



## Aristotle on Circular Proof

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### Abstract

In *Posterior Analytics* 1.3, Aristotle advances three arguments against circular proof. The third argument relies on his discussion of circular proof in *Prior Analytics* 2.5. This is problematic because the two chapters seem to deal with two rather disparate conceptions of circular proof. In *Posterior Analytics* 1.3, Aristotle gives a purely propositional account of circular proof, whereas in *Prior Analytics* 2.5 he gives a more complex, syllogistic account. My aim is to show that these problems can be solved, and that Aristotle's third argument in 1.3 is successful. I argue that both chapters are concerned with the same conception of circular proof, namely the propositional one. Contrary to what is often thought, the syllogistic conception provides an adequate analysis of the internal deductive structure of the propositional one. Aristotle achieves this by employing a kind of multiple-conclusion logic.

### Keywords

Aristotle; *Analytiks*; circular proof; deduction; multiple-conclusion logic

### 1. Introduction

A demonstrative science, for Aristotle, is based on first principles (*ἀρχαί*). All of its theorems are derived from these principles by means of proofs (*ἀποδείξεις*). The principles, on the other hand, are unproved; they are not derived by proofs from prior premisses. As Aristotle points out, some philosophers are sceptical about the existence of unproved principles. For example, some hold that 'nothing prevents there being proofs of everything; for it is possible for proofs to proceed in a circle or reciprocally' (*Posterior Analytics* 1.3, 72b16-18). On their view, proof may be circular

in such a way that all propositions covered by a given science are proved, with the result that there are no unproved principles.

In chapter 1.3 of the *Posterior Analytics* Aristotle aims to undermine the conception of circular proof. He advances three arguments to this effect (72b25–32, 72b32–73a6, 73a6–20). The third of these arguments is troublesome. The problem is that it is unclear how the third argument relates to the two previous ones, and whether it is concerned with the same conception of circular proof targeted in them. In the first two arguments, Aristotle takes a circular proof to consist of items  $P_1, \dots, P_n$  such that:

$P_1$  is proved from  $P_2$ ,  $P_2$  is proved from  $P_3, \dots$ , and  $P_n$  is proved from  $P_1$ .

This is a purely propositional characterization of circular proof, inasmuch as each ‘ $P_i$ ’ stands for a proposition or plurality of propositions. By contrast, Aristotle’s third argument relies on a sub-propositional, syllogistic characterization of circular proof developed in chapters 2.5–7 of the *Prior Analytics*. There, Aristotle uses his syllogistic theory to argue for the claim that circular proof is only possible ‘in the case of terms which convert’ (2.5, 58a12–14). This claim plays a central role in his third argument in *Posterior Analytics* 1.3 (73a11–17).

As we will see, the *Prior Analytics*’ syllogistic characterization of circular proof is complex and somewhat artificial. It does not correspond in any obvious way to the more natural propositional characterization used in the first two arguments in *Posterior Analytics* 1.3. Jonathan Barnes writes that their ‘correspondence is not exact... Here, as elsewhere, syllogistic does not readily provide a formalization of the argument structures considered in *APst*.’<sup>1</sup> Indeed, one may doubt whether the syllogistic characterization can be regarded as a reasonable analysis of circular proof at all. Accordingly, there is a question as to how Aristotle’s third argument in *Posterior Analytics* 1.3 fits into the context of this chapter. Robin Smith holds that the argument is ‘badly suited to the surrounding text’ and is ‘not a very successful one’.<sup>2</sup>

The aim of this paper is to show that and how these problems can be solved. I argue that throughout *Posterior Analytics* 1.3 and *Prior Analytics* 2.5–7 Aristotle is concerned with one and the same conception of circular

<sup>1</sup>) Barnes 1994, 106.

<sup>2</sup>) Smith 1982a, 115–16; similarly 1982b, 329–30 and 335.

proof, namely the propositional one. The *Prior Analytics*' syllogistic characterization, I argue, provides an analysis of the internal deductive structure implicit in the propositional conception. Contrary to what is often thought, the analysis turns out to be adequate. Recognition of this fact will shed new light on how Aristotle thought his syllogistic connects to what we would call propositional logic.

I begin with a brief discussion of Aristotle's first two arguments against circular proof in *Posterior Analytics* 1.3. We will see how they both rely on the propositional characterization of circular proof described above (Section 2). We will then turn to the third argument. This argument starts from the claim that every proof must have at least two premisses. Consequently, each 'P<sub>i</sub>' in the propositional characterization stands for a plurality of at least two propositions (Section 3). In order to prove a plurality of propositions from another plurality, every proposition in the first plurality must be deduced from the second. I show how Aristotle's discussion in *Prior Analytics* 2.5-7 conforms to this conception of circular proof (Sections 4 and 5). Finally, we will consider Aristotle's theorem that circular proof is only possible 'in the case of terms which convert'. Although Aristotle does not provide a fully general justification for it, the theorem holds true: it can be shown that every circular proof as described above contains at least two converting terms (Section 6). Thus, Aristotle's third argument against circular proof is ultimately successful. It builds on *Prior Analytics* 2.5-7 in an intelligible way, and it coheres well with the first two arguments in *Posterior Analytics* 1.3.

## 2. The First Two Arguments

A proof (ἀπόδειξις) is a deduction which confers knowledge of its conclusion (*Posterior Analytics* 1.2, 71b17-19). Someone who grasps a proof thereby obtains knowledge of its conclusion. Aristotle holds that in order for a proof to fulfil this function, its premisses must be epistemically prior to and more intelligible than the conclusion (71b29-72a5). Based on this, he argues against circular proofs as follows (*Posterior Analytics* 1.3, 72b25-8):

That it is impossible to prove *simpliciter* in a circle is plain, if proofs must proceed from what is prior and more intelligible. For it is impossible for the same thing at the same time to be both prior and posterior to something.

This is Aristotle's first argument against circular proof in *Posterior Analytics* 1.3. The argument seems to assume that every circular proof contains two items which are reciprocally proved from one another. Moreover, it assumes that if something is proved from something, the latter item is epistemically prior to the former. Hence, any circular proof contains two items that are epistemically prior to one another, which is something Aristotle takes to be impossible.

Aristotle's argument is only compelling if it is granted that the premisses of a proof must be epistemically prior to the conclusion. But the advocates of circular proof would presumably deny this assumption, in which case Aristotle's argument would not be effective against them.<sup>3</sup> This may be part of the reason why Aristotle goes on to give two more arguments against circular proof. His second argument appears as follows (*Posterior Analytics* 1.3, 72b34–73a4):

[i] Those who say that proof is circular say nothing more than that this is the case if this is the case—and it is easy to prove everything in this way. [ii] It will be clear that this follows if we posit three terms. (It makes no difference whether we say that the circle revolves through many terms or through few—or through few or two.) [iii] When if A is the case, of necessity B is, and if B then C, then if A is the case C will be the case. [iv] Thus given that if A is the case it is necessary that B is, and if B is that A is (for that is what being circular is), [v] let A be C. Hence to say that if B is the case A is, is to say that C is; and to say this is to say that if A is the case C is. But C is the same as A.

To aid the discussion of this passage, I have divided it into five points. In point [i], Aristotle states that circular demonstrators do nothing more than to prove A from A. That is, they prove A on the assumption that A. But if this is to count as a proof of A, Aristotle adds, then 'it is easy to prove everything' (cf. 73a4–6).

In order to justify his claim in [i], Aristotle goes on in [iii] to state that the relation of necessary consequence is transitive:

- (1) If C is a necessary consequence of B, and B is a necessary consequence of A, then C is a necessary consequence of A.

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<sup>3</sup> See Barnes 1976, 280; 1994, 104–8; Lear 1980, 80.

The letters ‘A’, ‘B’, and ‘C’ in this principle do not stand for subjects or predicates of categorical propositions. Instead, they are placeholders for whole propositions.<sup>4</sup> Each of them stands for a single proposition or for a plurality of propositions. Although this propositional use of schematic letters is not Aristotle’s usual practice, it does occur elsewhere in the *Analytics*.<sup>5</sup> We will consider more examples of it later on.

Aristotle proceeds in [iv] to introduce two items, A and B, each of which is a necessary consequence of the other:

- (2) A is a necessary consequence of B, and B is a necessary consequence of A.

As Aristotle explains in [v], (2) is an instance of the antecedent of (1): it is obtained from this antecedent by identifying C with A. So (1) and (2) imply that A is a necessary consequence of A. This lends support to Aristotle’s initial claim that circular demonstrators do nothing more than to prove A from A.<sup>6</sup>

Now, when Aristotle appeals to necessary consequence in the present passage, he seems to have in mind cases of necessary consequence that are established by a proof. For example, it seems clear that point [iv] is meant to indicate a circular proof of the following form:

- (3) A is proved from B, and B is proved from A.

Similarly, when Aristotle states the transitivity of necessary consequence in [iii], he seems to imply that the relation of being proved from something is transitive:<sup>7</sup>

- (4) If C is proved from B, and B is proved from A, then C is proved from A.

The schema in (3) represents a circular proof which involves two items, A and B. But as Aristotle makes clear in [ii], circular proofs may involve

<sup>4</sup> Philoponus, in *Post. An.* xiii.3, 50.23-51.4 Wallies; Ross 1949, 515.

<sup>5</sup> For example, *Pr. An.* 1.15, 34a5-33; 1.25, 41b36-42a31; 2.2, 53b11-20, 53b23-4.

