Aristotle on One-Sided Possibility
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The term ‘possible’, in Aristotle’s view, is ambiguous.\footnote{Prior Analytics 1.3 25a37–8.} It has two senses known as one-sided possibility and two-sided possibility (or contingency). Being two-sided possible means being neither impossible nor necessary, and being one-sided possible simply means being not impossible. Aristotle defines the two senses in Prior Analytics 1.13 (32a18–21). This definition is followed by a controversial passage which states several equivalences between modal expressions (32a21–8). It is often thought that this passage is spurious and should be excised even though it is found in all manuscripts.\footnote{Becker 1933: 11–14, Ross 1949: 327–8, Mignucci 1969: 112 and 295–6, Seel 1982: 163, Smith 1989: 18 and 125–6, Patterson 1995: 270, Huby 2002: 92, Striker 2009: 18 and 128–9.} I argue that the passage is not spurious, but contains a coherent argument justifying the one-sided sense of ‘possible’ (Section 1). This argument follows a pattern of proof explained by Aristotle elsewhere in the Prior Analytics and in the De caelo (Section 2). Moreover, it is closely related to a similar argument concerning one-sided possibility in De interpretatione 13 (Section 3). Examining this latter argument will help us better understand the force of Aristotle’s argument in Prior Analytics 1.13 (Section 4).

1. The problematic passage
At the beginning of Prior Analytics 1.13, Aristotle introduces the two senses of ‘possible’ as follows:

[i] I use the expressions ‘to be possible’ and ‘what is possible’ in application to something if it is not necessary but nothing impossible will result if it is put as being
the case; [ii] for it is only equivocally that we say that what is necessary is possible.3 (Pr. An. 1.13 32a18–21)

In point [i] of this passage, Aristotle characterizes two-sided possibility. Something is two-sided possible just in case it is not necessary and nothing impossible results from putting it as being the case. This characterization involves the term ‘impossible’. Unlike ‘possible’, ‘impossible’ is not ambiguous in the Prior Analytics but is consistently used in the sense of ‘not one-sided possible’ as opposed to ‘not two-sided possible’.4 Aristotle takes the condition that nothing impossible results from putting X as being the case to be equivalent to the condition that X is not impossible.5 Thus, he holds that something is two-sided possible just in case it is neither necessary nor impossible.

In point [ii] of the passage, Aristotle turns to one-sided possibility. He does not offer an explicit definition of one-sided possibility, but characterizes it by pointing out the specific case of application that distinguishes it from two-sided possibility. Thus, he states in point [ii] that whatever is necessary is one-sided possible. Accordingly, something is one-sided possible just in case it is not impossible. Two-sided possibility is, so to speak, bounded on two sides by

3 [i] λέγω δ’ ἐνδέχεσθαι καὶ τὸ ἐνδεχόμενον, οὗ μὴ ὄντος ἀναγκαίου, τεθέντος δ’ υπάρχειν, οὐδὲν ἔσται διὰ τούτ’ ἀδύνατον. [ii] τὸ γὰρ ἀναγκαῖον ὁμωνύμως ἐνδέχεσθαι λέγομεν.
4 In the De interpretatione, by contrast, ‘impossible’ (ἀδύνατον) is occasionally used in the sense of ‘not two-sided possible’ (see n. 38 below).
5 See Ross 1949: 327, Łukasiewicz 1957: 154–5, Ebert & Nortmann 2007: 470–1, and Beere 2009: 120–1. Aristotle holds that if something impossible results from putting X as being the case, then X is impossible (De caelo 1.12 281b14–15 and 23–5; Physics 7.1 243a1–2, Pr. An. 1.15 34a25–33; cf. Alexander in Pr. An. 157.7, Beere 2009: 121–2, Rosen & Malink 2012: 185–7, 196–200, and 210–11). Moreover, he is committed to the converse, that if X is impossible then something impossible results from putting X as being the case. Otherwise there would be an X which is impossible but satisfies Aristotle’s two conditions for being two-sided possible stated in point [i] (since nothing impossible is necessary; De int. 13 22b13–14). Consequently, something would be both impossible and two-sided possible, which is absurd (see De int. 13 22a15–17). This leaves open the question of whether, if X is impossible, X itself can be regarded as something impossible that results from putting X as being the case. Ebert & Nortmann (2007: 470–1) think that the answer is yes, whereas Beere (2009: 120 n. 4) is sceptical.
impossibility and necessity, whereas one-sided possibility is only bounded on one side by impossibility.

Aristotle often does not explicitly indicate whether a given occurrence of ‘possible’ is to be understood in the two-sided or one-sided sense, but in the Prior Analytics it is usually clear from the context which of them he means. In point [ii], the one-sided sense is qualified as being merely equivocal (ὁμωνύμως). This qualification indicates that the one-sided sense is not the preferred sense of ‘possible’ in the system of modal syllogisms expounded in Prior Analytics 1.13–22. Modal syllogisms concerning one-sided possibility play only a minor role in these chapters. Instead, Aristotle focuses on modal syllogisms concerning two-sided possibility. Accordingly, he refers to the characterization of two-sided possibility in point [i] as his official ‘definition’ (διορισμός) of possibility in the modal syllogistic.

