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FIGURES OF PROSLEPTIC SYLLOGISMS IN *PRIOR ANALYTICS* 2.7*

In chapters 2.5–7 of the *Prior Analytics* Aristotle is concerned with what he calls circular proof. He gives an account of circular proofs within the framework of his syllogistic theory, and discusses how they come about in the three figures of categorical syllogisms. The results of this discussion are summarized at the end of chapter 2.7, at 59a32–41. The summary contains several statements to the effect that certain circular proofs come about in the third figure. Some of these statements are problematic because the circular proofs in question are actually not in the third figure of categorical syllogisms; in fact, these circular proofs are not categorical syllogisms at all, but what Theophrastus called prosleptic syllogisms. Hence, the statements are incorrect if they are understood to refer to the third figure of Aristotle's categorical syllogisms. Since it seems natural to understand them in this way, Ross and others conclude that the passage at 59a32–41 is spurious and should be excised, although it is found in all MSS.[1]

By contrast, this paper aims to show that the passage is not spurious. Following Pacius, I argue that the problematic statements in it refer not to the third figure of categorical syllogisms, but to the third figure of prosleptic syllogisms. On this interpretation, the statements are correct and can be regarded as genuine. Given that they are genuine, they show that Aristotle was aware of a classification of prosleptic syllogisms into three figures, even though such a classification does not occur elsewhere in his writings. Thus, the passage at 59a32–41 appears to be the earliest evidence we have of figures of prosleptic syllogisms.

I begin with an overview of Aristotle's treatment of circular proofs in *Prior Analytics* 2.5–7, focussing on his use of prosleptic syllogisms (§1). Readers familiar with the contents of *Prior Analytics* 2.5–7 may wish to skip this overview. Next we consider the problematic statements in 59a32–41 (§2). I will argue that these statements refer to the third figure of prosleptic syllogisms, and that there is no reason to doubt Aristotle's authorship of the passage (§3).

1. CIRCULAR PROOF IN *PRIOR ANALYTICS* 2.5–7

The second book of the *Prior Analytics* relies on the assertoric syllogistic, developed in the first seven chapters of the first book. As is well known, the assertoric syllogistic deals primarily with four kinds of propositions, namely with a-, e-, i- and o-propositions:

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[1] W.D. Ross, *Aristotle's Prior and Posterior Analytics* (Oxford, 1949), 444; M. Mignucci, *Aristotele: Gli Analitici Primi* (Naples, 1969), 622–3; J. Barnes (ed.), *The Complete Works of Aristotle: The Revised Oxford Translation*, *Volume 1* (Princeton, 1984), 94; R. Smith, *Aristotle: Prior Analytics* (Indianapolis and Cambridge, 1989), 78.

AaB (A belongs to all B)
AeB (A belongs to no B)
AiB (A belongs to some B)
AoB (A does not belong to some B)

Propositions of this form are traditionally called categorical propositions. The assertoric syllogistic is concerned with syllogisms which consist of three categorical propositions, two of them serving as premisses and one as the conclusion. For example, syllogisms of the form Barbara consist of three a-propositions:

major premiss:	AaB
minor premiss:	BaC
conclusion:	AaC

Such syllogisms are traditionally called categorical syllogisms. Aristotle distinguishes three figures of categorical syllogisms, which can be represented as follows (with 'x', 'y', 'z' being placeholders for 'a', 'e', 'i', 'o'):

	first figure:	second figure:	third figure:
major premiss:	AxB	BxA	AxB
minor premiss:	ByC	ByC	CyB
conclusion:	AzC	AzC	AzC

In *Prior Analytics* 2.5–7, Aristotle uses the framework of the assertoric syllogistic to give an account of circular proofs. At the beginning of chapter 2.5, circular proof is defined as follows:

τὸ δὲ κύκλῳ καὶ ἐξ ἀλλήλων δείκνυσθαί ἐστι τὸ διὰ τοῦ συμπεράσματος καὶ τοῦ ἀνάπαλιν τῇ κατηγορίᾳ τὴν ἑτέραν λαβόντα πρότασιν συμπεράνασθαι τὴν λοιπήν, ἣν ἐλάμβανεν ἐν θατέρῳ συλλογισμῷ. (*An. pr.* 2.5 57b18–21)

Proving in a circle, or from one another, is concluding something which was taken in some other syllogism as a premiss by means of the conclusion of that syllogism and its other premiss taken as converted in predication.

Aristotle describes here a syllogism in which one of the premisses of some other syllogism is proved by means of the conclusion of this latter syllogism and its other premiss 'converted in predication'. In other words: given a categorical syllogism, Aristotle describes a syllogism (i) one of whose premisses is the conclusion of the original syllogism, (ii) whose other premiss is one of the premisses of the original syllogism 'converted in predication', (iii) and whose conclusion is the other premiss of the original syllogism. For any given categorical syllogism there may, in principle, be two syllogisms which satisfy Aristotle's description: one which proves its major premiss and another which proves its minor premiss.

original syllogism:	proof of major premiss P_1:	proof of minor premiss P_2:
P_1 (major premiss)	C	C
P_2 (minor premiss)	P_2 converted	P_1 converted
C (conclusion)	P_1	P_2

This diagram is not meant to determine the order of the premisses in the proofs of P_1 and P_2. As becomes clear from Aristotle's discussion in chapters 2.5–7, the conclusion of the original syllogism, C, can serve either as the major premiss or as the minor premiss in these proofs.

