of ae-3-QN and ie-3-QN.

[41] In the model given towards the end of Section 2 above, *a* is o-predicated of *b*, but not vice versa.

[42] It follows from Aristotle's conversions that a-propositions imply i-propositions. Given the assertoric square of opposition, which is valid in the y1- and y3-semantics, it follows from this that e-propositions imply o-propositions.

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In some cases, the inconcludence of a premiss pair follows from that of another premiss pair; for instance, the inconcludence of ie-1 follows by subalternation from that of ae-1. Given subalternation and the three valid assertoric conversions, the inconcludence of all assertoric premiss pairs held to be inconcludent by Aristotle follows from the inconcludence of five premiss pairs, namely, ae-1, ee-1, oa-1, aa-2, and oa-2.[43] In the following four Facts, these five premiss pairs are proved to be inconcludent in the y1- and y3-semantics.

Fact 1: The premiss pair ae-1 is inconcludent in the y1- and y3-semantics.

Proof. Let $\mathfrak{A}$ be a model with the domain A={ a, b, c }; $\Upsilon^{\mathfrak{A}}$ is the reflexive and symmetric closure of { ab, ac } on A. We have $\mathfrak{A} \vDash \mathbf{A}ab$. Moreover, we have $\mathfrak{A} \vDash \neg\exists z(\mathbf{A}cz \wedge \mathbf{A}bz)$ and $\mathfrak{A} \vDash \neg\Upsilon bc$. So we have $\mathfrak{A} \vDash \mathbf{E}bc$ in the y1- and y3-semantics. At the same time, we have $\mathfrak{A} \vDash \mathbf{A}ac$, hence $\mathfrak{A} \nvDash \mathbf{O}ac$.

Let $\mathfrak{B}$ be a model with B={ a, b, c }; $\Upsilon^{\mathfrak{B}}$ is the reflexive and symmetric closure of { ab } on B. We have $\mathfrak{B} \vDash \mathbf{A}ab$. Moreover, we have $\mathfrak{B} \vDash \neg\exists z(\mathbf{A}cz \wedge \mathbf{A}bz)$ and $\mathfrak{B} \vDash \neg\Upsilon bc$. So we have $\mathfrak{B} \vDash \mathbf{E}bc$ in the y1- and y3-semantics. At the same time, we have $\mathfrak{B} \vDash \neg\exists z(\mathbf{A}cz \wedge \mathbf{A}az)$ and $\mathfrak{B} \vDash \neg\Upsilon ac$. So we have $\mathfrak{B} \nvDash \mathbf{I}ac$ in the y1- and y3-semantics. □

Fact 2: The premiss pair ee-1 is inconcludent in the y1- and y3-semantics.

Proof. Let $\mathfrak{A}$ be a model with A={ a, b, c }; $\Upsilon^{\mathfrak{A}}$ is the reflexive and symmetric closure of { ac } on A. We have $\mathfrak{A} \vDash \neg\exists z(\mathbf{A}bz \wedge \mathbf{A}az)$ and $\mathfrak{A} \vDash \neg\Upsilon ab$. So we have $\mathfrak{A} \vDash \mathbf{E}ab$ in the y1- and y3-semantics. Moreover, we have $\mathfrak{A} \vDash \neg\exists z(\mathbf{A}cz \wedge \mathbf{A}bz)$ and $\mathfrak{A} \vDash \neg\Upsilon bc$. So we have $\mathfrak{A} \vDash \mathbf{E}bc$ in the y1- and y3-semantics. At the same time, we have $\mathfrak{A} \vDash \mathbf{A}ac$, hence $\mathfrak{A} \nvDash \mathbf{O}ac$.

Let $\mathfrak{B}$ be a model with B={ a, b, c }; $\Upsilon^{\mathfrak{B}}$ is the reflexive closure of the empty set on B, i.e., $\Upsilon^{\mathfrak{B}}$ is { aa, bb, cc }. We have $\mathfrak{B} \vDash \neg\exists z(\mathbf{A}bz \wedge \mathbf{A}az)$ and $\mathfrak{B} \vDash \neg\Upsilon ab$. So we have $\mathfrak{B} \vDash \mathbf{E}ab$ in the y1- and y3-semantics. Moreover, we have $\mathfrak{B} \vDash \neg\exists z(\mathbf{A}cz \wedge$

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43 The inconcludence of ae-1 implies that of ie-1, io-1, ao-1, ie-2, io-2, ae-3, ao-3, ie-3, and io-3. The inconcludence of ee-1 implies that of oe-1, eo-1, oo-1, ee-2, eo-2, oe-2, oo-2, ee-3, eo-3, oe-3, and oo-3. The inconcludence of oa-1 implies that of oi-1 and oi-3. The inconcludence of aa-2 implies that of ia-1, ii-1, ai-2, ii-2, ia-2, and ii-3. The inconcludence of oa-2 implies that of oi-2. These are all purely universal and particular premiss pairs held to be invalid by Aristotle in the assertoric syllogistic.

Abz) and $\mathfrak{B} \vDash \neg\Upsilon bc$. So we have $\mathfrak{B} \vDash \mathbf{E}bc$ in the y1- and y3-semantics. At the same time, we have $\mathfrak{B} \vDash \neg\exists z(\mathbf{A}cz \wedge \mathbf{A}az)$ and $\mathfrak{B} \vDash \neg\Upsilon ac$. So we have $\mathfrak{B} \nvDash \mathbf{I}ac$ in the y1- and y3-semantics. $\square$

Fact 3: The premiss pairs oa-1 and aa-2 are inconcludent in the y1- and y3-semantics.