Having introduced the two senses of ‘possible’ in points [i] and [ii], Aristotle proceeds as follows:

[iii] That this [viz., one-sided possibility] is what is possible is evident from opposed pairs of denials and affirmations. [iv] For ‘it is not possible to belong’ and ‘it is

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6 For this use of ὁμωνύμως, see Bonitz 1870: 514a49–61 and Striker 2009: 128.
7 Aristotle generally does not consider any modal syllogisms that contain a one-sided possibility premiss (the only exception is his use of a syllogism called Barbara MXM at 1.15 34b2–6; see Malink & Rosen 2013: 971–3). Aristotle discusses a number of modal syllogisms that have a one-sided possibility conclusion, but he does so only when the modal syllogism in question is established by reductio ad absurdum.
8 Thus, two-sided possibility is the preferred notion of possibility in the modal syllogistic in Prior Analytics 1.3 and 8–22. By contrast, one-sided possibility is preferred in the framework of propositional modal logic developed in Metaphysics Θ 4 and Prior Analytics 1.15. This framework is based on the principle that if B follows from A then the possibility of B follows from the possibility of A (Rosen & Malink 2012: 179–95). The principle is correct for one-sided possibility but not for two-sided possibility (Hintikka 1973: 59–60). Thus, the adjective ‘possible’ (δυνατόν) must be understood as expressing one-sided possibility in Metaphysics Θ 4 and Prior Analytics 1.15 (1047b14–30 and 34a5–33). On the other hand, the adjective is never used to indicate possibility in the premisses and conclusions of modal syllogisms in Prior Analytics 1.3 and 8–22 (instead, Aristotle uses the verbs ἐνδέχεσθαι and ἐγχωρεῖν; see Malink & Rosen 2013: 959–61).
9 Pr. An. 1.14 33b23, 1.15 33b28, 33b30, 34b27; see Malink 2013: 257 n. 14.
impossible to belong’ and ‘it is necessary not to belong’ are either the same or follow one another, [v] so that their opposites, ‘it is possible to belong’ and ‘it is not impossible to belong’ and ‘it is not necessary not to belong’, will also either be the same or follow one another; [vi] for either the affirmation or the denial holds of everything. [vii] Therefore, what is possible will not be necessary and what is not necessary will be possible.10 (Pr. An. 1.13 32a21–9)

This passage has been the subject of controversy. The main question is whether the pronoun ‘this’ in point [iii] refers to two-sided or one-sided possibility. Since Philoponus, most commentators have taken it to refer to two-sided possibility.11 Thus, they take points [iii]–[vii] to justify the notion of two-sided possibility introduced in point [i]. However, this is problematic because Aristotle’s claims in point [v] are true for one-sided but not for two-sided possibility (e.g., the claim that ‘it is possible to belong’ is equivalent to ‘it is not necessary not to belong’). Because of this problem, it is often thought that points [iii]–[vii] are spurious and should be excised even though they are found in all manuscripts (see n. 2 above).

On the other hand, some have argued that the passage can be retained if the pronoun ‘this’ in point [iii] is taken to refer to one-sided possibility.12 On this interpretation, points [iii]–[vi] are intended to explain the notion of one-sided possibility introduced in [ii]. Aristotle does so by establishing, in point [v], a number of theses characteristic of one-sided but not two-sided possibility. By contrast, point [vii] is again about two-sided possibility,
drawing a consequence from the initial characterization of two-sided possibility in [i]. Thus, points [i] and [vii] deal with Aristotle’s preferred notion of possibility, whereas [ii]–[vi] constitute a parenthetical argument introducing and justifying the secondary notion of one-sided possibility.

On this interpretation, there is no reason to think that points [ii]–[vi] are spurious, even if they are parenthetical. In what follows, I elaborate and vindicate this interpretation by showing how Aristotle’s argument in [iii]–[vi] succeeds in establishing the one-sided sense of ‘possible’, and how it is related to a similar argument concerning one-sided possibility in De interpretatione 13.

2. Establishing one-sided possibility in Prior Analytics 1.13 (32a21–8)

As Aristotle indicates in point [iii], his argument for one-sided possibility appeals to opposed pairs of denials and affirmations. The denials he has in mind are linguistic expressions such as ‘it is not possible to belong’; the corresponding affirmations are expressions such as ‘it is possible to belong’. Aristotle considers the following six expressions in points [iv] and [v]:

- it is not possible to belong
- it is impossible to belong
- it is necessary not to belong
- it is possible to belong
- it is not impossible to belong
- it is not necessary not to belong

Table 1: Opposed pairs of affirmations and denials at Prior Analytics 1.13, 32a21–8

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14 τὸ (οὐκ) ἐνδέχεται ὑπάρχειν. The combination of the definite article τὸ with the finite verb ἐνδέχεται in points [iv] and [v] indicates that Aristotle is concerned with linguistic expressions rather than with non-linguistic items signified by expressions; cf. Categories 10 12b5–16.
Each row of this table contains an opposed pair of affirmation and denial. The denials in the left-hand column are the contradictory opposites of the corresponding affirmations in the right-hand column, and vice versa. Aristotle does not simply assume that these expressions are contradictory opposites, but argues for this in detail in De interpretatione 12.\textsuperscript{15} In earlier chapters of the De interpretatione, affirmations and denials are defined as complete declarative sentences affirming or denying something of something.\textsuperscript{16} In De interpretatione 12–13 and in points [iii]–[vi] above, however, Aristotle uses the terms ‘affirmation’ and ‘denial’ to refer not to complete sentences but to incomplete expressions that can be completed into sentences.\textsuperscript{17} Thus, the six expressions listed above are sentential fragments that can be completed into sentences of the form ‘It is possible for A to belong to B’ and ‘It is not possible for A to belong to B’ (or, in other words, ‘It is possible for B to be A’ and ‘It is not possible for B to be A’). In what follows, I use ‘affirmation’ and ‘denial’ to refer to such incomplete expressions rather than to complete sentences.

The affirmations and denials listed in Table 1 can be regarded as sentential forms such as ‘It is possible for . . . to belong to . . .’ and ‘It is not possible for . . . to belong to . . .’. Each of them has two free slots. Let X be a pair of expressions such that the result of filling the two slots with these expressions is a well-formed sentence. If the resulting sentence is true, let us say that the affirmation or denial in question holds of X. For example, if X is the pair of expressions ‘walking’ and ‘Socrates’, then the affirmation ‘it is possible to belong’ holds of X, whereas the corresponding denial does not hold of X (since it is possible for Socrates to be walking).