Aristotle's definition suggests that the proofs of P_1 and of P_2 each count as an instance of 'proving in a circle', that is, as a circular proof.[2] The definition is rather technical and artificial, especially in view of the 'conversion in predication' required by it. Aristotle does not explain why this is a reasonable definition of circular proof, nor does he explain what the 'circle' is by virtue of which his circular proofs are called 'circular' (*κύκλῳ*).[3] For present purposes, however, it is not necessary to enter into a discussion of these questions.

Let us consider Aristotle's requirement that one of the premisses of the original syllogism be 'converted in predication'. By this he means a special kind of conversion different from the standard conversions introduced in *Prior Analytics* 1.2 (according to which AeB can be converted to BeA, and AaB to BiA, and AiB to BiA). Unlike these standard conversions, the conversions used by Aristotle in his circular proofs in 2.5–7 are not in general truth-preserving. Aristotle applies this latter kind of conversion only to a- and e-propositions. When applied to a-propositions, it simply consists in interchanging the predicate and subject term, converting AaB to BaA. Thus, for example, Aristotle's circular proofs of the two premisses of Barbara are as follows:

original syllogism (Barbara):	circular proof of the major premiss (2.5 57b22–5):	circular proof of the minor premiss (2.5 57b25–8):
AaB	AaC	BaA
BaC	CaB	AaC
AaC	AaB	BaC

Both circular proofs are themselves syllogisms in Barbara. In the circular proof of the major premiss, the minor premiss of the original syllogism, BaC, is converted to CaB. In the other circular proof, the major premiss of the original syllogism, AaB, is converted to BaA.

[2] Alternatively, Barnes takes a circular proof to be a set of three syllogisms, consisting of the original syllogism and the two circular proofs; J. Barnes, *Aristotle*: *Posterior Analytics* (Oxford, 1994[2]), 106. Smith (n. 1), 193, takes a circular proof to be a more complex structure consisting of six syllogisms (his view is supported by 2.5 57b32–58a20). For present purposes, we may set aside this issue, and follow Ross (n. 1), 438–44 in regarding each of the two proofs of the original premisses as a circular proof.

[3] In *An. post.* 1.3 Aristotle considers a more natural notion of circular proof, which requires there to be a finite sequence of items such that the first item is deduced from the second, the second from the third and so on, and finally the last item is deduced from the first (72b25–73a6). In the context of this discussion, at 73a11–16, Aristotle also refers to his treatment of circular proof in *An. pr.* 2.5–7. However, it is not clear how the notion of circular proof from *An. post.* 1.3 is related to that from *An. pr.* 2.5–7; see R. Smith, 'Immediate propositions and Aristotle's proof theory', *AncPhil* 6 (1986), 47–68, at 60–1; Barnes (n. 2), 106.

The conversion in question is more complex when it is applied to e-propositions. To see why, consider a possible circular proof of the minor premiss of Celarent:

original syllogism (Celarent):	circular proof of the minor premiss:
AeB	AeB converted
BaC	AeC
AeC	BaC

Such a circular proof needs to deduce the a-premiss of Celarent, BaC, from the e-conclusion, AeC, and a converted version of the e-premiss, AeB. If the conversion simply consisted in interchanging the predicate and subject term, the circular proof would need to deduce an a-proposition from two e-propositions. But this is impossible; there is no such valid deduction. Nevertheless, Aristotle wants to give a circular proof of the a-premiss of Celarent. To this end, he converts the e-premiss, AeB, to a more complex proposition, namely to:

ᾧ τὸ A μηδενὶ ὑπάρχει, τὸ B παντὶ ὑπάρχειν (*An. pr.* 2.5 58a29–30)

whatever A belongs to none of, B belongs to all of it

We know that Theophrastus called such propositions prosleptic propositions (*κατὰ πρόσληψιν προτάσεις*).[4] In modern notation, the prosleptic proposition invoked by Aristotle can be expressed as follows:

(1) for every X, if AeX then BaX

Aristotle regards the transition from AeB to (1) as a conversion.[5] He does not explain why it counts as a conversion, but a possible explanation can be found in an anonymous scholium edited by Brandis (*Σ* 190a5–8 Brandis). According to the scholiast, the transition from AeB to (1) consists of two steps. First, AeB is transformed into the following prosleptic proposition:

(2) for every X, if BaX then AeX

The validity of Celarent guarantees that AeB entails (2). Thus, the transition from AeB to (2) is a valid inference. The converse transition, too, seems to be a valid inference: given that a-propositions of the form BaB are always true, (2) entails AeB.[6] As a result, the categorical proposition AeB and the prosleptic proposition in (2) are equivalent: necessarily the one is true if and only if the other is true. This equivalence was endorsed by Theophrastus (see *Σ* 190a1–4 Brandis).

[4] Alex. Aphr. *In An. pr.* 378.14; see also *Σ* 189b43–4 Brandis. Aristotle himself does not use the expression 'prosleptic' (unless one accepts the phrase *διὰ προσλήψεως* at *An. pr.* 2.5 58b9, which is only attested by some MSS and is generally regarded as spurious).

[5] Cf. *An. pr.* 2.5 58a26–30, in conjunction with the reference to conversion (*ἀντιστρέφειν*) at 58b8.

[6] Aristotle appears to hold that all a-propositions of the form BaB are true, see *An. pr.* 2.15 64b7–13, in conjunction with 64a4–7, 23–30; cf. J. Łukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic* (Oxford, 1957[2]), 9; P. Thom, *The Syllogism* (Munich, 1981), 92.