Proof. Let $\mathfrak{A}$ be a model with A=$\{a, b, c, d\}$; $\Upsilon^{\mathfrak{A}}$ is the reflexive and symmetric closure of $\{ab, bc, ac, bd\}$ on A. We have $\mathfrak{A} \vDash \neg\mathbf{A}ab$, hence $\mathfrak{A} \vDash \mathbf{O}ab$; and we have $\mathfrak{A} \vDash \mathbf{A}ba$. Moreover, we have $\mathfrak{A} \vDash \mathbf{A}bc$. At the same time, we have $\mathfrak{A} \vDash \mathbf{A}ac$, hence $\mathfrak{A} \nvDash \mathbf{O}ac$.

Let $\mathfrak{B}$ be a model with B=$\{a, b, c\}$; $\Upsilon^{\mathfrak{B}}$ is the reflexive and symmetric closure of $\{ab, bc\}$ on B. We have $\mathfrak{B} \vDash \neg\mathbf{A}ab$, hence $\mathfrak{B} \vDash \mathbf{O}ab$; and we have $\mathfrak{B} \vDash \mathbf{A}ba$. Moreover, we have $\mathfrak{B} \vDash \mathbf{A}bc$. At the same time, we have $\mathfrak{B} \vDash \neg\exists z(\mathbf{A}cz \wedge \mathbf{A}az)$ and $\mathfrak{B} \vDash \neg\Upsilon ac$. So we have $\mathfrak{B} \nvDash \mathbf{I}ac$ in the y1- and y3-semantics. $\square$

Fact 4: The premiss pair oa-2 is inconcludent in the y1- and y3-semantics.

Proof. Let $\mathfrak{A}$ be a model with A=$\{a, b, c, d\}$; $\Upsilon^{\mathfrak{A}}$ is the reflexive and symmetric closure of $\{ab, ac, ad, bc\}$ on A. We have $\mathfrak{A} \vDash \neg\mathbf{A}ba$, hence $\mathfrak{A} \vDash \mathbf{O}ba$. Moreover, we have $\mathfrak{A} \vDash \mathbf{A}bc$. At the same time, we have $\mathfrak{A} \vDash \mathbf{A}ac$, hence $\mathfrak{A} \nvDash \mathbf{O}ac$.

Let $\mathfrak{B}$ be a model with B=$\{a, b, c\}$; $\Upsilon^{\mathfrak{B}}$ is the reflexive and symmetric closure of $\{bc\}$ on B. We have $\mathfrak{B} \vDash \neg\mathbf{A}ba$, hence $\mathfrak{B} \vDash \mathbf{O}ba$. Moreover, we have $\mathfrak{B} \vDash \mathbf{A}bc$. At the same time, we have $\mathfrak{B} \vDash \neg\exists z(\mathbf{A}cz \wedge \mathbf{A}az)$ and $\mathfrak{B} \vDash \neg\Upsilon ac$. So we have $\mathfrak{B} \nvDash \mathbf{I}ac$ in the y1- and y3-semantics. $\square$

This completes the proof that the y1- and y3-semantics are adequate for Aristotle's assertoric syllogistic.

Let us now consider the y2-semantics. As far as purely universal and particular moods and conversions are concerned, the y2-semantics is identical with the y1-semantics. As to indeterminate propositions, it suffices to show that every assertoric mood and conversion held to be invalid by Aristotle remains invalid when every particular proposition in it is replaced by the corresponding indeterminate proposition of the same quality. As mentioned earlier, this condition can explain Aristotle's statements of invalidity which involve indeterminate propositions in the assertoric syllogistic. The y2-semantics can be shown to satisfy this

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condition. Aristotle denies that o-propositions are convertible. So indeterminate negative propositions should not be convertible in the y2-semantics – which can be proved by means of the model given towards the end of Section 2 above.[44]

Now for the premiss pairs held to be inconcludent by Aristotle. Affirmative (or negative) indeterminate propositions follow from i- (or o-) propositions in the y2-semantics.[45] Inconcludence means that no assertoric conclusion follows from a given premiss pair. Thus, in order to establish inconcludence in the y2-semantics, we should show that no indeterminate conclusion follows from the premiss pair under consideration. This is true in the y2-semantics for all purely universal and particular premiss pairs held to be inconcludent by Aristotle; for in each of the foregoing four Facts, $\exists z(\Upsilon cz \wedge \neg\exists u(Azu \wedge Aau))$ is false in the model $\mathfrak{A}$, and Υac is false in the model $\mathfrak{B}$. As a result, every assertoric mood held to be invalid by Aristotle remains invalid in the y2-semantics when every particular proposition in it is replaced by the corresponding indeterminate proposition of the same quality.

This completes the proof that the y2-semantics is adequate for Aristotle's assertoric syllogistic.

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[44] In this model, $\Upsilon bc \wedge \neg\exists u(Acu \wedge Aau)$ is true, and hence also $\exists z(\Upsilon bz \wedge \neg\exists u(Azu \wedge Aau))$. But $\exists z(\Upsilon az \wedge \neg\exists u(Azu \wedge Abu))$ is false in it.

[45] Notes 23 and 26 above.

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