In point [vi], Aristotle states a version of the law of excluded middle (LEM), according to which ‘either the affirmation or the denial holds of everything’. The same version is stated in De interpretatione 12 (21b4).\textsuperscript{18} This version of LEM seems to presuppose that the

\textsuperscript{15} 21b23–6 and 21b34–22a13.
\textsuperscript{16} De int. 4 17a2–3, 5 17a8–9, 6 17a25–6; see Crivelli 2004: 86–9.
\textsuperscript{18} Similarly, Pr. An. 1.46 51b32–3, 2.11 62a13–14, Topics 6.6 143b15–16.
affirmations and denials in question are not complete sentences, but incomplete expressions that are capable of holding of something. In *De interpretatione* 12, Aristotle adds a corresponding version of the principle of non-contradiction (PNC), according to which ‘it is impossible for opposite expressions (τὰς ἀντικειμένας φάσεις) to be true of the same thing’ (21b17–18). Thus, each of the contradictory pairs listed in Table 1 satisfies the following two principles:

Where X is a pair of expressions as described above:

- **PNC:** Not both the affirmation and the denial hold of X
- **LEM:** Either the affirmation or the denial holds of X

As we will see shortly, both PNC and LEM play an important role in Aristotle’s argument in points [iii]–[vi].

In point [iv], Aristotle states that the three expressions on the left-hand side of Table 1 are ‘the same or follow one another (ἀκολούθει ἀλλήλοις)’. By this he means that the expressions are mutually equivalent. In the *Prior Analytics*, Aristotle uses the verb ‘follow’ (ἀκολούθειν or ἕπεσθαι) to express that a term is predicated universally of another term in the sense that the former term holds of everything of which the latter holds.19 For example, when Aristotle says that animal follows man, he means that ‘animal’ holds of everything of which ‘man’ holds (*Pr. An.* 1.27 43b31). Correspondingly, A and B follow one another just in case they are equivalent in the sense that, for any X, A holds of X just in case B holds of X.20 In point [iv], Aristotle states that the three expressions on the left-hand side of Table 1 are mutually equivalent in this sense. He does not argue for these equivalences but seems to take them for granted (see Section 4 below).

20 For this use of ‘follow one another’ (ἀκολούθει ἀλλήλοις), see *Metaph.* Γ 2 1003b22–5 and *De caelo* 1.12 282a30–b2; similarly, *De int.* 12 21b35–6.
Aristotle goes on, in point [v], to infer that the three expressions on the right-hand side of Table 1 are mutually equivalent. He infers their equivalence from that of their contradictory opposites on the left-hand side by appealing to LEM in point [vi]. Aristotle does not spell out the details of this inference in *Prior Analytics* 1.13. However, he provides an account of the inference elsewhere, in chapters 1.46 and 2.22 of the *Prior Analytics* and in *De caelo* 1.12. Let us consider these accounts in turn, beginning with the one in chapter 1.46:

Whenever A and B are so related that it is not possible for them to belong to the same thing at the same time, but necessarily one or the other of them belongs to everything, and again C and D are related in the same way, and A follows C, . . . then D will follow B. . . . For, since necessarily one or the other of C and D belongs to everything, but it is not possible for C to belong to anything to which B belongs, because C brings along with it A, and it is not possible for A and B to belong to the same thing, it is evident that D will follow B. (*Pr. An.* 1.46 52a39–b8)

In this passage, Aristotle considers four terms A, B, C, and D. He assumes that A and B do not both hold of the same thing (Premiss 1), but that one or the other of them holds of everything (Premiss 2). The same is true for C and D (Premisses 3 and 4). Moreover, Aristotle assumes

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21 These equivalences hold for one-sided possibility but not for two-sided possibility. Accordingly, some commentators who take points [i]–[vi] to be about two-sided possibility suggest that Aristotle does not state any equivalences between modal expressions in [iv] and [v] (Hintikka 1973: 33, Nortmann 1996: 162–6, Ebert & Nortmann 2007: 474–5; cf. n. 11 above). However, once [ii]–[vi] are taken to be about one-sided possibility, there is no reason to deny that Aristotle states these equivalences in [iv] and [v].

22 In points [iv] and [v], Aristotle states that the expressions in each column of Table 1 ‘are either the same or follow one another’. If they are taken to be the same (or to signify the same), Aristotle’s inference is underwritten by the principle that ‘if things are the same, their opposites also will be the same, in any of the recognized forms of opposition; for since they are the same, there is no difference between taking the opposite of the one and that of the other’ (*Topics* 7.1 151b33–6). Aristotle’s account in *Prior Analytics* 1.46, 2.22, and *De caelo* 1.12 justifies the inference on the weaker assumption that the expressions are not the same (or do not signify the same) but are equivalent in the sense that they follow one another.
that A follows C, that is, that A holds of everything of which C holds (Premiss 5). He concludes that D follows B:

Premiss 1: For any X, not both A and B hold of X
Premiss 2: For any X, either A or B holds of X
Premiss 3: For any X, not both C and D hold of X
Premiss 4: For any X, either C or D holds of X
Premiss 5: For any X, if C holds of X then A holds of X
Conclusion: For any X, if B holds of X then D holds of X

This argument is valid. Its conclusion follows logically from Premisses 1, 4, and 5. It is worth noting that Premisses 2 and 3 are dispensable in the present argument; but they are required for the corresponding argument inferring the converse of the conclusion from the converse of Premiss 5. Although Aristotle does not mention this converse argument in *Prior Analytics* 1.46, it is a straightforward version of the one he presents in the passage just quoted. Taken together, the two arguments establish that, given Premisses 1–4 and given that A and C follow one another, it may be inferred that B and D follow one another:

Premiss 1: For any X, not both A and B hold of X
Premiss 2: For any X, either A or B holds of X
Premiss 3: For any X, not both C and D hold of X
Premiss 4: For any X, either C or D holds of X
Premiss 5*: For any X, A holds of X if and only if C holds of X
Conclusion*: For any X, B holds of X if and only if D holds of X

Aristotle justifies the inference from Premisses 1–5* to Conclusion* in *De caelo* 1.12 as follows:

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23 In this converse argument, the conclusion ‘For any X, if D holds of X then B holds of X’ is inferred from Premisses 2–3 and the converse of Premiss 5 (i.e., ‘For any X, if A holds of X then C holds of X’).
It has been shown that A and C follow one another. When terms stand to one another as these do, that is, when A and C follow [one another], B and A never belong to the same thing, but one or the other of them belongs to everything, and D and C likewise, then it is necessary that B and D follow one another. For suppose that B does not follow D. Then A will follow, since either B or A belongs to everything. But C belongs to [everything] to which A belongs. Consequently, C will follow D; but it was assumed that this is impossible. And the same argument shows that D follows B. (De caelo 1.12 282b25–283a1)

This argument proceeds by reductio ad absurdum. The assumption for reductio is that B does not follow D. In other words, the assumption is that it is not the case that, for any X, if D holds of X then B holds of X. This means that there is something – call it Y – such that D but not B holds of Y. Aristotle proceeds to draw a consequence from the assumption for reductio. He describes this consequence by the phrase ‘A will follow’ (τὸ ἄρα Α ἀκολουθήσει). Contrary to what might be thought, this does not mean that A will follow D; for this would not be warranted by the assumption for reductio. Instead, Aristotle’s intended meaning is that A will hold of Y, of which D holds. Similarly, Aristotle concludes the reductio by claiming that ‘C will follow D’ (τῷ ἄρα Δ τὸ Γ ἀκολουθήσει). Again, this should not be taken to mean that C holds of everything of which D holds. Rather, Aristotle uses the phrase ‘C will follow D’ to express that C holds of something, namely Y, of which D holds. By contrast, the phrase ‘D follows B’ in the last sentence of the passage just quoted expresses the claim that D holds of everything of which B holds. Thus, Aristotle’s use of the verb ‘follow’ in the reductio at 282b30–2 is potentially misleading (as we will see, the same misleading use of ‘follow’ occurs in De interpretatione 13). Nevertheless, his argument is a lucid proof, in two halves, of the validity of the inference from Premisses 1–5* to Conclusion*.

Aristotle presents the same proof in *Prior Analytics* 2.22 without the misleading use of ‘follow’:

If either A or B and either C or D belong to everything, but they cannot belong at the same time, then, if A and C convert, B and D also convert. For if B does not belong to something to which D belongs, it is clear that A belongs to it; and if A belongs, then also C does; for they convert. Consequently, C and D belong to it together; but this is impossible. (*Pr. An.* 2.22 68a11–16)

In this version of the argument, Aristotle uses the verb ‘convert’ instead of the phrase ‘follow one another’. He avoids the misleading use of ‘follow’ in the *reductio* by writing ‘A belongs to it’ instead of ‘A will follow’, and writing ‘C and D belong to it together’ instead of ‘C will follow D’.

In sum, then, Aristotle has demonstrated that Premises 1–5* entail Conclusion*. In *De caelo* 1.12, he uses this pattern of argument to establish the equivalence of the terms ‘imperishable’ and ‘ungenerable’ based on the equivalence of their contradictory opposites by interpreting the schematic letters as follows:25

A generable
B ungenerable
C perishable
D imperishable

In *Prior Analytics* 1.13, he uses the same pattern of argument in point [v] to establish the equivalence of modal affirmations and denials. In this application of the pattern, the quantified variable ‘X’ ranges over suitable pairs of expressions that yield complete sentences when combined with incomplete affirmations and denials, as explained above. The schematic letters are interpreted as follows:

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In this interpretation of the schematic letters, A is the contradictory opposite of B, and C is the contradictory opposite of D. Consequently, Premisses 1–4 of the inference are guaranteed by LEM and PNC, respectively. Thus, Aristotle’s appeal to LEM in point [vi] of his argument is a statement of Premisses 2 and 4 of the inference. Aristotle does not explicitly assert PNC in Prior Analytics 1.13, but takes it for granted since he regards it as the firmest of all principles.

3. Establishing one-sided possibility in De interpretatione 13 (22b11–14)

In De interpretatione 13, Aristotle discusses various implications (ἀκολουθήσεις, 22a14) between modal expressions. He presents these implications by means of the following square:26

<table>
<thead>
<tr>
<th>Possible to be</th>
<th>Not possible to be</th>
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<tbody>
<tr>
<td>Not impossible to be</td>
<td>Impossible to be</td>
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<tr>
<td>Not necessary to be</td>
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<table>
<thead>
<tr>
<th>Possible not to be</th>
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<tr>
<td>Not impossible not to be</td>
<td>Impossible not to be</td>
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<tr>
<td>Not necessary not to be</td>
<td>Necessary to be</td>
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26 Aristotle’s square contains separate entries for two distinct expressions of possibility: δυνατὸν εἶναι and ἐνδεχόμενον εἶναι (22a24–31). Aristotle takes these two expressions to be equivalent in De interpretatione 13 (22a15–16; see Ammonius in De int. 231.9–16, 237.14–15, Seel 1982: 149, Weidemann 2002: 424–5); thus it suffices for our purposes to consider only one expression of possibility.
For the sake of brevity, I shall write ‘necessary’ as shorthand for ‘necessary to be’, ‘possible not’ as shorthand for ‘possible not to be’, and so on. Aristotle asserts that the first expression in each quadrant of the square implies the second and third expressions (22a14–23). For example, ‘possible’ implies ‘not impossible’ in the sense that, for any X, if the former holds of X then the latter holds of X. Likewise, ‘possible’ implies ‘not necessary’, and so on. Aristotle does not assert that the expressions in each quadrant are mutually equivalent, nor does he assert that the second or third expression implies the first expression. Rather, his focus is on determining what follows from the first expression in each quadrant.