In the second step described by the scholiast, the two categorical propositions which occur in (2), BaX and AeX, are interchanged. The result is the prosleptic proposition in (1). This transformation is not truth-preserving, and hence does not constitute a valid inference. The scholiast regards this transformation as a conversion (ἀντιστρέφειν, *Σ* 190a7 Brandis). The transition from AeB to (1) can then be analysed as follows: first, AeB is transformed into the equivalent prosleptic proposition in (2), and then (2) is converted to (1). By virtue of the latter step, the whole transition from AeB to (1) may be regarded as a kind of conversion.[7]

As mentioned above, the conversion from AeB to (1) is applied in connection with the e-premiss of Celarent in order to give a circular proof of the a-premiss. This circular proof is as follows:

original syllogism (Celarent):	circular proof of the minor premiss (2.5 58a26–32):
AeB	for every X, if AeX then BaX
BaC	AeC
AeC	BaC

The circular proof is an argument which consists of a prosleptic premiss, a categorical premiss and a categorical conclusion. We know from a scholium entitled 'On all the kinds of syllogism' that Theophrastus called such arguments prosleptic syllogisms.[8]

Among the circular proofs given by Aristotle in *Prior Analytics* 2.5–7, there are four prosleptic syllogisms, namely the circular proofs of the minor premiss of Celarent, Ferio, Festino and Ferison. In all of them, the major premiss of the original syllogism is an e-proposition. This e-proposition is converted into a prosleptic proposition, which is then used in the prosleptic syllogism. In the case of Celarent, Aristotle employs the conversion from AeB to (1) described above. In the other three cases, Aristotle employs two slightly different conversions, converting AeB to the prosleptic propositions in (3) and (4) respectively:

(3) for every X, if BoX then AiX (Festino, 2.6 58b36–8)
(4) for every X, if AoX then BiX (Ferio, 2.5 58b7–10; Ferison, 2.7 59a25–9)

Like the earlier conversion from AeB to (1), these two conversions are not truth-preserving; but they can be justified by means of the earlier conversion and some additional truth-preserving transformations.[9]

[7] Similarly, Aristotle's conversion from AaB to BaA can be analysed into three steps as follows. First, AaB is transformed into the prosleptic proposition 'for every X, if BaX then AaX'. Given the validity of Barbara and given that BaB is true for all B, this prosleptic proposition is equivalent to AaB (again, this equivalence was stated by Theophrastus, see *Σ* 190a4–5 Brandis). Secondly, the two categorical propositions occurring in the prosleptic proposition are interchanged, which leads to: 'for every X, if AaX then BaX'. Thirdly, this latter prosleptic proposition is transformed into the equivalent categorical proposition BaA.

[8] *Σ CAG* 4.6 XII.4 Wallies.

[9] Aristotle holds that an a-proposition is true if and only if the corresponding o-proposition is false, and likewise for e- and i-propositions. Consequently, (3) is equivalent to (1), since by contraposition, 'if BoX then AiX' is equivalent to 'if AeX then BaX'. So, given that AeB can be converted to (1), it can also be converted to (3). This accounts for the conversion from AeB to (3). Given this conversion, BeA can be converted to (4). Now, AeB is equivalent to BeA,

In chapters 2.5–7, Aristotle examines the categorical syllogisms of the assertoric syllogistic, and determines whether a circular proof of their major and minor premiss is possible. If it is possible, he specifies the circular proof, either as a categorical or as a prosleptic syllogism. In some cases, Aristotle applies an extended version of circular proof which involves an additional truth-preserving conversion (this occurs in the circular proofs of the major premiss of Cesare and of the minor premiss of Datisi and Ferison).

Aristotle's discussion of circular proofs in chapters 2.5–7 can be summarized as follows:

Original syllogism in the first figure (2.5)

original syllogism:	circular proof of the major premiss:	circular proof of the minor premiss:
Barbara: AaB BaC AaC	first figure (57b22–5): AaC CaB AaB	first figure (57b25–8): BaA AaC BaC
Celarent: AeB BaC AeC	first figure (58a23–6): AeC CaB AeB	prosleptic syllogism (58a26–32): for every X, if AeX then BaX AeC BaC
Darii: AaB BiC AiC	not possible (58a36–b2)	first figure (58b2–6): BaA AiC BiC
Ferio: AeB BiC AoC	not possible (58b6–7)	prosleptic syllogism (58b7–12): for every X, if AoX then BiX AoC BiC

Original syllogism in the second figure (2.6)

Camestres: AaB AeC BeC	not possible (58b13–18)	second figure (58b18–22): BaA BeC AeC

due to the truth-preserving conversions from *An. pr.* 1.2. So, given that BeA can be converted to (4), AeB can also be converted to (4).