<sup>6</sup> As Barnes (1994, 109) points out, however, Aristotle’s argument falls short of fully establishing this claim.

<sup>7</sup> Irwin 1988, 127-8; Detel 1993, ii. 96-7.

more than two items. They may involve any finite number of items.<sup>8</sup> Thus, writing 'P<sub>1</sub>', 'P<sub>2</sub>', 'P<sub>3</sub>' instead of Aristotle's 'A', 'B', 'C', the general structure of a circular proof can be represented as follows:<sup>9</sup>

- (5) P<sub>1</sub> is proved from P<sub>2</sub>, P<sub>2</sub> is proved from P<sub>3</sub>, ..., P<sub>n-1</sub> is proved from P<sub>n</sub>, and P<sub>n</sub> is proved from P<sub>1</sub> (n ≥ 1).

This schema also seems to underlie Aristotle's first argument against circular proof discussed above. At least, it justifies one of the assumptions on which that argument relies, namely that every circular proof contains two items which are proved from each other. For, given the transitivity of proof stated in (4), any items P<sub>i</sub> and P<sub>j</sub> in (5) are proved from each other.<sup>10</sup>

In Aristotle's first two arguments, then, circular proofs are taken to be of the form specified in (5). As we will see in the next two sections, Aristotle's third argument relies on another, more complex characterization of circular proof.

### 3. Pluralities of Propositions

Aristotle's third argument against circular proof begins as follows (*Posterior Analytics* 1.3, 73a6-11):

Moreover, even this [viz. circular proof] is only possible for things which follow one another, as *propria* do. If a single thing is laid down, I have proved that it is never necessary that anything else be the case (by a single thing I mean that neither if one term nor if one posit is posited): two posits are the first and fewest from which a necessary consequence is possible, since two posits are also the fewest from which it is possible to deduce anything.<sup>11</sup>

<sup>8</sup> They presumably cannot involve infinitely many items. According to *Posterior Analytics* 1.3, knowledge cannot be obtained by an infinite series of proofs because 'it is impossible to go through infinitely many items' (72b10-11). Aristotle presents the advocates of circular proof as accepting this view (72b15-18; see Barnes 1976, 279-80; 1994, 103 and 105).

<sup>9</sup> See Irwin 1988, 125-6.

<sup>10</sup> If n=1 in (5) then P<sub>1</sub> is proved from itself. More generally, every P<sub>i</sub> is proved from itself in (5). For further discussion of how an item P<sub>i</sub> can be proved from itself, see Section 5 below.

<sup>11</sup> The last clause of this passage reads: ἐκ δύο δὲ θέσεων πρώτων καὶ ἐλαχίστων ἐνδέχεται εἶπερ καὶ συλλογίσασθαι. The above translation of this clause follows Tredennick (1960,

In the first sentence of this passage, Aristotle claims that circular proof is only possible for terms which ‘follow one another’ (that is, for terms which convert).<sup>12</sup> He takes himself to have justified this claim in *Prior Analytics* 2.5, and we will examine his justification later. For now, I wish to focus on the second sentence of the passage. I will argue that this sentence initiates a train of thought that eventually leads to the claim made in the first sentence.

Aristotle states that no necessary consequences can be drawn from a single premiss, but that at least two premisses are required for this. He states the same thesis in several places in the *Prior Analytics*.<sup>13</sup> Although Aristotle claims to have proved this thesis, it is not clear whether or where he actually did so.<sup>14</sup> In any case, it seems clear that Aristotle takes the thesis to be justified by his account of deduction (συλλογισμός). His official definition of deduction, to be found in the opening chapter of the *Prior Analytics*, requires that a deduction have at least two premisses.<sup>15</sup> Moreover, all deductions discussed in the syllogistic in *Prior Analytics* 1.1-22 have exactly two premisses. Nevertheless, Aristotle’s thesis that at least two premisses are needed for a necessary consequence is problematic. For example, it seems to conflict with the fact that he accepts conversion rules according to which, for instance, ‘B belongs to no A’ is a necessary consequence of

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41). Barnes (1994, 6) gives a different translation: ‘two posits are the first and fewest from which it is possible, if at all, to deduce something.’ For present purposes we need not decide between the two translations.

<sup>12</sup> Two terms A and B ‘follow one another’ just in case everything that falls under B falls under A and vice versa. An example are *propria* (ἴδια). According to the *Topics*, ‘a *proprium*... counterpredicates with the subject. For example, it is a *proprium* of man to be capable of learning grammar; for if he is a man, then he is capable of learning grammar, and if he is capable of learning grammar, he is a man’ (*Top.* 1.5, 102a18-22).

<sup>13</sup> *Pr. An.* 1.15, 34a16-19; 1.23, 40b35-7; 2.2, 53b16-20.

<sup>14</sup> It has been thought that Aristotle takes himself to have proved the statement in chapter 1.15 of the *Prior Analytics* (Waitz 1846, 312), or in chapter 1.24 (Pacius 1597, 421), or in chapter 1.25 (Ross 1939, 256; Tredennick 1960, 41, Barnes 1994, 110). But none of these chapters actually seems to contain a proof of it. Thus, Smith (1982b, 329) holds that Aristotle does not offer any proof of the statement in the *Analytics*.

<sup>15</sup> In this definition the premisses of a deduction are referred to by the plural phrase ‘certain things having been assumed’ (τεθέντων τινῶν, *Pr. An.* 1.1, 24b19). This is usually taken to indicate that a deduction must have at least two premisses; see Alexander, in *Pr. An.* ii.1, 17.10-18.7, 257.8-13 Wallies, in *Top.* ii.2, 8.14-9.19 Wallies; Ammonius, in *Pr. An.* iv.6, 27.14-33 Wallies; Frede 1974, 20; Striker 2009, 79-80.

‘A belongs to no B’ (*Prior Analytics* 1.2). Fortunately, we need not enter into a discussion of these problems here. The important point for us is that Aristotle endorses the thesis on the basis of the *Prior Analytics*’ account of deduction.

Thus, when Aristotle says that B is a necessary consequence of A, the letter ‘A’ stands for a plurality of at least two propositions. As Aristotle puts it elsewhere, ‘A is posited as if one thing, the two premisses being taken together’ (*Prior Analytics* 2.2, 53b23–4).<sup>16</sup> This use of the letter ‘A’ is also found in a passage from *Prior Analytics* 1.15, 34a19–24, in which Aristotle argues that if B is a necessary consequence of A then the possibility of B follows from the possibility of A:

If C is predicated of D and D of F, then necessarily C is also predicated of F; and if each of the two premisses is possible, then the conclusion is also possible—as, if someone should put A as the premisses and B as the conclusion, it would result not only that when A is necessary then B is simultaneously also necessary, but also that when A is possible B is possible.

At the beginning of this passage, Aristotle sketches a deduction, in which, as he emphasizes, the conclusion is a necessary consequence of the two premisses. The deduction has two premisses and one conclusion, which are composed of the terms D, E, and F. Aristotle goes on to stipulate that ‘A’ should stand for the two premisses of this deduction and ‘B’ for its conclusion. Thus B is a necessary consequence of A. Finally, he infers from this that if A is necessary then B is also necessary, and that if A is possible then B is also possible.<sup>17</sup>

In this particular example, ‘A’ stands for two propositions, whereas ‘B’ stands for a single proposition. In such a case, A and B cannot constitute a circular proof. For this would require that A be a necessary consequence of B, and hence that ‘B’ stand for more than one proposition. Similar problems arise for all circular proofs of the form (5). Every item  $P_i$  in such a circular proof is a necessary consequence of something, and something is a

<sup>16</sup> This use of ‘A’ comes close to the idea of a conjunctive proposition in which a number of propositions is unified into a single proposition, and which is true just in case each of the constituent propositions is true (see Geach 1963, 44). However, it is doubtful whether Aristotle actually recognized such conjunctive propositions in his logical theory (Patterson 1995, 158–9).

<sup>17</sup> For further discussion of 34a19–24, see Rosen and Malink 2012, 182–5.



necessary consequence of it. Inasmuch as the latter is the case, Aristotle requires  $P_i$  to be a plurality of at least two propositions. But inasmuch as  $P_i$  is a necessary consequence of something, it seems reasonable to think that  $P_i$  must be a single proposition. If so, then the conception of circular proof specified in (5) is incoherent.<sup>18</sup>

Now, Aristotle does not say that in order for B to be a necessary consequence of A, the former must be a single proposition. It is true that in the vast majority of cases in the *Analytics*, the consequent of a necessary consequence is a single proposition. But this does not mean that it has to be a single proposition. It seems therefore best to give up this assumption, and to accept that a plurality of propositions can be a necessary consequence of another plurality. Of course, Aristotle does not explain what it means to say that a plurality  $P_i$  is a necessary consequence of a plurality  $P_j$ . But it seems natural to take it to mean that *every* proposition which is a member of plurality  $P_i$  is a necessary consequence of  $P_j$ . Accordingly, saying that  $P_i$  is proved from  $P_j$  means that *every* proposition in  $P_i$  is proved from  $P_j$ . In what follows, I argue that this is indeed how Aristotle conceived of circular proof, both in *Posterior Analytics* 1.3 and in *Prior Analytics* 2.5.