Unfortunately, as Aristotle goes on to explain at 22b10–28, the above square turns out to be incoherent. The incoherence is best described as being due to the ambiguity of ‘possible’ between the two-sided and the one-sided sense. Although Aristotle does not explicitly distinguish between these two senses in *De interpretatione* 13, the distinction underlies his discussion of the incoherence. In the left-hand column of the square, ‘possible (not)’ is taken to imply ‘not necessary (not)’. This implication requires that ‘possible’ be understood not in the one-sided sense but in the two-sided sense. In the right-hand column, by contrast, ‘not possible’ is taken to imply ‘necessary not’, and ‘not possible not’ is taken to imply ‘necessary’. These implications require that the occurrences of ‘possible’ in these phrases be understood not in the two-sided but in the one-sided sense. For if ‘possible’ is understood in the two-sided sense, then ‘not possible’ does not imply ‘necessary not’ but only the disjunctive phrase

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‘necessary or necessary not’. Thus, ‘possible’ expresses two-sided possibility in the left-hand column of the square, but one-sided possibility in the right-hand column.

This does not mean that Aristotle’s use of ‘possible’ in the square is random or haphazard. There is a systematic difference between the two columns that helps to explain the distribution of one-sided and two-sided uses of ‘possible’: in the left-hand column ‘possible’ occurs unnegated in affirmations, and in the right-hand column it occurs negated in denials. When introducing the square at the beginning of chapter 13, Aristotle states the implications in question without argument (22a14–23). Thus he takes it for granted that ‘possible’ expresses two-sided possibility when it occurs unnegated in affirmations, whereas it expresses one-sided possibility when it occurs negated in denials. He does not, at this point of chapter 13, recognize any unnegated one-sided uses of ‘possible’, nor any negated two-sided uses of ‘possible’.

The double use of ‘possible’ threatens not only the coherence of Aristotle’s square, but also the main results of chapter 12. There, Aristotle argues at length that ‘possible’ and ‘not possible’ constitute an opposed pair of affirmation and denial. He requires that such opposed pairs satisfy LEM, according to which ‘the affirmation or the negation holds of everything’. However, Aristotle’s square in Table 2 violates LEM. For example, consider the affirmative sentence ‘It is possible for every man to be an animal’ and the corresponding negative sentence ‘It is not possible for every man to be an animal’. LEM requires that at least one of them be true. But according to Aristotle’s square, ‘possible’ is to be understood as expressing two-sided possibility in the affirmative sentence, and as expressing one-sided possibility in the negative sentence. As a result, LEM is violated because neither sentence is true (since it is necessary for every man to be an animal).

31 De int. 12 21b23–5, 21b37–22a1, 22a11–13.
32 De int. 12 21b4; see n. 18 above.
Due to the homonymy of ‘possible’, the occurrence of ‘possible’ and ‘not possible’ in Aristotle’s square fail to be a genuine opposed pair of affirmation and denial.33 For in a genuine opposed pair the same thing must be denied and affirmed of the same thing not homonymously:

Let us call an affirmation and a denial which are opposite a contradiction (ἀντίφασις). I speak of sentences as opposite when they affirm and deny the same thing of the same thing – not homonymously, together with all other such conditions that we add to counter the troublesome objections of sophists. (De int. 6 17a33–7)

According to this definition, ‘not possible’ fails to be the contradictory opposite of ‘possible’ in Aristotle’s square. On the other hand, Aristotle holds that for every affirmation there is a corresponding denial which is its contradictory opposite, and that for every denial there is a corresponding affirmation which is its contradictory opposite (17a30–3). Hence, if ‘possible’ has the two-sided sense in a given affirmation, there must be a corresponding denial in which it has the same sense. And if ‘possible’ has the one-sided sense in a given denial, there must be a corresponding affirmation in which it has the same sense. Given this, Aristotle is in a position to construct a revised square in which ‘possible’ has the same sense throughout, and which therefore does not suffer from the problems described above.

In De interpretatione 13, Aristotle chooses to construct a revised square for one-sided possibility (he does not give a square for two-sided possibility). As a first step toward this revised square, Aristotle argues that ‘necessary’ implies ‘possible’, as follows:

[i] What is necessary to be is possible to be. [ii] For otherwise the denial will follow, [iii] since it is necessary either to affirm or to deny; [iv] and then, if it is not possible to

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be, it is impossible to be; [v] so what is necessary to be is impossible to be – [vi] which is absurd.\(^{34}\) (*De int.* 13 22b11–14)

In point [i] of this passage, Aristotle states the thesis to be proved, that everything necessary is possible: for any X, if ‘necessary’ holds of X then ‘possible’ holds of X. Aristotle’s proof of this thesis proceeds by *reductio*. The assumption for *reductio* is that there is something – call it Y – such that ‘necessary’ holds of Y but ‘possible’ does not hold of Y.