Cesare: AeB AaC BeC	first figure (58b22–7): BeC CaA BeA AeB additional conversion	not possible (58b13–18)
Baroco: AaB AoC BoC	not possible (58b27–9)	second figure (58b29–33): BaA BoC AoC
Festino: AeB AiC BoC	not possible (58b27–9)	prosleptic syllogism (58b33–8): for every X, if BoX then AiX BoC AiC

Original syllogism in the third figure (2.7)

Darapti: AaC BaC AiB	not possible (58b39–59a3)	not possible (58b39–59a3)
Felapton: AeC BaC AoB	not possible (58b39–59a3)	not possible (58b39–59a3)
Datisi: AaC BiC AiB	not possible (58a36–b2, 58b27–9)	first figure (59a3–14): CaA AiB CiB BiC additional conversion
Disamis: AiC BaC AiB	third figure (59a15–18): AiB CaB AiC	not possible (58a36–b2, 58b27–9)
Bocardo: AoC BaC AoB	third figure (59a18–23): AoB CaB AoC	not possible (58a36–b2, 58b27–9)
Ferison: AeC BiC AoB	not possible (58a36–b2, 58b27–9)	prosleptic syllogism (59a24–31): for every X, if AoX then CiX AoB CiB BiC additional conversion

In a number of cases, Aristotle denies the existence of circular proofs. Some of these denials are open to question. For example, consider Aristotle's claim that there is no circular proof of the a-premiss of Cesare. Pseudo-Philoponus argues that this a-premiss can be proved by means of a prosleptic syllogism as follows:[10]

original syllogism (Cesare):	circular proof of the minor premiss:
AeB	for every X, if BeX then AaX
AaC	BeC
BeC	AaC

This circular proof is based on the assumption that the e-premiss of Cesare, AeB, can be converted to the following prosleptic proposition:

(5) for every X, if BeX then AaX

This prosleptic proposition is, by contraposition, equivalent to (4). Since Aristotle accepts that AeB can be converted to (4) in circular proofs, he should also accept that AeB can be converted to (5). Thus, Pseudo-Philoponus' circular proof of the a-premiss of Cesare seems to be in accordance with Aristotle's standards; it would be difficult to reject this prosleptic circular proof without rejecting Aristotle's own prosleptic circular proofs. In view of this, Aristotle's claim that there is no circular proof of the a-premiss of Cesare does not seem to be true without qualification; it should be understood to be restricted to categorical circular proofs, so that it does not rule out a prosleptic circular proof. This is supported by two passages from chapters 2.6–7 which suggest that only categorical circular proofs, but not prosleptic ones, are circular proofs in the proper sense.[11]

Similar problems arise with Aristotle's claim that there is no circular proof of the a-premiss of Camestres. Pseudo-Philoponus argues that this a-premiss can be proved by means of a prosleptic syllogism in a similar way to the a-premiss of Cesare.[12] Again, Aristotle's claim can be understood to be restricted to categorical circular proofs.

All other cases in which Aristotle denies the existence of a circular proof concern categorical syllogisms whose conclusion is an i- or o-proposition: Aristotle states that there is no circular proof of an a- or e-premiss of such a categorical syllogism. Pseudo-Philoponus argues that even in these cases circular proofs can be constructed by means of suitable prosleptic propositions.[13] However, it is not clear whether these circular proofs would be in accordance with Aristotle's standards.[14]

[10] Ps.-Philop. *In An. pr.* 420.1–3 (*CAG* 13.2); see also Cz. Lejewski, 'On prosleptic syllogisms', *Notre Dame Journal of Formal Logic* 2 (1961), 158–76, at 167.

[11] 2.6 58b33–8, 2.7 59a24–6; see also Ps.-Philop. *In An. pr.* 419.16–17, 420.22, 422.6–7. On the other hand, Aristotle seems to accept prosleptic syllogisms as circular proofs without qualification at 2.5 58a26–35.

[12] Ps.-Philop. *In An. pr.* 419.16–19. As in the case of Cesare, the e-premiss of Camestres, AeC, is converted to 'for every X, if CeX then AaX'. The conclusion of Camestres is BeC. This is converted to CeB by means of an additional truth-preserving conversion. Finally, 'for every X, if CeX then AaX' and CeB allow us to infer AaB, which is the a-premiss of Camestres.

[13] Ps.-Philop. *In An. pr.* 418.3, 418.16–20, 420.11–13, 421.5–6, 422.1–2; see also T. Waitz, *Aristotelis Organon graece, vol. 1* (Leipzig, 1844), 498; Lejewski (n. 10), 167.

[14] For example, Pseudo-Philoponus' circular proof of the a-premiss of Baroco makes use of the prosleptic proposition 'for every X, if XoC then AaX' (*In An. pr.* 420.11–13). However,

2. THE PROBLEMATIC PASSAGE

We are now in a position to consider the problematic passage which is the main subject of this paper. The passage occurs at the end of chapter 2.7, and is a summary of chapters 2.5–7. It can be divided into four parts. The first three parts provide an overview of the figures in which circular proofs come about when the original syllogism is in the first, second and third figure, respectively. The fourth part states that some circular proofs are deficient:

> (1) It is evident, then, that in the first figure, proof by means of one another comes about both through the third and through the first figure. For when the conclusion is affirmative, the proof is through the first figure, and when the conclusion is negative it is through the last figure; for it is assumed that whatever this belongs to none of, the other belongs to all of. (2) And in the middle figure, when the syllogism is universal then proof by means of one another is both through the second and through the first figure (*δι᾽ αὐτοῦ τε καὶ διὰ τοῦ πρώτου σχήματος*); and when the syllogism is particular, the proof is through both the second and the last figure (*δι᾽ αὐτοῦ τε καὶ τοῦ ἐσχάτου*). (3) And in the third figure, all proofs by means of one another are through the third figure (*δι᾽ αὐτοῦ πάντες*). (4) It is also evident that in the third and the middle figure, the syllogisms which do not come about through the same figure (*οἱ μὴ δι᾽ αὐτῶν γινόμενοι συλλογισμοί*) are either not in accordance with circular proof or are incomplete. (*An. pr.* 2.7 59a32–41)

This passage contains numerous references to figures of syllogisms. When Aristotle refers to figures of syllogisms elsewhere in the *Prior Analytics*, he always means figures of categorical syllogisms. This is also true for many of the references in the present passage, but, as we will see, probably not for all of them.