#### 4. Circular Proof in *Prior Analytics* 2.5

In *Prior Analytics* 2.5-7 Aristotle gives an account of circular proof within the framework of his assertoric syllogistic. As is well known, the assertoric syllogistic deals primarily with four kinds of categorical propositions, namely a-, e-, i-, and o-propositions:

- AaB (A belongs to all B)
- AeB (A belongs to no B)
- AiB (A belongs to some B)
- AoB (A does not belong to some B)

<sup>18</sup> In view of these problems, Smith writes: ‘The analysis of 72b32-73a6 [i.e. (5) above], which seemed to assume that a circular deduction might involve deriving premisses and conclusion from each other, is here [at 73a6-11] rejected in the light of the *Prior Analytics*’ result that every argument has at least two premisses’ (Smith 1986, 60; see also 66-7 n. 33; cf. Smith 1982a, 116; Lear 1980, 80-1). However, Aristotle gives no indication of such a break within chapter 1.3. On the contrary, the pronoun τοῦτο at the beginning of the third argument at 73a6 seems to refer to the conception of circular proof described in the second argument, i.e. to (5).

Aristotle focuses on deductions which consist of three categorical propositions: two premisses and one conclusion. These deductions are usually called ‘syllogisms’. Typical examples are the following:

	Barbara:	Celarent:	Darii:	Ferio:
major premiss:	AaB	AeB	AaB	AeB
minor premiss:	BaC	BaC	BiC	BiC
conclusion:	AaC	AeC	AiC	AoC

With this framework in place, Aristotle begins his discussion of circular proof in *Prior Analytics* 2.5 by giving the following definition (57b18–21):

Proving in a circle, or from one another, is concluding something which was taken in some other syllogism as a premiss by means of the conclusion of that syllogism and its other premiss taken as converted in predication.

Aristotle is describing a syllogism in which one of the premisses of some other syllogism is deduced by means of the conclusion of this latter syllogism and its other premiss ‘converted in predication’. In other words: given some syllogism, Aristotle describes a syllogism (i) one of whose premisses is the conclusion of the original syllogism, (ii) whose other premiss is one of the premisses of the original syllogism ‘converted in predication’, and (iii) whose conclusion is the other premiss of the original syllogism. For any given syllogism there can, in principle, be two syllogisms which satisfy this description: one which infers its major premiss and another which infers its minor premiss:

original syllogism:	syllogism inferring $M_1$ :	syllogism inferring $M_2$ :
$M_1$ (major premiss)	C	C
$M_2$ (minor premiss)	$M_2$ converted	$M_1$ converted
C (conclusion)	$M_1$	$M_2$

This diagram is not meant to determine the order of the premisses in the syllogisms inferring  $M_1$  and  $M_2$ . The conclusion of the original syllogism, C, can serve either as the major premiss or as the minor premiss in these syllogisms.

Aristotle requires that one of the premisses of the original syllogism be ‘converted in predication’. When applied to a-propositions, this conversion simply consists in interchanging the predicate and subject term, converting AaB to BaA.<sup>19</sup> Unlike the conversion rules stated in *Prior Analytics* 1.2, this kind of conversion is not in general truth-preserving. When the conversion is applied to the two premisses of a syllogism in Barbara, we obtain the following two syllogisms:

original syllogism (Barbara):	syllogism inferring the major premiss (2.5, 57b22-5):	syllogism inferring the minor premiss (2.5, 57b25-8):
AaB	AaC	BaA
BaC	CaB	AaC
AaC	AaB	BaC

In the syllogism inferring the major premiss, the minor premiss of the original syllogism, BaC, has been converted to CaB. In the other syllogism, the major premiss of the original syllogism, AaB, has been converted to BaA.

Clearly, this definition of circular proof is somewhat technical and artificial, especially in view of the conversion required by it. Aristotle does not explain how he arrived at it, and why it constitutes a reasonable definition of circular proof. The question arises: On what grounds is the structure described by the definition called ‘circular’ (κύκλω)? What is the relevant ‘circle’? Although these are natural questions to ask, they are rarely addressed by commentators. An exception is Robin Smith, who writes as follows (1989, 192; see also 1986, 60-1):

The exact relationship of ‘circular’ proof as defined here to the circular argumentation of *Posterior Analytics* 1.3 is not as straightforward as at first appears... But with some speculation, we may imagine how Aristotle might have been led from one case to the other. The simplest case of a circular deduction would be two propositions which can be deduced from one another. But... every deduction has (at least) two premisses. But since no deduction is possible in which *both* the premisses can, in turn, be deduced from the conclusion, the next possibility would be a deduction either premise of

<sup>19)</sup> The conversion in question is more complex when it is applied to e-propositions (see 2.5, 58a29-30, 58b7-10; 2.6, 58b36-8; 2.7, 59a25-9; cf. Malink 2012, 166-8).

which could be deduced from the conclusion and the other premise. However, an investigation of all deductions in the figures shows that this can never happen. If we suppose him to have gotten to this point, Aristotle may then have asked: is anything *close* to this situation possible? A simple modification is replacing ‘and the other premise’ by ‘and *the converse* of the other premise’: and here we find that under some circumstances such circular deductions are possible.

On this view, chapters 2.5–7 of the *Prior Analytics* do not deal with circular proof proper, but only with an approximate substitute. If this were correct, these chapters would be of limited value as a discussion of circular proof. Ross would be right in regarding them ‘simply as a mental gymnastic’ (1949, 441). Correspondingly, Aristotle’s third argument in *Posterior Analytics* 1.3, which relies on *Prior Analytics* 2.5–7, would be questionable since it would only target an artificial substitute for circular proof (see Barnes and Smith cited in nn. 1 and 2 above).

Such an interpretation is not very satisfactory. A more satisfactory interpretation can, I think, be obtained by considering another passage from *Prior Analytics* 2.5. The passage I have in mind contains an argument to the effect that circular proof is only possible ‘in the case of terms which convert’. This argument, which follows shortly after Aristotle’s initial definition of circular proof, reads as follows (57b32–58a15):

[i] In the case of terms which do not convert with one another, the deduction comes about from one unproved premiss; for it cannot be proved by means of these terms that the third belongs to the middle or the middle to the first. [ii] But in the case of terms which do convert, they can all be proved through each other, as for example if A, B, and C convert with each other. [iii] For let conclusion AC have been proved by means of middle B, [iv] and again AB by means of the conclusion together with premiss BC converted, [v] and likewise also BC by means of the conclusion and premiss AB converted. [vi] Both premiss CB and premiss BA need to be proved, for we have used only these premisses as unproved. [vii] Accordingly, if B is taken to belong to all C and C to all A, then there will be a deduction of B in relation to A. [viii] Also, if C is taken to belong to all A and A to all B, then it is necessary for C to belong to all B. [ix] Now, in both of these deductions, premiss CA was taken as unproved; for the others had been proved. Consequently, if we can prove this premiss, then all of the premisses will have been proved through one another. [x] Accordingly, if C is taken to belong to all B and B to all A, then both the premisses taken have been proved, and also it is necessary for C to belong to A. [xi] It is evident, then, that only in the case of terms which convert is it possible for circular or reciprocal proofs to come about; in the case of others, it is as we said earlier [in [i]].

In points [i] and [ii] of this passage, Aristotle states the desired conclusion of the argument, that circular proof is only possible ‘in the case of terms

which convert'. He repeats it in [xi]. By 'terms which convert' he means terms which are a-predicated of each other: A and B convert if and only if both AaB and BaA.

Points [iii]-[x] are intended to establish Aristotle's conclusion. They involve the following six syllogisms in Barbara:

- [iii] AaB, BaC, therefore AaC
- [iv] AaC, CaB, therefore AaB
- [v] BaA, AaC, therefore BaC
- [vii] BaC, CaA, therefore BaA
- [viii] CaA, AaB, therefore CaB
- [x] CaB, BaA, therefore CaA

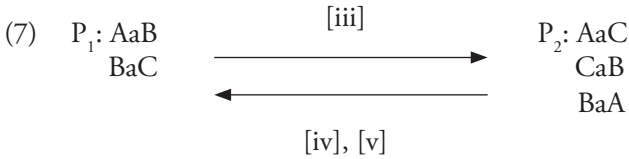
The syllogism in [iii] is the original syllogism which serves as the starting point of the circular proof. The syllogisms in [iv] and [v] are obtained from the original one by the transformation described by Aristotle in his definition of circular proof. This definition, taken on its own, suggests that each of the last two syllogisms counts as a circular proof with respect to the original syllogism, or that the three syllogisms in [iii]-[v] together constitute a circular proof. The present argument, however, shows that Aristotle took a circular proof to be a more complex structure consisting, at least in this case, of the six syllogisms in [iii]-[x].<sup>20</sup> But on what grounds does Aristotle take these six syllogisms to constitute a circular proof?

In order to answer this question, let us recall that in *Posterior Analytics* 1.3, Aristotle took a circular proof to consist of items  $P_1, \dots, P_n$  which are proved from each other in a circle. As we have seen, each of these items is a plurality of propositions. I suggested that a plurality  $P_i$  is proved from  $P_j$  just in case *every* proposition in  $P_i$  is proved from  $P_j$ . Accordingly,  $P_i$  is deduced from  $P_j$  just in case *every* proposition in  $P_i$  is deduced from  $P_j$ . Given this conception of circular proof, the present argument from *Prior Analytics* 2.5 can be taken to involve two pluralities,  $P_1$  and  $P_2$ . The premisses of the original syllogism in [iii] belong to one plurality, and the conclusion to the other:

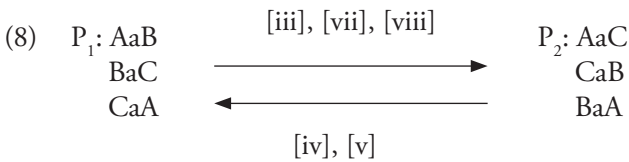
$$(6) \quad \begin{array}{l} P_1: AaB \\ \quad BaC \end{array} \quad \xrightarrow{\text{[iii]}} \quad P_2: AaC$$

<sup>20</sup> Pacius 1597, 325-6; Lear 1980, 80; Smith 1989, 193-4. Pace Barnes (1981, 38; 1994, 106), who takes a circular proof to consist of three syllogisms such as those in [iii]-[v].