In point [ii], Aristotle draws a consequence from the assumption for *reductio*. He indicates this consequence by the phrase ‘the denial will follow’ (ἡ ἀπόφασις ἀκολουθήσει). Contrary to what is sometimes thought, this is not intended to mean that ‘not possible’ follows ‘necessary’ in the sense that the former holds of *everything* of which the latter holds; for this is not warranted by the assumption for *reductio*.\(^{35}\) Rather, point [ii] exhibits the same slightly misleading use of ‘follow’ that we saw in *De caelo* 1.12 (282b30–2). When Aristotle writes that ‘the denial will follow’, his intended meaning is that ‘not possible’ holds of something, namely Y, of which ‘necessary’ holds.\(^{36}\) Aristotle justifies this claim by appealing to LEM in point [iii]: since, according to the assumption of *reductio*, ‘possible’ does not hold of Y, it follows by LEM that ‘not possible’ holds of it.

In point [iv], Aristotle goes on to infer that, since ‘not possible’ holds of Y, so does ‘impossible’. This is justified by one of the implications stated in Aristotle’s original square (in the top right-hand quadrant, 22a19–20). Finally, Aristotle infers in point [v] that ‘what is necessary to be is impossible to be’. Again, this does not mean that ‘impossible’ holds of


\(^{35}\) *Pace* Ackrill (1963: 152), who holds that when Aristotle ‘claims that ‘necessary’ must imply ‘possible’ since otherwise it would have to imply ‘not possible’ he is, of course, misusing the principle of excluded middle’. Similarly, Fitting & Mendelsohn (1998: 34) think that in point [ii] ‘Aristotle confused the negation of the conditional with the negation of the consequent’.

\(^{36}\) Weidemann 2002: 437–41. The same use of ‘follow’ is found at 22b30, where Aristotle recapitulates point [ii] of the present argument.
everything of which ‘necessary’ holds. Rather, it means that ‘impossible’ holds of something, namely Y, of which ‘necessary’ holds. ³⁷ In point [vi], Aristotle states that this last consequence is absurd, thereby concluding the reductio.

Aristotle’s argument in points [i]–[vi] can thus be reconstructed as follows:

1. There is an X such that ‘necessary’ holds of X and ‘possible’ does not hold of X [assumption for reductio]
2. ‘Necessary’ holds of Y and ‘possible’ does not hold of Y [from 1: existential instantiation]
3. ‘Necessary’ holds of Y [from 2]
4. ‘Possible’ does not hold of Y [from 2]
5. Either ‘possible’ holds of Y or ‘not possible’ holds of Y [premiss: LEM]
6. ‘Not possible’ holds of Y [from 4, 5]
7. For any X, if ‘not possible’ holds of X then ‘impossible’ holds of X [premiss: 22a19–20]
8. ‘Impossible’ holds of Y [from 6, 7]
9. There is an X such that ‘necessary’ holds of X and ‘impossible’ holds of X [from 3, 8]
10. Not: there is an X such that ‘necessary’ holds of X and ‘impossible’ holds of X [premiss]
11. Not: there is an X such that ‘necessary’ holds of X and ‘possible’ does not hold of X [reductio: 1–9, 10]
12. For any X, if ‘necessary’ holds of X then ‘possible’ holds of X [from 11]

³⁷ The use of the phrase ‘what is necessary to be’ (τὸ ἀναγκαῖον εἶναι) in [v] differs from its use in [i]. In [i], this phrase indicates a general term true of everything of which ‘necessary to be’ holds. In [v], by contrast, the phrase picks out a particular item, Y, of which ‘necessary to be’ holds (Weidemann 2012: 441).
This argument relies on three premisses. First, it relies on LEM (line 5, stated in [iii]). The second premiss is that ‘not possible’ implies ‘impossible’ (line 7, stated in [iv]). The third premiss is that ‘impossible’ and ‘necessary’ are disjoint in that they do not hold of the same thing (line 10, stated in [vi]). This last premiss makes it clear that ‘impossible’ is taken to mean ‘not one-sided possible’ rather than ‘not two-sided possible’. This is a substantive assumption because ‘impossible’ is also occasionally used in the latter sense in *De interpretatione* 13.\(^{38}\) It follows that ‘not possible’, since it implies ‘impossible’, is disjoint from ‘necessary’. This means that the negated occurrence of ‘possible’ in ‘not possible’ does not have the two-sided sense but the one-sided sense; for if it had the two-sided sense, ‘not possible’ would mean ‘not two-sided possible’ and would fail to be disjoint from ‘necessary’ (in fact, it would follow from it). Thus, the second and third premisses taken together entail that a *negated* occurrence of ‘possible’ has the one-sided sense.

Based on this, Aristotle’s argument establishes, by means of LEM, that an *unnegated* occurrence of ‘possible’ has the one-sided sense. For if the argument’s conclusion in line 12 is to be true, the unnegated occurrence of ‘possible’ in it cannot have the two-sided sense but must have the one-sided sense.\(^{39}\) This is the first unnegated occurrence of ‘possible’ that is used in the one-sided sense in *De interpretatione* 12 and 13. Prior to Aristotle’s argument at 22b11–14, all unnegated occurrences of ‘possible’ in these chapters are understood in the two-sided sense.\(^{40}\) Aristotle’s argument shows that unnegated occurrences of ‘possible’ can have the one-sided sense too.

4. The revised square of modal expressions (*De interpretatione* 13 22b10–28)

Having shown that ‘necessary’ implies ‘possible’, Aristotle goes on to argue that the left-hand column of his original square is incorrect and in need of revision. In particular, he argues that, contrary to what he said at the beginning of chapter 13, ‘possible’ does not imply ‘not

\(^{38}\) For example, ‘impossible’ presumably means ‘not two-sided possible’ in Aristotle’s claim that ‘not impossible’ implies ‘not necessary’ (22b15–16); see Seel 1982: 161–3, Weidemann 2002: 443.

\(^{39}\) Provided that ‘necessary’ holds of something; otherwise the conclusion in line 12 would be vacuously true.