Part (1) states that when the original syllogism is in Barbara or Darii, circular proofs are in the first figure. This is correct, since these circular proofs are themselves categorical syllogisms in Barbara and Darii. Part (1) also states that when the original syllogism is in Celarent or Ferio, circular proofs are in the third figure. There are two problems with this: (i) the circular proof of the major premiss of Celarent is in the first figure, and (ii) the circular proofs of the minor premiss of Celarent and Ferio are not categorical syllogisms in the third figure, but prosleptic syllogisms.

Part (2) states, correctly, that when the original syllogism is in Camestres or Cesare, circular proofs are in the second and first figure respectively. It also states that when the original syllogism is in Baroco or Festino, circular proofs are in the second and third figure respectively. There is one problem with this: (iii) while the passage states that the circular proof of the minor premiss of Festino is in the third figure, this circular proof is not a categorical but a prosleptic syllogism.

Part (3) states that when the original syllogism is a categorical syllogism in the third figure, all circular proofs are also in the third figure. There are two problems with this: (iv) the circular proof of the minor premiss of Datisi is a categorical

Aristotle does not consider such prosleptic propositions in the *Prior Analytics*. He only considers prosleptic propositions of the form ‘for every X, if ByX then AzX’, with the quantified variable ‘X’ serving as the subject both in the if-clause and in the then-clause. Thus, Pseudo-Philoponus’ circular proof is not in accordance with Aristotle’s usage of prosleptic propositions. The same is true for the other categorical syllogisms which have an i- or o-conclusion: there is no prosleptic circular proof of an a- or e-premiss of such a syllogism by means of a prosleptic proposition of the form ‘for every X, if ByX then AzX’.

syllogism in the first figure, and (v) the circular proof of the minor premiss of Ferison is not a categorical but a prosleptic syllogism.

Finally, part (4) is concerned with circular proofs whose original syllogism is in the second or third figure and which are not in the same figure as the original syllogism (*οἱ μὴ δι᾽ αὐτῶν γινόμενοι συλλογισμοί*). These circular proofs are said to be deficient in a certain way. They include the circular proofs of the major premiss of Cesare and of the minor premiss of Datisi, neither of which is in the same figure as the original syllogism. These two circular proofs are regarded as deficient because they involve an additional truth-preserving conversion.[15]

Let us now consider the five problems (i)–(v) identified above. Ross points out that problems (i) and (iv) are less serious and may be a mere oversight.[16] But problems (ii), (iii) and (v) are more serious. They concern precisely the four cases in which Aristotle gives a prosleptic circular proof. In each case, the passage states that the prosleptic circular proof is in the third figure. These statements are incorrect if they are understood to refer to the third figure of categorical syllogisms. Since it seems natural to understand them in this way, Ross and others conclude that, in view of problems (ii), (iii) and (v), the whole passage at 59a32–41 is a gloss and should be excised (see n. 1 above).

However, there are at least two reasons to think that the passage is not a gloss. First, the passage is found in all MSS, and there is no indication of problems with its textual transmission. The second reason has to do with the context provided by chapters 2.2–14 of the *Prior Analytics*. These chapters deal with five topics: true conclusions from false premisses (2.2–4), circular proofs (2.5–7), conversion of deductions (2.8–10), *reductio ad impossibile* (2.11–13), and the relation between direct deductions and *reductio ad impossibile* (2.14). The discussion of these topics is structured by the three figures of categorical syllogisms; in the first four topics, each chapter is devoted to one figure. The discussion of each of the five topics is concluded by a brief summary, introduced by the words 'it is evident, then'

[15] See J. Pacius, *Aristotelis Stagiritae Peripateticorum Principis Organum* (Frankfurt, 1597[2]), 335; Waitz (n. 13), 498; H. Tredennick, *Aristotle: The Organon I. The Categories, On Interpretation, Prior Analytics* (Cambridge, MA, 1938), 448–9. Part (4) might also be taken to apply to the prosleptic circular proof of the minor premiss of Festino; for in part (2) this circular proof is said to be in the third figure, whereas Festino is in the second figure. However, it is not clear whether this circular proof is really meant to be included among the deficient circular proofs discussed in part (4). Since it does not involve an additional truth-preserving conversion, it would probably be regarded as deficient on the grounds that it is prosleptic. But in this case, the prosleptic circular proofs whose original syllogism is in the first figure should also be regarded as deficient. So it would be difficult to explain why part (4) does not take into account circular proofs whose original syllogism is in the first figure (see *Σ* 190b4–10 Brandis).

Ross (n. 1), 444 takes part (4) to apply to the two circular proofs of the minor premiss of Festino and Ferison. The latter circular proof is a prosleptic syllogism and involves an additional truth-preserving conversion; but as I argue below, it is in the same figure as the original syllogism (namely, in the third figure). So, given that part (4) is only concerned with circular proofs which are not in the same figure as the original syllogism, it does not explicitly apply to the circular proof of the minor premiss of Ferison, although this circular proof is deficient because of its additional truth-preserving conversion.

[16] Ross (n. 1), 444. Pacius (n. 15), 335, solves problem (iv) by taking the statement in part (3) to be restricted to circular proofs in the proper sense, so that it does not apply to the deficient circular proofs discussed in part (4), and in particular not to the deficient circular proof of the minor premiss of Datisi. He thereby also resolves an apparent conflict between parts (3) and (4): namely, that according to part (3) all circular proofs whose original syllogism is in the third figure are also in the third figure, whereas part (4) implies that this is not true.