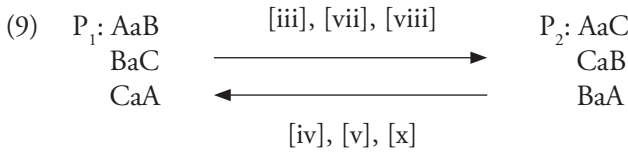
Clearly,  $P_2$  is deduced from  $P_1$ . But in order for a circular proof to come about,  $P_1$  should also be deduced from  $P_2$ . That is, every proposition in  $P_1$  should be deduced from  $P_2$ . Obviously, neither proposition in  $P_1$  can be deduced from AaC alone. But they can be deduced from AaC in combination with CaB and BaA, respectively. So, if the last two propositions are added to  $P_2$ , then  $P_1$  can be deduced from  $P_2$  by means of the syllogisms in [iv] and [v]:



$P_2$  now contains CaB and BaA. In point [vi], Aristotle states that these two propositions ‘need to be proved, for we have used only these premisses as unproved’. By this he seems to mean that these two propositions, unlike AaC, have not been deduced from  $P_1$  so far. Thus, point [vi] states that in order for  $P_1$  and  $P_2$  to constitute a circular proof, CaB and BaA need to be deduced from  $P_1$ . In points [vii] and [viii], Aristotle shows that this can be achieved if CaA is added to  $P_1$ . With this addition, both CaB and BaA can be deduced from  $P_1$  by means of the syllogisms in [vii] and [viii]:



Finally, Aristotle points out in [ix] that the proposition which was added to  $P_1$ , CaA, ‘was taken as unproved; for the others had been proved’. As before, this seems to mean that, unlike AaB and BaC, the proposition CaA has not been deduced from  $P_2$  so far. Aristotle writes: ‘if we can prove this premiss, then all of the premisses will have been proved through one another.’ In other words, if CaA can be deduced from  $P_2$  without adding further propositions to  $P_2$ , then the circular proof will be complete. As Aristotle explains in [x], this is indeed the case: CaA can be deduced from the propositions already present in  $P_2$  by means of the syllogism in [x]. Thus we obtain a complete circular proof:



Every proposition in  $P_2$  is deduced from  $P_1$ , and vice versa. In other words,  $P_2$  is deduced from  $P_1$  and vice versa. Moreover, the pluralities  $P_1$  and  $P_2$  each imply that the terms A, B and C convert: both AaB and BaA are deducible from either plurality, and so are BaC and CaB, and AaC and CaA.<sup>21</sup> This justifies Aristotle’s claim in [ii] and [xi] that circular proof is possible ‘in the case of terms which convert’.

Thus, I take the phrase ‘in the case of terms which convert’ (ἐν τοῖς ἀντιστρέφουσιν, 57b35, 58a13) to mean: in the case of terms whose mutual convertibility is deducible from each of the pluralities  $P_1, \dots, P_n$  in the circular proof under consideration. In other words, any terms A and B convert with each other in a circular proof if and only if both AaB and BaA are deducible from each plurality  $P_i$  in the circular proof. Being convertible in this way does not necessarily mean that the terms actually convert; for, if AaB and BaA are deducible from each plurality, this does not mean that the two propositions are actually true. The terms may fail to convert if some propositions in  $P_1, \dots, P_n$  are false. In this case, the circular proof would remain intact, although some deductions in it would proceed from false premisses.<sup>22</sup>

Aristotle also claims, in [i] and [xi], that circular proof is not possible ‘in the case of terms which do not convert’. This seems to mean that if pluralities  $P_1, \dots, P_n$  do not imply the mutual convertibility of the terms involved, these pluralities cannot constitute a circular proof. Aristotle takes himself to have established this claim in the passage quoted above. But it is not obvious whether or to what extent he succeeds in doing so. I will discuss this issue in Section 6 below.

In regarding (9) as a circular proof, Aristotle is not concerned whether the six deductions in it are proofs (ἀποδείξεις) as opposed to mere deductions. His first argument in *Posterior Analytics* 1.3 relied on the requirement

<sup>21</sup>) For the notion of deducibility employed here, see Section 5 below.

<sup>22</sup>) Unless it is assumed that all these deductions have true premisses on the grounds that proofs (ἀποδείξεις) must proceed from true premisses (*Post. An.* 1.2, 71b19-26).

that the premisses of a proof be epistemically prior to the conclusion—a requirement that does not apply to deductions in general. As mentioned above, the advocates of circular proof would presumably reject this requirement. They would likely regard every deduction as a proof. In his second and third argument in chapter 1.3, Aristotle seems to adopt their position, with the aim of showing that even in this case circular proof is not a viable option. Accordingly, he does not seem to distinguish between proofs and deductions in these arguments, nor does he distinguish between them in *Prior Analytics* 2.5–7. Thus, given the dialectical context of his argument, he may regard (9) as a circular proof of the form (5): he may take it that  $P_2$  and  $P_1$  are *proved* from each other simply because they are *deduced* from each other.

There is a close connection between (9) and Aristotle's initial definition of circular proof given at the beginning of chapter 2.5. The definition captures not only the three syllogisms in [iii]–[v], but also the other parts of (9). For each of the six syllogisms in (9), there are two other syllogisms in it which are related to that syllogism in the way described by Aristotle's definition. For example, [viii] and [vii] are so related to [x]; and [vii] and [iii] are so related to [v]. Thus, Aristotle's initial definition captures the internal syllogistic structure of the circular proof in (9). However, it does not capture its global, propositional structure, mainly because it does not mention the division of propositions into the two pluralities  $P_1$  and  $P_2$ . Nor does the definition explain why the circular proof should be called 'circular'. If I am correct, it is so called because  $P_1$  is deduced (or proved) from  $P_2$  and vice versa. The circle by virtue of which it is called 'circular' is a circle of deductions (or proofs) between pluralities  $P_1, \dots, P_n$ , as described in (5). Aristotle uses the term 'circular' in his initial definition in chapter 2.5 because this definition is intended to model circular proofs of the form (5). The definition is successful as an analysis of the internal structure of a specific circular proof of the form (5), namely of (9). Whether it provides a syllogistic analysis of *all* circular proofs of the form (5) remains to be seen.

It should be noted that the above interpretation of 57b32–58a15 differs from the traditional interpretation given by Pacius, Lear, and Smith.<sup>23</sup> They agree that the six syllogisms mentioned in [iii]–[x] constitute a circular proof, but they do not arrange them as in (9). Instead, they take the

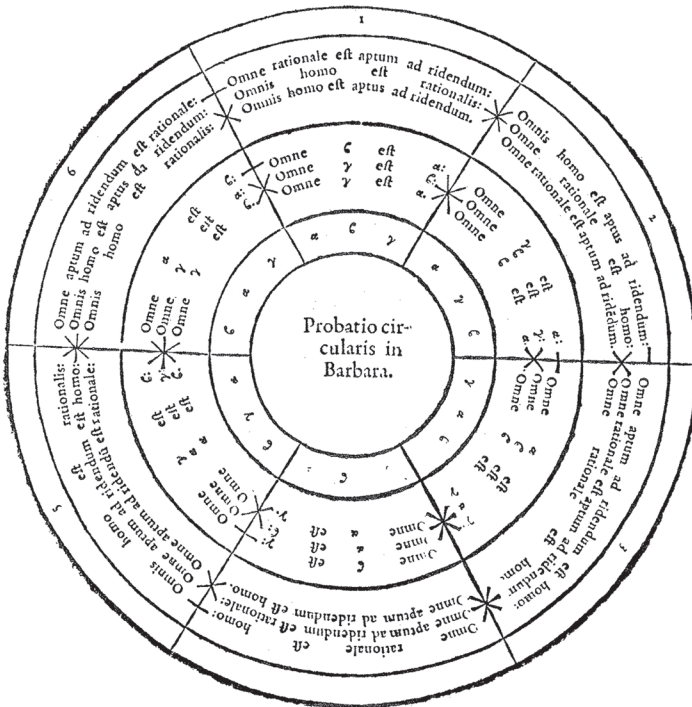
<sup>23</sup> See n. 20 above for references to Pacius, Lear, and Smith.



circular proof to be a sequence of the six syllogisms in the following order:

- [iii] AaB, BaC, therefore AaC
- [iv] AaC, CaB, therefore AaB
- [viii] CaA, AaB, therefore CaB
- [x] CaB, BaA, therefore CaA
- [vii] BaC, CaA, therefore BaA
- [v] BaA, AaC, therefore BaC (and from [v] back to [iii])

In this sequence, every syllogism is related to its predecessor and successor in the way described by Aristotle’s definition at the beginning of chapter 2.5. For example, [x] is so related to [viii] and [vii]; and [iii] to [v] and [iv]. As a result, the circular proof may proceed in two directions: downward and upward. Pacius (1597, 326) presents the traditional interpretation in an especially vivid manner, printing the six deductions in a circle as follows:



Despite this impressive diagram, the interpretation given in (9) seems preferable to the traditional interpretation for two reasons. First, the sequence of the six syllogisms postulated by the traditional interpretation does not match the order of Aristotle's presentation at 57b32–58a15. If Aristotle took the circular proof to be this sequence, why did he not present the six syllogisms in the order demanded by it? By contrast, the interpretation given above exactly matches the order in which Aristotle mentions the syllogisms. It is the order in which a full-blown circular proof between  $P_1$  and  $P_2$  is constructed step by step in (6)–(9).