\(^{40}\) See *De int.* 12 21b12–17, 21b35–7, 13 22a15–31; cf. Seel 1982: 160.
necessary’. For otherwise – since ‘necessary’ implies ‘possible’ – it would follow that ‘necessary’ implies ‘not necessary’, which is absurd (22b14–17). Nor does ‘possible’ imply ‘necessary’ or ‘necessary not’ (b17–22). From this Aristotle concludes that the only expression concerning necessity that can be implied by ‘possible’ is ‘not necessary not’ (b22–8).

Thus, Aristotle revises his original square by transposing ‘not necessary’ and ‘not necessary not’ in the left-hand column:

<table>
<thead>
<tr>
<th>possible to be</th>
<th>not possible to be</th>
</tr>
</thead>
<tbody>
<tr>
<td>not impossible to be</td>
<td>impossible to be</td>
</tr>
<tr>
<td>not necessary not to be</td>
<td>necessary not to be</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>possible not to be</th>
<th>not possible not to be</th>
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<tbody>
<tr>
<td>not impossible not to be</td>
<td>impossible not to be</td>
</tr>
<tr>
<td>not necessary to be</td>
<td>necessary to be</td>
</tr>
</tbody>
</table>

Table 3: Revised square of modal expressions (De interpretatione 13 22b10–28)

In this revised square, ‘possible’ is not ambiguous but is consistently used in the one-sided sense throughout. Accordingly, ‘impossible’ means ‘not one-sided possible’. As before, Aristotle holds that the first expression in each quadrant implies the second and third expressions.

The top half of the revised square corresponds exactly to the two groups of modal expressions discussed by Aristotle in his argument for one-sided possibility in Prior Analytics 1.13 (see Table 1). This argument can easily be adapted to capture the bottom half of the revised square. Thus, Aristotle’s discussion in Prior Analytics 1.13 is closely connected to the revised square in De interpretatione 13. Nevertheless, there are differences between the two chapters. In De interpretatione 13, Aristotle does not give a full direct proof showing that ‘possible’ implies ‘not necessary not’. Instead, he argues that ‘possible not’ does not imply any of the other three expressions concerning necessity in the square. From this he infers that it
implies ‘not necessary not’. However, since he has not shown that it must imply at least one of the four expressions, his argument remains incomplete. In Prior Analytics 1.13, by contrast, Aristotle does give a full proof that ‘possible’ implies ‘not necessary not’. In fact, he proves that the two expressions are equivalent.

In De interpretatione 13, Aristotle states that ‘not possible’ implies ‘necessary not’ (in the top right-hand quadrant of the square). By contraposition, it follows that ‘not necessary not’ implies ‘possible’, and hence that the two expressions are equivalent. Crucially, however, Aristotle does not state the latter implication nor does he perform such inferences by contraposition in De interpretatione 13. Although he is committed to the equivalence of the three expressions in each quadrant of the revised square, Aristotle does not explicitly state these equivalences in De interpretatione 13. Thus, the argument in Prior Analytics 1.13 is not only more concise and straightforward than the one in De interpretatione 13; it is also more powerful in that it establishes the equivalence of the three expressions in each quadrant of the square. In particular, it establishes the two equivalences that guarantee the interdefinability of one-sided possibility and necessity, namely, the equivalence of ‘possible’ and ‘not necessary not’, and – provided that the argument is adapted to the bottom half of the square – the equivalence of ‘necessary’ and ‘not possible not’.

Despite these differences, the arguments in Prior Analytics 1.13 and De interpretatione 13 share important similarities. First, they are both of an extensional nature in that they prove implications (or equivalences) between modal expressions by showing that the one expression holds of everything of which the other holds (and vice versa). Moreover, they both aim to establish the one-sided sense of unnegated occurrences of ‘possible’ by appealing to the one-sided sense of negated occurrences of ‘possible’. They do so by invoking the law of excluded middle (LEM). Among other things, this law encodes Aristotle’s commitment to the principle that for every denial in which something is denied of something there is a corresponding affirmation in which the same thing is affirmed of the same thing not homonymously. It

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follows from this principle that for every negated occurrence of ‘possible’ in a denial there is a corresponding unnegated occurrence in an affirmation such that both occurrences have the same sense. If the negated occurrences has the one-sided sense, the unnegated occurrence must have the one-sided sense too.

Aristotle’s strategy in both arguments shows that, in his view, negated occurrences of ‘possible’ are most naturally interpreted in the one-sided sense, whereas unnegated occurrences are more naturally interpreted in the two-sided sense. For Aristotle, one-sided readings of unnegated occurrences are somewhat artificial and stand in need of justification. Aristotle does not explain why this is so. Contemporary theorists have provided an explanation on his behalf by appealing to the Gricean concept of conversational implicature. These theorists disagree with Aristotle that there are two distinct senses of ‘possible’. Instead, they argue that there is only the one-sided sense, and that the two sided interpretation is a pragmatic phenomenon resulting from a conversational implicature. For example, Lauri Karttunen writes:

Aristotle distinguished between one-sided possibility and two-sided possibility. . . . However, I doubt that there is any need to postulate these two distinct senses. As far as I understand it, Aristotle’s distinction is designed to account for the same facts that are also covered by Grice’s conversational postulates. Assuming that the speaker is following the cooperative principle by saying that something is possible, he indicates that he is not in the position to make a stronger statement [to the effect that that thing is necessary]. . . . Therefore, for all he knows, the contrary is also possible. The two-sided interpretation of possible arises from these considerations; it is not part of the meaning of possible.42 (Karttunen 1972: 6 n. 2)

One of Grice’s postulates governing cooperative communication is the requirement that the speaker is to provide the addressee with the strongest relevant information available. Thus, if the speaker asserts that something is one-sided possible she must have reasons to withhold

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from the stronger claim that it is necessary. On the assumption that the speaker knows whether or not the stronger claim is true, her assertion of the weaker claim gives rise to a conversational implicature to the effect that the stronger claim is false.\(^\text{43}\) This helps explain why the two-sided interpretation of unnegated occurrences of ‘possible’ is usually preferred over the one-sided interpretation in ordinary language. It may also help explain why Aristotle felt the need to include an argument justifying the one-sided sense of unnegated ‘possible’ in Prior Analytics 1.13.