(*φανερὸν οὖν*).[17] One of these five summaries is our problematic passage at the end of chapter 2.7. In view of this context, it seems unlikely that the passage is a gloss; for given that Aristotle wrote summaries for the other four topics, it is unlikely that he did not write one for the topic of circular proof. Since it is also unlikely that Aristotle's original summary was later replaced by a gloss, there is good reason to think that the passage is genuine.

Some commentators accept that the passage is genuine, but hold that Aristotle made an error in the statements that give rise to problems (ii), (iii) and (v).[18] Pacius, on the other hand, argues that Aristotle did not make an error there, but had in mind the third figure of hypothetical syllogisms (*syllogismus hypotheticus*). In commenting on the part of the passage which gives rise to problem (ii), Pacius writes: 'cave intelligas tertiam figuram syllogismorum categoricorum […] efficitur hypotheticus: qui dicitur esse in tertia figura'.[19] At the same time, Pacius makes clear that by 'hypothetical syllogisms' he means prosleptic syllogisms; in commenting on Aristotle's prosleptic circular proof of the minor premiss of Celarent, he writes: 'hic est syllogismus hypotheticus *κατὰ πρόσληψιν*'.[20] Thus, Pacius thinks that Aristotle takes his prosleptic syllogisms to be in the third figure.

The Greek commentators, too, take Aristotle's prosleptic syllogisms to be in the third figure. For example, in Pseudo-Philoponus' commentary on the second book of the *Prior Analytics* (*CAG* 13.2), Aristotle's prosleptic circular proof of the minor premiss of Celarent is described as being in the third figure:

ἵνα δείξῃ τὴν ἐλάττω πρότασιν τοῦ πρώτου σχήματος, διὰ προσλήψεως ἔδειξεν· ἡ δὲ πρόσληψις ἔδειξε τὴν ἐλάττω πρότασιν διὰ τοῦ τρίτου σχήματος.
(Ps.-Philop. *In An. pr.* 417.27–9, cf. also 418.19–20)

In order to prove the minor premiss of the first figure [that is, the minor premiss of Celarent], he proved it through *proslepsis*; the *proslepsis* proved the minor premiss through the third figure.

Similarly, Aristotle's prosleptic circular proof of the minor premiss of Festino is described by an anonymous scholiast as follows:

διὰ τοῦ τρίτου δείκνυσθαι, διὰ τῆς κατὰ πρόσληψιν προτάσεως (*Σ* 190b7 Brandis)

proved through the third figure, through a prosleptic proposition

Consequently, the Greek commentators did not deem it necessary to discuss or explain the references to the third figure that lead to problems (ii), (iii) and (v) in our passage – at least there is no indication of it in Pseudo-Philoponus' com-

[17] The summaries are found at the end of chapters 4, 7, 10, 13 and 14, at 2.4 57a36–57b17, 2.7 59a32–41, 2.10 61a5–16, 2.13 62b25–8 and 2.14 63b12–21. In the first of them, only 57a36–40 summarizes chapters 2.2–4, while the rest provides an explanation of what Aristotle says at 57a40.

[18] Waitz (n. 13), 498; J.H. von Kirchmann, *Erläuterungen zu den ersten Analytiken des Aristoteles* (Leipzig, 1877), 191.

[19] Pacius (n. 15), 335; similarly O.F. Owen, *The Organon, or Logical Treatises, of Aristotle, vol. 1* (London, 1889), 198–9.

[20] Pacius (n. 15), 327; see also his comment (p. 329) on the prosleptic circular proof of the minor premiss of Ferio.

mentary and in Brandis's scholia. They seem to have thought, like Pacius, that Aristotle took his prosleptic syllogisms to be in the third figure.

Of course, this line of interpretation faces the objection that Aristotle does not elsewhere classify prosleptic syllogisms as being in the third figure or in any other figure; there is no trace of figures of prosleptic syllogisms in Aristotle's writings, except perhaps in the present passage.[21] However, as I will argue in the next section, it is nevertheless not unlikely that Aristotle was aware of, and was ready to appeal to, a classification of prosleptic syllogisms into three figures.

3. FIGURES OF PROSLEPTIC SYLLOGISMS

The author of the scholium 'On all the kinds of syllogism' distinguishes three main kinds of syllogisms, namely categorical, hypothetical and prosleptic syllogisms. He writes that prosleptic syllogisms were so called by Theophrastus, and that they are divided into three figures:

ἔστιν γὰρ καὶ τρίτον εἶδος συλλογισμοῦ μετὰ τὸ κατηγορικὸν καὶ ὑποθετικὸν τὸ λεγόμενον παρὰ Θεοφράστῳ κατὰ πρόσληψιν, ὃ κατὰ τὰ τρία σχήματα πλέκεται οὕτως (*Σ CAG* 4.6 XII.3–5 Wallies)

There is also a third kind of syllogism besides the categorical and the hypothetical, called in the work of Theophrastus 'prosleptic'; it is formed according to the three figures as follows.