Secondly, the traditional interpretation cannot explain why the sequence postulated by it represents a reasonable conception of circular proof. (Just printing it in a circle does not help.) The sequence does not match the natural conception of circular proof employed in the first two arguments in *Posterior Analytics* 1.3. It would, at best, be an imperfect substitute for this more natural conception. Yet, as Barnes points out, Aristotle 'evidently thinks that his remarks there [in *Prior Analytics* 2.5–7] yield a formal analysis of the sort of reasoning implicit in [the second argument of *Posterior Analytics* 1.3]' (Barnes 1994, 106). The interpretation given above is in a better position to explain this. For, on this interpretation, the circular proof in 57b32–58a15, viz. (9), is a special instance of (5)—although it involves additional syllogistic complexity. Given the framework of Aristotle's syllogistic, (9) is a very natural instance of the pattern specified in (5). Thus, *Prior Analytics* 2.5–7 and *Posterior Analytics* 1.3 are concerned with the very same kind of circular proof, namely (5).<sup>24</sup>

## 5. Deducibility

Before returning to Aristotle's third argument in *Posterior Analytics* 1.3, let us have a closer look at the conception of circular proof that has emerged so far. A circular proof consists of pluralities  $P_1, \dots, P_n$  which are proved

<sup>24</sup> Barnes (1990, 61) takes a circular proof to be 'a sequence of arguments such that the conclusion of one argument is a premiss for the next argument, and the conclusion of the last argument is a premiss for the first argument'. The sequence of the six syllogisms postulated by the traditional interpretation satisfies this description, and hence counts as a circular proof on Barnes' view. Consequently, Barnes' proposal is open to the same objections raised against the traditional interpretation. For further criticism of his proposal, see n. 42 below.

from each other in a circle. As we have seen, Aristotle takes proof to be transitive: if  $P_1$  is proved from  $P_2$ , and  $P_2$  is proved from  $P_3$ , then  $P_1$  is proved from  $P_3$  (cf. (4) above). Consequently, any two pluralities in a circular proof are proved from each other. Moreover, it follows that every plurality is proved from itself. Since proofs are deductions, every plurality is deduced from itself.

How can a plurality be deduced from itself? Consider  $P_1$  in (9), which contains the propositions  $AaB$ ,  $BaC$ , and  $CaA$ . In order for this plurality to be deduced from itself, every proposition in  $P_1$  should be deduced from  $P_1$ . For example,  $AaB$  should be deduced from the plurality  $AaB$ ,  $BaC$ ,  $CaA$ . Although this might seem trivial, it is important to be clear about how such a deduction proceeds. It might be thought that  $AaB$  can simply be deduced from itself by means of a rule of iteration, as follows:<sup>25</sup>

1.  $AaB$  [premiss]
2.  $AaB$  [from 1; by iteration]

However, such a derivation does not count as a deduction in Aristotle's sense. For one thing, it violates his requirement that any deduction have more than one premiss (see nn. 13 and 15 above). Moreover, it violates another requirement of Aristotle's, namely that the conclusion of a deduction must not be identical with any of the premisses.<sup>26</sup> The conclusion is required to be distinct from every premiss. In view of this,  $AaB$  may instead be deduced from  $P_1$  by means of the following complex derivation:

1.  $AaB$  [premiss]
2.  $BaC$  [premiss]
3.  $CaA$  [premiss]
4.  $AaC$  [from 1, 2; by Barbara]
5.  $CaB$  [from 1, 3; by Barbara]
6.  $AaB$  [from 4, 5; by Barbara]

<sup>25</sup> Alternatively, the first line of this derivation might be regarded as a deduction of  $AaB$  from itself; see Smiley 1973, 141.

<sup>26</sup> Aristotle's definition of deduction requires that the conclusion be 'something different from the premisses' (ἕτερόν τι τῶν κειμένων, *Pr. An.* 1.1, 24b19; *Top.* 1.1, 100a25-6). See also *Soph. el.* 5, 167a25-6; 6, 168b25-6; Alexander, in *Pr. An.* ii.1, 18.12-20.29 Wallies; Ammonius, in *Pr. An.* iv.6, 27.34-28.20 Wallies; Frede 1974, 20-1; Barnes 2007, 487-90; Striker 2009, 80.

This derivation involves three applications of Barbara, each of them entirely unobjectionable. Yet the whole derivation is problematic because its conclusion, AaB, is among the premisses. If deduction is transitive, then the derivation should count as a deduction; for the conclusion in line 6 is deduced from the two propositions in lines 4-5, and these are deduced from the premisses in lines 1-3. Thus the transitivity of deduction conflicts with Aristotle's requirement that the conclusion of a deduction be distinct from all its premisses.<sup>27</sup>

Can this conflict be resolved? It seems difficult to give up transitivity of deduction, at least in the context of Aristotle's discussion of circular proof. In his second argument in *Posterior Analytics* 1.3, Aristotle makes it clear that he takes proof to be transitive. He is therefore committed to the transitivity of those deductions that occur in circular proofs, e.g. in (9). Given this, he should accept the above complex derivation of line 6 from lines 1-3 as a proper deduction. Accordingly, we should assume that his requirement concerning the distinctness of the conclusion from the premisses does *not* apply to complex deductions such as the one above. Aristotle seems to suspend the requirement for complex deductions in the context of his discussion of circular proof.<sup>28</sup> On the other hand, we may still assume that the requirement applies to simple (i.e. non-complex) deductions, such as Barbara and Celarent. For if the requirement is restricted in this way, it does not conflict with the transitivity of deduction. Thus, for example, the following instances of Barbara and Celarent are still excluded:

AaA, AaA, therefore AaA  
 AaB, BaB, therefore AaB  
 AeB, BaB, therefore AeB

By excluding such deductions, we exclude instances of (9) in which A is identified with B, or in which B is identified with C. Moreover, we exclude circular proofs such as the following:

(10)  $P_1: AeB$   $\xrightarrow{\hspace{2cm}}$   $P_2: BeA$   
           BaB  $\xleftarrow{\hspace{2cm}}$  BaB

<sup>27</sup> See Barnes 2007, 498-500.

<sup>28</sup> This is confirmed by *Pr. An.* 2.5, 58a15-20, where Aristotle seems to acknowledge that circular proofs such as (9) involve complex deductions that violate this requirement.

If the above three deductions were admissible, (10) would be an acceptable circular proof. For every proposition in  $P_1$  would be deducible from  $P_2$  and vice versa. But (10) does not contain two distinct converting terms. Recall that any terms  $A$  and  $B$  convert in a circular proof if and only if  $AaB$  and  $BaA$  are deducible from each plurality in the circular proof. Since  $AaB$  and  $BaA$  are not deducible from either plurality in (10), the terms  $A$  and  $B$  do not convert in this circular proof. This would be a problem for Aristotle's theorem that circular proof is only possible 'in the case of terms which convert'.<sup>29</sup> It is therefore preferable to exclude the above three simple deductions. By excluding them we will be able to establish, in Section 6, that every circular proof of the form (5) contains at least two converting terms.

Thus, I suggest that in Aristotle's discussion of circular proof the requirement that the conclusion be distinct from all premisses applies to simple deductions but not to complex ones. If the requirement is so restricted, there are no problems with the transitivity of deduction. We can take it that if  $P_1$  is deduced from  $P_2$ , and  $P_2$  is deduced from  $P_3$ , then  $P_1$  is deduced from  $P_3$ . It follows from this that, if a given proposition is deducible from *one* of the pluralities  $P_1, \dots, P_n$  in a circular proof, then it is deducible from *each* of them. As a result, it is not important to indicate from which plurality a given proposition is deducible in a circular proof. If it is deducible from one of them, we may simply say that the proposition is 'deducible in the circular proof'. Every proposition contained in one of the pluralities is deducible in the circular proof.