On the other hand, negation in ordinary language typically operates exclusively on the meaning of expressions but not on conversational implicatures that might be associated with them on a given occasion of use.\(^\text{44}\) Thus, if a negation is applied to ‘possible’, it only negates its one-sided meaning but not the implicature that gives rise to its two-sided interpretation. Consequently, ‘not possible’ is usually understood as ‘not one-sided possible’ rather than ‘not two-sided possible’ in ordinary language. This is to say, negated occurrences of ‘possible’ are usually interpreted as expressing one-sided possibility in ordinary language. In the Prior

\(^{43}\) For the same reason, an assertion of a particular affirmative sentence such as ‘Some As are B’ gives rise to a conversational implicature to the effect that the corresponding universal sentence ‘All As are B’ is false. Accordingly, particular affirmative sentences can be used in a two-sided sense (‘Some but not all As are B’) and in a one-sided sense (‘Some, perhaps all, As are B’). In Prior Analytics 1.1–22, particular sentences are used only in the one-sided sense, whereas ‘possible’ is frequently used in the two-sided sense (see Horn 1989: 209–10). In the second book of the Prior Analytics, however, particular affirmative sentences are sometimes used in the two-sided sense, when Aristotle takes ‘Some As are B’ to imply the particular negative sentence ‘Some As are not B’ (Pr. An. 2.2 55a15, 2.3 56a15; see Waitz 1844: 490–1, Ross 1949: 431 and 433, Smith 1989: 188). Even in chapters 1.1–22, Aristotle is trying to avoid examples in which a particular sentence is true together with the corresponding universal sentence (Pr. An. 1.4 26a39–b10; see Patzig 1968: 177–8, Brunschwig 1969: 14–15, Ebert & Nortmann 2007: 304–5). When he must use such examples, he finds it necessary to explain this fact by pointing out that particular sentences are to be understood in the one-sided sense rather than in the two-sided sense (1.4 26b14–20, 1.5 27b20–23, 1.6 28b28–30), just as he finds it necessary to explain the one-sided sense of ‘possible’ in chapter 1.13. It is sometimes thought that Aristotle connects the one-sided use of ‘possible’ to the one-sided use of particular sentences at De interpretatione 13 23a16–18 (Seel 1982: 167–9, Whitaker 1996: 169; pace Pacius 1597b: 104, Weidemann 2002: 455). Whatever the correct interpretation of 23a16–18, the parallel between the one-sided uses of ‘possible’ and particular sentences is an apt one.

\(^{44}\) See, e.g., Horn 1989: 362–89.
Aristotle sometimes uses the phrase ‘not possible’ in the sense of ‘not two-sided possible’. However, these uses are restricted to special contexts in which Aristotle explicitly indicates that the kind of possibility that is being negated is two-sided possibility. For example, he often says that ‘what is necessary was not possible’ in chapters 1.14–19 of the Prior Analytics.\(^4\) By using the past tense form ‘was’, Aristotle indicates that he is referring to his official definition of two-sided possibility as the preferred notion of possibility introduced in chapter 1.13 (32a18–20).\(^5\)

Absent such explicit markers, the usual interpretation of negated occurrences of ‘possible’ is the one-sided one. Aristotle takes advantage of this fact in De interpretatione 13 when he takes it for granted that ‘not possible’ implies both ‘impossible’ and ‘necessary not’, and in Prior Analytics 1.13 when he takes it for granted that these three expressions are equivalent.

Finally, we are in a position to respond to an objection raised by Gisela Striker, who rejects the above interpretation of the argument in Prior Analytics 1.13 on the following grounds:

One might then think that these lines [1.13 32a21–8] are intended to explain the one-sided sense of ‘possible’. If so, however, one would expect something like the consideration mentioned at de Int. 13, 22b11–14 or 29–33: it would seem absurd to say that what is necessary is not possible (and hence impossible). But this does not seem to have worried Aristotle in the Analytics (cf. 33b17; 37a8–9; 38a35). (Striker 2009: 129)

First, it is important to note that the passages mentioned by Striker at the end of this quotation are the ones in which Aristotle uses the past tense ‘was’ to make it clear that he is negating two-sided possibility but not one-sided possibility (see n. 45). These passages do not show that Aristotle is comfortable calling what is necessary ‘not possible’ when he has not

\(^{4}\) 1.14 33b17, 1.17 37a8–9, 37b9–10, 1.19 38a35–6. See also 37a14–24, where it is clear from 37a8–9 that two-sided possibility is under consideration.

\(^{5}\) See n. 9 above.
made it clear that what is being negated is two-sided possibility. Moreover, there is no need for Aristotle in *Prior Analytics* 1.13 to invoke the consideration from *De interpretatione* 13, because the structure of his argument differs in the two chapters. The two arguments share a common goal, namely, to establish the one-sided sense of unnegated occurrences of ‘possible’. In the *De interpretatione*, Aristotle achieves this goal by invoking Striker’s consideration to argue that ‘possible’ does not imply ‘not necessary’ but ‘not necessary not’. In the *Prior Analytics*, he achieves the goal in a more powerful and elegant way by showing that ‘possible’ is equivalent to ‘not necessary not’.

**References:**


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