The scholiast goes on to specify the three figures of prosleptic syllogisms. They differ from each other in the position of the quantified variable 'X' in the prosleptic premiss, and can be represented as follows (again, 'y', 'z' are placeholders for 'a', 'e', 'i', 'o'):

first figure: for every X, if XyB then AzX
CyB
AzC

second figure: for every X, if XyB then XzA
CyB
CzA

third figure: for every X, if ByX then AzX
ByC
AzC

This corresponds to a classification of prosleptic propositions into three figures: prosleptic propositions belong to the same figure as the prosleptic syllogisms in which they can serve as a premiss. For example, prosleptic propositions of the form 'for every X, if XyB then AzX' belong to the first figure, and so on.[22] The prosleptic propositions used by Aristotle in chapters 2.5–7 and in other places of

[21] This kind of objection is raised by Ross (n. 1), 444.
[22] See *Σ* 189b44–8 Brandis; Anon. *CAG* 4.6 69.30–3; Lejewski (n. 10), 159–60.

the *Prior Analytics* are all in the third figure.[23] Aristotle does not, in his extant works, consider prosleptic propositions in the first or second figure.

Now, problems (ii), (iii) and (v) can be solved by taking the relevant statements in the problematic passage to refer to the third figure of prosleptic syllogisms. For example, consider problem (ii): the passage states, at 59a35, that the prosleptic circular proofs of the minor premiss of Celarent and Ferio are in the third figure, and both circular proofs are indeed prosleptic syllogisms in the third figure. The statement at 59a35 is followed by a reminder of the prosleptic premiss used in the circular proof of the minor premiss of Celarent: 'for it is assumed (*λαμβάνεται γάρ*) that whatever this belongs to none of, the other belongs to all of' (59a35–6). This reminder, introduced by the particle *γάρ*, shows that the author of the passage was aware that the circular proof under consideration is not a categorical but a prosleptic syllogism (whether or not he knew the expression 'prosleptic'). In fact, the author may have added the reminder precisely in order to make it clear that he is referring to the third figure, not of categorical syllogisms, but of prosleptic syllogisms. Thus, it is difficult to think that he made an incorrect reference to the third figure of categorical syllogisms at 59a35.

Problem (iii) can be solved in the same way, by taking the reference in question to be to the third figure of prosleptic syllogisms.[24] As to problem (v), the passage states that when the original syllogism is in the third figure, all circular proofs are also in the third figure (59a39). This statement covers two categorical circular proofs in the third figure, namely those of the major premiss of Disamis and Bocardo. At the same time, it covers a prosleptic circular proof in the third figure, namely that of the minor premiss of Ferison. Thus, one and the same general reference to the third figure at 59a39 includes both a reference to the third figure of categorical syllogisms and one to the third figure of prosleptic syllogisms. This might seem unusual and confusing, but it can be explained by the fact that there is a close connection between the three figures of categorical syllogisms and the three figures of prosleptic syllogisms and propositions.

The connection between them can be described as follows. A prosleptic proposition contains two categorical propositions, one in the if-clause and one in the then-clause. For example, a first figure prosleptic proposition of the form 'for every X, if XyB then AzX' contains the two categorical propositions AzX and XyB. These constitute the premiss pair of a categorical syllogism, with the categorical proposition in the then-clause being the major premiss. This categorical syllogism is in the same figure as the original prosleptic proposition. Likewise for the other two figures. Thus, the classification of prosleptic propositions and syllogisms into three figures appears to have been derived from Aristotle's three figures of categorical syllogisms.[25]

[23] See e.g. the prosleptic propositions at *An. pr.* 1.41 49a14–32, 1.46 51b41, 52a6, 52b18–19, 52b24, 2.2 53b20–1, 54a15, 2.21 66b40, 67a9–12, 2.22 67b29–31, 2.27 70b35–6.

[24] Some MSS read *καὶ διὰ τοῦ ἐσχάτου* after *σχήματος* at 2.7 59a38. If this reading is accepted, it can also be taken to refer to the third figure of prosleptic syllogisms, indicating prosleptic circular proofs of the a-premiss of Camestres and Cesare. Although Aristotle does not mention these two circular proofs, they appear to be possible (see nn. 10 and 12 above).

[25] W. and M. Kneale, 'Prosleptic propositions and arguments', in S. Stern et al. (edd.), *Islamic Philosophy and the Classical Tradition* (Oxford, 1972), 189–207, at 194; see also *Σ* 190a22–4 Brandis.

This is supported by an anonymous text entitled 'On prosleptic syllogisms', preserved as an appendix to Ammonius' commentary on the first book of the *Prior Analytics*. The text states that prosleptic syllogisms are similar to categorical ones in that they fall into 'the three figures':[26]

οὗτοι τοίνυν τῶν μὲν κατηγορικῶν ἔχουσι τὸ ἐν πᾶσι τοῖς σχήμασιν εἶναι.
(Anon. *CAG* 4.6 69.30)

These [i.e. prosleptic syllogisms] have in common with categorical syllogisms that they occur in all the figures.

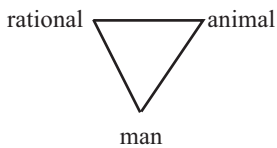
οἱ κατὰ πρόσληψιν κατηγορικοί εἰσιν ὡς γινόμενοι κατὰ τὰ τρία σχήματα.
(Anon. *CAG* 4.6 69.38)

Prosleptic syllogisms are categorical inasmuch as they come about according to the three figures.