Finally, it will be helpful to be more specific about the deduction rules by means of which a conclusion may be derived from given premisses. As is well known, Aristotle's assertoric syllogistic is based on two kinds of rule: conversion rules and simple deductions such as Barbara or Celarent. More specifically, there are three conversion rules (*Prior Analytics* 1.2), and fourteen simple deductions. The latter are the familiar syllogisms introduced

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<sup>29</sup> Also, neither  $AaA$  nor  $BaB$  are deducible from  $P_1$  and  $P_2$ . This means that neither  $A$  nor  $B$  converts with itself in (10). One might argue that  $B$  converts with itself in (10) on the grounds that  $BaB$  is contained in (though not deducible from)  $P_1$  and  $P_2$ . In this case, (10) would contain one term that converts with itself. However, it is doubtful whether this would suffice to satisfy the condition stated in Aristotle's theorem. When Aristotle speaks of 'terms which convert' ( $\acute{\epsilon}\nu\ \tau\omicron\upsilon\varsigma\ \acute{\alpha}\nu\tau\iota\sigma\tau\acute{\rho}\epsilon\phi\omicron\upsilon\sigma\iota\nu$ , 57b35, 58a13), he seems to have in mind at least two distinct terms which convert with each other. If so, then Aristotle's condition is not satisfied even if every term converts with itself.

in *Prior Analytics* 1.4–6: four in Aristotle’s first figure, four in the second figure, and six in the third figure.<sup>30</sup> For concreteness, let us say that a *deduction* is a direct deduction using exclusively these seventeen deduction rules. Direct deductions start from a plurality of propositions assumed as premisses; each of the subsequent propositions is derived from the propositions preceding it by one of the seventeen rules.<sup>31</sup> Let us say that a proposition is *deducible* from a plurality of propositions just in case it can be derived from the plurality by means of such a direct deduction (using only the seventeen rules).

Inferences which consist of a single application of a conversion rule do not count as deductions, since they have only one premiss. If such inferences counted as deductions, the following would be an acceptable circular proof:

$$(11) \quad \begin{array}{ccc} P_1: AeB & \longrightarrow & P_2: BeA \\ CiD & \longleftarrow & DiC \end{array}$$

The terms in (11) do not convert: neither are AaB and BaA deducible in the circular proof, nor CaD and DaC or any other pair of a-propositions. Again, this would be a problem for Aristotle’s theorem. Such problems are avoided by requiring that deductions have at least two premisses.

It follows from this requirement that every deduction involves at least one application of a simple syllogism; for Aristotle does not accept deductions in which a single conclusion is derived from two or more premisses using conversion rules but no syllogisms.<sup>32</sup> Now, all of Aristotle’s fourteen

<sup>30</sup> For a list of these fourteen syllogisms, see e.g. Smith 1989, 230. The three conversion rules are the following: from AeB to BeA, from AiB to BiA, and from AaB to BiA.

<sup>31</sup> We may disregard indirect deductions (i.e. those by *reductio ad impossibile*). For Aristotle holds that, once the fourteen simple syllogisms are established, indirect deductions can be dispensed with in the assertoric syllogistic: given the fourteen syllogisms, every conclusion that can be derived from some premisses by an indirect deduction can also be derived from them by a direct deduction (*Pr. An.* 1.29, 45a23–b11; 2.14, 62b38–63b21; see Ross 1949, 454–6).

<sup>32</sup> This is because he does not accept deductions containing superfluous premisses which are not needed to infer the conclusion; see *Top.* 8.11, 161b28–30, in combination with *Pr. An.* 1.1, 24b18–20; cf. also Alexander, in *Top.* 2.2, 13.28–14.2 and 568.18–23 Wallies; Frede 1974, 22; Barnes 1980, 168–9; Striker 2009, 81–2. For example, if DiC is inferred by means of a conversion rule from the two premisses AeB and CiD, this is not a deduc-

simple syllogisms have at least one universal premiss, either an a-premiss or an e-premiss. Moreover, in Aristotle's syllogistic, a universal premiss can only be deduced by means of a deduction which contains an a-premiss.<sup>33</sup> Hence, every circular proof contains at least one a-proposition. More precisely, in every circular proof there is a plurality  $P_i$  which contains an a-proposition (in fact, every plurality contains one).<sup>34</sup> In the next section, we will use this result to establish that every circular proof contains at least two converting terms.

## 6. Converting Terms

In *Prior Analytics* 2.5, Aristotle argues that circular proof is only possible 'in the case of terms which convert'. He appeals to this theorem in *Posterior Analytics* 1.3, in his third argument against circular proof (73a11-20):

[i] Now if A follows B and C, and these follow one another and A, in this way it is possible to prove reciprocally in the first figure everything that was assumed — as has been proved in my account of deduction [in *Prior Analytics* 2.5]. [ii] And it has also been proved that in the other figures either no deduction at all comes about or else none about what was assumed. [iii] But items which are not counterpredicated cannot ever

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tion, because the first premiss is superfluous. More generally, if a single conclusion is inferred from two or more premisses using only conversion rules, then all but one of the premisses will be superfluous. However, when I say that a proposition is deduced or deducible from a plurality  $P_i$  in a circular proof, this is not meant to imply that all propositions in  $P_i$  are needed to deduce that proposition. It only means that *some* propositions in  $P_i$  can serve as the premisses of a deduction which does not have any superfluous premisses and whose conclusion is the proposition in question.

<sup>33</sup> Specifically, a-propositions can only be deduced by means of Barbara (see *Pr. An.* 1.26, 42b32-3); e-propositions can only be deduced by means of Celarent, Cesare, or Camestres.

<sup>34</sup> In addition, it follows that every circular proof contains an a-proposition whose predicate term is distinct from the subject term (this will be relevant in n. 40 below). For, given that every circular proof contains an a-proposition, this a-proposition is deducible in the circular proof. Since a-propositions can only be deduced by means of Barbara, the circular proof must contain two a-propositions which constitute the premiss pair of a syllogism in Barbara. In each of these a-propositions the predicate term is distinct from the subject term (otherwise there would be a simple syllogism in Barbara whose conclusion is identical with one of the premisses, inferring AaB from AaA and AaB or from AaB and BaB; but such simple syllogisms are not admissible).

be proved in a circle. [iv] Hence, since there are few counterpredicated items in proofs (*ἐν ταῖς ἀποδείξεσι*), it is clear that it is both empty and impossible to say that proofs may be reciprocal and that because of this there can be proofs of everything.

In point [i] of this passage, Aristotle refers to his discussion in *Prior Analytics* 2.5 of the circular proof in (9). Aristotle takes this circular proof to be in the first figure since it is constituted by six first-figure syllogisms in Barbara.

In [ii], Aristotle states that there are no proper circular proofs in the second and third figure. He takes himself to have established this claim in *Prior Analytics* 2.6-7. These chapters are based on the definition of circular proof given at the beginning of chapter 2.5, according to which circular proof consists in deducing one premiss of a syllogism from the conclusion and the converse of the other premiss. Aristotle examines whether this conception of circular proof is applicable when the original syllogism is in the second or third figure. As he indicates in point [ii], the answer is negative: this conception of circular proof is not properly applicable when the original syllogism is in the second or third figure.<sup>35</sup>

In point [iii], Aristotle restates his theorem from *Prior Analytics* 2.5, 'that only in the case of terms which convert is it possible for circular or reciprocal proofs to come about' (58a12-14). Finally, in [iv], he dismisses circular proof on the grounds that converting terms are rare in proofs or demonstrations (*ἀποδείξεις*). This completes his third argument against circular proof. The argument does not exclude the existence of any circular proofs, but merely shows that there are few circular proofs in demonstrative contexts. It thereby undermines any view that has circular proofs play a central role in demonstrative science. In the present passage, Aristotle does not justify the claim that converting terms are rare in demonstrations; but he seems to provide some justification for it later on, in *Posterior Analytics* 1.22 (83a36-b12).<sup>36</sup> At any rate, I am not concerned here so much with this last claim as I am with Aristotle's theorem from *Prior Analytics* 2.5. What exactly does the theorem amount to, and how is Aristotle justified in asserting it?

<sup>35</sup> Commentators offer different interpretations of the phrase 'or else none about what was assumed' in point [ii]; see Philoponus, in *Post. An.* xiii.3, 56.3-23 Wallies; Pacius 1597, 422; Ross 1949, 516; Barnes 1994, 110. For our present purposes, it is not necessary to determine the precise meaning of this phrase.

<sup>36</sup> See Malink 2009, 132-5.



As we have seen, the general form of a circular proof is as follows:

- (5)  $P_1$  is proved from  $P_2$ ,  $P_2$  is proved from  $P_3, \dots, P_{n-1}$  is proved from  $P_n$ , and  $P_n$  is proved from  $P_1$  ( $n \geq 1$ ).

Aristotle's example in (9) is an instance of this schema. In this particular example, all terms convert with each other (that is, every a-proposition formed from these terms is deducible in the circular proof). However, this is not true of all circular proofs of the form (5). There are instances in which not all terms convert. For example, we may extend (9) by adding the universal negative propositions  $DeA$  and  $DeB$  as follows:

- |      |         |       |   |         |       |
|------|---------|-------|---|---------|-------|
| (12) | $P_1$ : | $DeA$ | → | $P_2$ : | $DeB$ |
|      |         | $AaB$ |   |         | $AaC$ |
|      |         | $BaC$ | ← |         | $CaB$ |
|      |         | $CaA$ |   |         | $BaA$ |

The two new propositions can be deduced from  $P_1$  and  $P_2$ , respectively, by means of two syllogisms in Celarent:

$DeA, AaB, \text{ therefore } DeB$   
 $DeB, BaA, \text{ therefore } DeA$

Thus, (12) is an instance of (5): every proposition in  $P_2$  is deducible from  $P_1$ , and vice versa. Yet, not all the terms involved convert. For obviously it cannot be deduced in the circular proof that  $D$  converts with any of the other three terms; for neither are  $AaD$  and  $DaA$  deducible in the circular proof nor any other a-proposition involving  $D$ .