Both passages suggest that prosleptic syllogisms belong to the same three figures as categorical ones. This view may be related to the use of diagrams as representations of syllogisms. We do not know what kind of diagrams Aristotle and the early Peripatetics used. But we do know that the later scholiasts used the same diagrams to represent categorical and prosleptic syllogisms in the third figure. For example, consider the following prosleptic syllogism, taken from the scholium 'On all the kinds of syllogism':

prosleptic premiss:	For every X, if animal belongs to all X then rational belongs to all X
categorical premiss:	Animal belongs to all man
conclusion:	Rational belongs to all man

Although its prosleptic premiss is obviously false, this is a valid prosleptic syllogism in the third figure.[27] The scholiast represents this syllogism by the following diagram:[28]



The same kind of diagram is typically used by scholiasts to represent categorical syllogisms in the third figure, such as the following syllogism in Darapti:

major premiss:	Rational belongs to all man
minor premiss:	Animal belongs to all man
conclusion:	Rational belongs to some animal

[26] For a somewhat similar statement, see Ps.-Philop. *In An. pr.* 417.13–15.

[27] Cf. Kneale and Kneale (n. 25), 206.

[28] *Σ CAG* 4.6 XII.9–10 Wallies. The same kind of diagram is also used by a scholiast in the tenth-century MS Vaticanus Barberinianus gr. 87 (102v) to represent the prosleptic syllogism in Aristotle's circular proof of the minor premiss of Ferio (*An. pr.* 2.5 58b7–12).

Given that the same diagrams are used to represent categorical and prosleptic syllogisms in the third figure, it seems tempting to assume that both kinds of syllogisms belong to the same third figure, that is, that there is a single third figure comprising both kinds of syllogisms. This assumption also seems to underlie our problematic passage from *Prior Analytics* 2.7; it explains why the passage does not distinguish between the third figure of categorical syllogisms and the third figure of prosleptic syllogisms, and why one and the same general reference to the third figure, at 59a39, covers both kinds of syllogisms.

We may now turn to the question of the authorship of the passage. The author takes certain prosleptic syllogisms to be in the third figure. He does not mention prosleptic syllogisms in the first or second figure; as noted above, such prosleptic propositions do not occur in the *Prior Analytics*. However, given the systematic connection between the three figures of prosleptic and categorical syllogisms, someone recognizing prosleptic syllogisms in the third figure is also likely to recognize prosleptic syllogisms in the first and second figure. Thus, the author of the passage was probably aware of the classification of prosleptic syllogisms into three figures.

It is not clear when and by whom the classification of prosleptic syllogisms into three figures was introduced. Lejewski and the Kneales suggest that it was introduced by Theophrastus.[29] Theophrastus studied prosleptic propositions in his treatise *On Assertion*, and argued that certain prosleptic propositions are equivalent to categorical ones.[30] As mentioned above (nn. 4 and 8), he also used the expressions 'prosleptic proposition' and 'prosleptic syllogism'. At the same time, Theophrastus divided another class of syllogisms, namely what are known as wholly hypothetical syllogisms, into three figures; this classification, too, is based on an analogy with Aristotle's figures of categorical syllogisms.[31] Thus, Theophrastus may also have divided prosleptic syllogisms into three figures.

In her paper 'Did Aristotle reply to Eudemus and Theophrastus on some logical issues?', P. Huby suggests that the answer to this question is affirmative.[32] She argues that Aristotle knew Eudemus' and Theophrastus' logical work, and responded to it in some passages of the *Prior Analytics*. If this is correct, Aristotle may also have been familiar with Theophrastus' work on prosleptic propositions and may have discussed it with him. Lejewski suggests that Theophrastus' work on prosleptic propositions was inspired and influenced by Aristotle, especially by his discussion of prosleptic propositions in *Prior Analytics* 1.41 and by his treatment of circular proof in 2.5–7.[33] It might even have been Aristotle who introduced the idea that prosleptic propositions and syllogisms can be classified into three figures. But even if this classification was introduced by Theophrastus, Aristotle could have been familiar with it, and could have appealed to it in *Prior Analytics* 2.7.

In conclusion, let us summarize our findings regarding the passage at 59a32–41 and its problematic references to the third figure. As we have seen, the problems

[29] Lejewski (n. 10), 167; Kneale and Kneale (n. 25), 205; Cz. Lejewski, 'On prosleptic premisses', *Notre Dame Journal of Formal Logic* 17 (1976), 1–18, at 1. However, P.M. Huby challenges this view; see her *Theophrastus of Eresus, Commentary, vol. 2, Logic* (Leiden, 2007), 133–4.

[30] Alex. Aphr. *In An. pr.* 378.18–20; *Σ* 190a1–4 Brandis.

[31] Alex. Aphr. *In An. pr.* 328.2–5; see S. Bobzien, 'Wholly hypothetical syllogisms', *Phronesis* 45 (2000), 87–137, at 104.

[32] In I. Bodnár and W.W. Fortenbaugh (edd.), *Eudemus of Rhodes* (New Brunswick, 2002), 85–106, at 90.

[33] See Lejewski (n. 10), 164–7.

with these references can be solved by taking them to refer to the third figure, not of categorical syllogisms, but of prosleptic syllogisms. This does not mean that the passage was not written by Aristotle. Although he does not mention figures of prosleptic syllogisms elsewhere in his writings, he may well have been familiar with them. His pupil and collaborator Theophrastus worked extensively on prosleptic propositions and syllogisms, and probably knew the classification of prosleptic syllogisms into three figures. So Aristotle, too, may have known it, and there is no reason to doubt that he was the author of the passage. If he was, then the passage is the earliest evidence we have of figures of prosleptic syllogisms. Of course, it is also conceivable that the passage was written by another Peripatetic author; but if I am correct, there is no positive reason to think so.

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