Here is another example, which results from (9) by adding the a-propositions  $DaC$  and  $DaA$ :

- |      |         |       |   |         |       |
|------|---------|-------|---|---------|-------|
| (13) | $P_1$ : | $DaC$ | → | $P_2$ : | $DaA$ |
|      |         | $AaB$ |   |         | $AaC$ |
|      |         | $BaC$ | ← |         | $CaB$ |
|      |         | $CaA$ |   |         | $BaA$ |

Again, it is easy to see that every proposition in  $P_2$  is deducible from  $P_1$ , and vice versa. Yet it cannot be deduced that  $D$  converts with any of the other terms; for none of  $AaD$ ,  $BaD$ , and  $CaD$  is deducible in (13).

In view of this, it is not true to say that in every circular proof *all* terms convert. But it may still be true to say that in every circular proof *some* terms convert. At least, this is the case in (12) and (13). And it turns out that this is indeed the case in every circular proof. More precisely, it can be shown that every circular proof of the form (5) contains two distinct terms  $A$  and  $B$  such that both  $AaB$  and  $BaA$  are deducible in it.<sup>37</sup> This claim is established by an argument (given in n. 37) which can be outlined as follows. The argument starts from the fact that every circular proof contains an a-proposition. It goes on to consider a circular proof which contains an

<sup>37</sup> To see this, recall that every circular proof contains an a-proposition, say  $B_0aB_1$  (see end of Section 5). This proposition is deducible in the circular proof. The only way to deduce it is by means of a syllogism in Barbara (cf. n. 33). To this end, there must be a term, say  $B_{1/2}$ , such that both  $B_0aB_{1/2}$  and  $B_{1/2}aB_1$  are deducible in the circular proof. Since the last two propositions constitute the premiss pair of a syllogism in Barbara, the term  $B_{1/2}$  is distinct from  $B_0$  and from  $B_1$  (see Section 5).

Now, consider the proposition  $B_0aB_{1/2}$ . Since it is deducible in the circular proof, there is a term, say  $B_{1/4}$ , such that both  $B_0aB_{1/4}$  and  $B_{1/4}aB_{1/2}$  are deducible in the circular proof. As before, this term is distinct from  $B_0$  and from  $B_{1/2}$ . But  $B_{1/4}$  may be identical with  $B_1$ . If it is, then the circular proof contains two distinct converting terms, since both  $B_{1/4}aB_{1/2}$  and  $B_{1/2}aB_{1/4}$  (i.e.  $B_{1/2}aB_1$ ) are deducible in it. On the other hand, if  $B_{1/4}$  is not identical with  $B_1$ , then we may consider the next term in the regress, say  $B_{1/8}$ , such that  $B_0aB_{1/8}$  and  $B_{1/8}aB_{1/4}$  are deducible in the circular proof. If  $B_{1/8}$  is not identical with either  $B_{1/2}$  or  $B_1$ , we may consider the next term,  $B_{1/16}$ , and so on. We continue as long as the new terms are not identical with any of the previous ones. Thus, there is a potential regress of mutually distinct terms  $B_1, B_{1/2}, B_{1/4}, B_{1/8}, \dots$ , such that if  $B_i$  comes after  $B_j$  in this sequence then  $B_iaB_j$  is deducible in the circular proof. But this regress cannot go on to infinity. For neither Aristotle nor the advocates of circular proof would accept an infinite regress of mutually distinct terms and deductions in a circular proof (see n. 8 above; cf. also *Post. An.* 1.22, 84a29-b2).

Thus, the sequence  $B_1, B_{1/2}, B_{1/4}, B_{1/8}, \dots$  stops at some point and has a last term, say  $B_r$ . This term is identical with one of the preceding terms in the sequence (otherwise the regress would continue beyond  $B_r$ ). But  $B_r$  cannot be identical with its immediate predecessor, say  $B_s$ , in the sequence; for the proposition  $B_riaB_s$  is the minor premiss of a syllogism in Barbara and must therefore consist of two distinct terms (Section 5). Hence,  $B_r$  is identical with a term which precedes  $B_s$  in the sequence, say  $B_t$ . So  $B_r$  is both after and before  $B_s$  in the sequence. This means that both  $B_riaB_s$  and  $B_saB_r$  (i.e.  $B_saB_t$ ) are deducible in the circular proof. Since  $B_r$  is distinct from  $B_s$ , it follows that the circular proof contains two distinct converting terms.

a-proposition but no two terms that convert with each other. It shows that any attempt to complete such a circular proof by means of the deduction rules specified above leads to an infinite regress of deductions. Since neither Aristotle nor the advocates of circular proof would accept such a regress, this means that there cannot be a circular proof of the form (5) that does not contain two converting terms.<sup>38</sup> The details of the argument are a little cumbersome to spell out. But the underlying idea is intuitive enough and Aristotle may well have been aware of it. Thus, Aristotle may have been in a position to see that all circular proofs of the form (5) contain at least two converting terms.

In fact, the large majority of circular proofs of the form (5), if not all, will contain more than two converting terms. But for present purposes it suffices to note that they contain at least two such terms. This provides a general justification of Aristotle's theorem that circular proof is only possible 'in the case of terms which convert'. Of course, Aristotle himself does not give any such general justification. The only evidence he offers in *Prior Analytics* 2.5 is the discussion of the circular proof in (9), at 57b32-58a12. Aristotle presents his theorem as a consequence of this discussion ( $\varphi\alpha\nu\epsilon\rho\delta\nu$   $\omicron\upsilon\upsilon\nu$ , 58a12). This may seem odd for two reasons. First, one may wonder how a particular example of a circular proof, namely (9), can help to establish a universal theorem about all circular proofs. The structure in (9) will be an integral part of a large number of circular proofs, for example of (12) and (13). But there are more complex circular proofs of the form (5) which do not contain (9) as a part.<sup>39</sup> Thus, the example in (9) is far from establishing a theorem about all circular proofs. The second difficulty is that Aristotle states the theorem in the middle of chapter 2.5, without taking into account the rest of his discussion of circular proof in chapters 2.5-7. How can he make a universal claim about circular proofs before he has completed (or even properly begun) his survey of all the other potential cases of circular proof that he is going to discuss in 2.5-7?

<sup>38</sup>) Thus the argument in n. 37 relies on the premiss that there is no infinite regress of mutually distinct terms and deductions. Since both Aristotle and the advocates of circular proof accept this premiss (see n. 8 above; cf. also *Post. An.* 1.22, 84a29-b2), it can be taken for granted in the argument.

<sup>39</sup>) For instance, let  $P_1$  consist of the propositions AaC, CaB, BaE, EaC, CaD, DaA, while  $P_2$  consists of AaB, BaC, EaD, CaA, DaC, CaE. Another example is the following: let  $P_1$  consist of AaB, AaD, DaC, CaE, EaB, EaD, BaD, DaE, EaA, EaC, CaD, while  $P_2$  consists of AaC, CaB, DaA, CaD, EaC, BaE, DaC, CaE, EaD.

These are puzzling problems, but they can be mitigated. One way to do so is to assume that, throughout chapters 2.5-7, Aristotle is only concerned with circular proofs which contain exactly three terms. His example in (9) is of this kind: it contains exactly three terms, since the terms A, B and C in it are distinct from each other. (If they were not mutually distinct, (9) would involve applications of Barbara whose conclusion is among the premisses.) Moreover, it can be shown that (9) captures the internal deductive structure of every circular proof of this kind: if a circular proof of the form (5) contains exactly three terms, then it employs the six syllogisms which occur in (9).<sup>40</sup> It follows from this that, if a circular proof contains exactly three terms, then these terms all convert (i.e. their convertibility is deducible in the circular proof). Aristotle's discussion of (9) at 57b32-58a12 can be read as an argument establishing this claim. That is, it can be read as presenting the argument given in n. 40. On this interpretation, 57b32-58a12 does not only give a specific example of a circular proof, namely (9), but also establishes that (9) captures the deductive structure of all circular proofs which contain exactly three terms.<sup>41</sup> Aristotle's discussion in this

<sup>40</sup> To see this, consider a circular proof of the form (5) which contains exactly three terms. We know that it contains an a-proposition whose predicate term is distinct from the subject term (see n. 34 above). Let this a-proposition be AaC. The proposition must be deducible in the circular proof. The only way to deduce it is by means of a syllogism of the form 'AaB, BaC, therefore AaC'. This is syllogism [iii] in (9). The middle term B must be distinct from A and C (see Section 5). Hence, A, B, and C are distinct from each other, and B is the third term contained in the circular proof.

Now, both premisses of the syllogism must be deducible in the circular proof (since each premiss either is contained in one of the pluralities  $P_1, \dots, P_n$  in the circular proof, or has been deduced from one of the pluralities in order to deduce AaC). Consequently, the circular proof involves a syllogism in Barbara whose conclusion is AaB. Again, the middle term of this syllogism is distinct from A and B; but it cannot be distinct from C (otherwise the circular proof would contain four terms). So the syllogism in question is 'AaC, CaB, therefore AaB', which is [iv] in (9). For the same reason, the syllogism deducing BaC is 'BaA, AaC, therefore BaC', which is [v] in (9). Now, the last two syllogisms contain the premisses CaB and BaA, which must also be deducible in the circular proof. The only way to deduce them is by means of the syllogisms 'BaC, CaA, therefore BaA' and 'CaA, AaB, therefore CaB', which are [vii] and [viii] in (9). Finally, these syllogisms contain the premiss CaA, and this can only be deduced by means of 'CaB, BaA, therefore CaA', which is [x] in (9). Thus, the circular proof involves the six syllogisms [iii]-[x] that occur in (9).

<sup>41</sup> However, (9) does not necessarily capture the whole deductive structure of such circular proofs; for they may involve more propositions and more syllogisms than (9). For instance, consider a circular proof in which  $P_1$  consists of AaB, BaA, BaC, CaB, while  $P_2$  consists of

passage is therefore suitable to establish the theorem that circular proof is only possible ‘in the case of terms which convert’—as long as the theorem is restricted to circular proofs which contain exactly three terms. Aristotle is justified in presenting the restricted version of the theorem as a consequence of his discussion of (9). And he is justified in doing so without regard to the rest of chapters 2.5-7.<sup>42</sup>

It is worth pointing out that the argument given in n. 40 does not appeal to any particular pluralities  $P_i$  or  $P_j$ . The argument is not concerned with whether a given proposition is deducible from a certain plurality, but only whether it is deducible in the circular proof as a whole. For, as we have seen, whenever a proposition is deducible from *some* plurality in a circular proof, it is deducible from *each* of them. Therefore, it is usually not necessary to specify the plurality from which a given proposition is deduced. It suffices to know that the proposition is deducible in the circular proof. This may explain why Aristotle himself does not appeal to pluralities such as  $P_1$  or  $P_2$  at 57b32-58a12 or elsewhere in chapters 2.5-7. There is no need for him to appeal to them. Nevertheless, if I am correct, Aristotle’s discussion throughout chapters 2.5-7 is concerned with circular proofs of the form (5) consisting of pluralities  $P_1, \dots, P_n$ .

The above interpretation assumes that Aristotle’s discussion in *Prior Analytics* 2.5-7 is confined to circular proofs which contain exactly three terms. In the remainder of this section, I provide further evidence for this assumption. According to Aristotle’s definition at the beginning of 2.5, circular proof consists in deducing a premiss of a syllogism from the conclusion and the converse of the other premiss. For example, the minor premiss of ‘AaB, BaC, therefore AaC’ is deduced by the syllogism ‘BaA,

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AaB, BaC, CaA, BiA, CiA. In any case, what is important for us is that every circular proof which contains exactly three terms will employ the six syllogisms that occur in (9).

<sup>42</sup> By contrast, Barnes’ account of circular proof mentioned in n. 24 above (Barnes 1990, 61) cannot explain why Aristotle is justified in doing so. For example, on Barnes’ account, the following sequence of syllogisms is a circular proof: ‘AaB, BiC, therefore AiC; BaA, AiC, therefore BiC.’ This sequence involves two applications of Darii, and it contains two converting terms (namely A and B). More generally, it can be shown that *every* Barnesian circular proof that involves an application of Darii contains converting terms. But this does not follow from the discussion at 57b21-58a12 of circular proofs involving only applications of Barbara. Instead, it would need to be established separately, in Aristotle’s discussion of circular proofs involving applications of Darii, at 58a36-b6. Hence, Aristotle would not be justified in stating his theorem already at 58a12-14.

AaC, therefore BaC' (57b25-8). The middle term of the latter syllogism, A, is the major term of the original syllogism. In order to justify this choice of the middle term, Aristotle writes (*Prior Analytics* 2.5, 57b28-30):

Proving from one another is not possible otherwise. For if another middle term is taken, it will not be in a circle, for none of the same things is taken.

If understood as a universal claim about any circular proof of the form (5), this statement is false. For if the circular proof contains more than three terms, then the middle term of the syllogism deducing BaC need not be A, but may be a fourth term.<sup>43</sup> But if Aristotle's statement is taken to be confined to circular proofs which contain exactly three terms, then it is true. For, given that the terms A, B, and C are mutually distinct and that the circular proof does not contain a fourth term, the proposition BaC can only be deduced through the middle term A in the circular proof (see n. 40).<sup>44</sup>

Moreover, when Aristotle discusses a potential circular proof starting from an application of Celarent, 'AeB, BaC, therefore AeC', he takes it that the minor premiss BaC must be deduced by means of a syllogism which involves the term A (2.5, 58a26-35). Again, this is not correct if the circular proof contains more than three terms, as shown by (12) above. But it is correct under the assumption that the circular proof does not contain more than three terms.

In sum, then, Aristotle's discussion in 2.5-7 seems to be restricted to circular proofs which contain exactly three terms. These are the simplest cases of circular proofs. For, as a matter of fact, every circular proof of the

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<sup>43</sup> For example, consider the circular proof in (13). DaC is deducible in (13) by means of the syllogism 'DaA, AaC, therefore DaC'. But AaC is not deducible through the middle term D (since AaD is not deducible in the circular proof). Instead, AaC can be deduced through the middle term B.

<sup>44</sup> Is Aristotle justified in assuming that the terms A, B, and C are mutually distinct? A circular proof may involve syllogisms whose major term is identical with the minor term (see *Prior Analytics* 2.15 for such syllogisms). For example, it may involve an instance of 'AaB, BaC, therefore AaC' in which A is identical with C. However, every circular proof also involves a syllogism in Barbara in which the major, middle, and minor terms are mutually distinct (see n. 34 and the first paragraph of n. 40 above). In the passage just quoted, I take it, Aristotle is assuming (without loss of generality) that 'AaB, BaC, therefore AaC' is such a syllogism.

form (5) contains at least three distinct terms.<sup>45</sup> Aristotle leaves it to the reader to extend his syllogistic analysis of circular proof to cases containing more than three terms. Once he has indicated the general strategy for analyzing circular proofs of the form (5) within the framework of his syllogistic, it is straightforward to construct more complex circular proofs containing any number of terms.

## 7. Conclusion

In *Prior Analytics* 2.5, Aristotle argues that circular proof is only possible 'in the case of terms which convert'. However, he does not establish this theorem universally for all circular proofs, but only for those which contain exactly three terms. Nevertheless, he seems to rely on the universal version of the theorem in *Posterior Analytics* 1.3, in his third argument against circular proof. Thus, there is a gap between *Prior Analytics* 2.5 and *Posterior Analytics* 1.3. But the gap is not as radical as has been thought, and it can be filled. For the universal version of the theorem holds true: as we have seen (n. 37), every circular proof of the form (5) contains at least two converting terms (that is, two distinct terms A and B such that both AaB and BaA are deducible in the circular proof). Although the kind of proof he gives for the restricted version of the theorem in 2.5 is not applicable to the more general case, Aristotle may well have been in a position to see that the universal theorem is true. In any case, since it is true, Aristotle is justified in appealing to it in his third argument in *Posterior Analytics* 1.3. Thus the argument is successful: given Aristotle's claim that converting terms are rare in demonstrations, it succeeds in undermining the notion that circular proofs should play a central role in demonstrative science.

Despite appearances, then, Aristotle's two treatments of circular proof in *Prior Analytics* 2.5-7 and *Posterior Analytics* 1.3 form a coherent unity. The syllogistic analysis provided by the former plays a vital role in Aristotle's third argument in the latter. We are now in a better position to see how Aristotle's third argument proceeds, and how it relates to the first two arguments. With regard to the syllogistic treatment in *Prior Analytics*

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<sup>45</sup> See n. 34 and the first paragraph of n. 40 above.

2.5-7, we are in a better position to see how it is motivated, and on what grounds it can be viewed as a reasonable account of circular proof.

As mentioned above, the characterization of circular proof given in (5) is purely propositional; it does not appeal to sub-propositional features of the inferences involved. In order to give a syllogistic analysis of this propositional conception, Aristotle takes each of the items  $P_i$  in (5) to be a plurality of propositions. He thereby accepts that a *plurality* of propositions can be deduced from another plurality, and that a *plurality* can be a necessary consequence of another plurality. Thus, Aristotle seems to employ a form of multiple-conclusion logic.

However, his form of multiple-conclusion logic differs from the modern one. Following the work of Gerhard Gentzen (1935, 180), modern logicians usually employ a disjunctive reading of multiple conclusions: saying that a plurality  $A$  is a consequence of a plurality  $B$  means, roughly, that whenever all propositions in  $B$  are true necessarily *at least one* proposition in  $A$  is true. By contrast, Aristotle understands the conclusions conjunctively: for him, saying that a plurality  $A$  is a consequence of a plurality  $B$  means, roughly, that, whenever all propositions in  $B$  are true, necessarily *all* propositions in  $A$  are true. This conjunctive reading of multiple conclusions is also found in the work of Bernard Bolzano (1837, §155).<sup>46</sup> The reason why modern logicians prefer the disjunctive reading is mainly technical: it allows for constructing more elegant calculi by exploiting a certain formal symmetry between premisses and conclusions.<sup>47</sup> But from an intuitive point of view Aristotle's and Bolzano's conjunctive reading is arguably more natural.<sup>48</sup>

If I am correct, this reading is at the heart of Aristotle's account of circular proof. It is this reading that allows him to give a syllogistic analysis of the propositional conception of circular proof. Thus, for Aristotle, conjunctive multiple-conclusion logic serves as a bridge between propositional logic and syllogistic term logic.<sup>49</sup>

<sup>46</sup> Cellucci (1992, 179-80) suggests that Bolzano's conjunctive reading is motivated by a passage from *Prior Analytics* 2.1 in which Aristotle argues that some syllogisms 'yield more than one conclusion' (πλείω συλλογίζονται, 53a3-14).

<sup>47</sup> See Shoesmith and Smiley 1978, p. ix and 129-32; cf. also Schröter 1955, 60-1.

<sup>48</sup> See Siebel 1996, 150-2.

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