5. Schematic Reasoning v. Compositional Semantics. In Section 3 I stressed that Schema (T) as standardly understood is wholly inadequate as a theory of truth, and that inductive theories built upon compositional principles such as

For any formulas A and B and assignment function s, the formula disj(A,B) that results by putting ‘or’ between them is true relative to s if and only if either A is true relative to s or B is true relative to s

are much better because much more powerful. There is however a possibility I’d like to raise, that we might understand schematic principles like (T) in a different way which makes a theory based on them more adequate. The issue of whether this can be successfully done is not crucial to the rest of the book, so I will be brief; the Bibliographical Notes for this chapter contain a reference to a fuller account.

On the standard account of how schemas function, a schema is simply a collection of its instances; so to use a schema in axiomatizing a theory is simply to take as axioms all instances of the schema that exist in ones language. But various authors have raised the question of whether this does justice to the role that schematic principles play for us. A minimal point to make is that typically when we advocate a schema such as the law of excluded middle

(LEM) \[ p \text{ or not } p \]

or the truth schema

(T) \[ ‘p’ \text{ is true if and only if } p, \]

we are not merely advocating the collection of instances that happen to be instantiated in our language, we are expressing a commitment to continue to accept new instances as we expand the language. Of course, in saying that we implicitly commit ourselves to new instances I don’t mean to say that the commitment couldn’t be rescinded; the point is simply that this would be rescinding something we had previously been implicitly committed to.

There is a complication to be raised here: it could be that there are limitations as to how far the commitment extends. For instance, it is not absurd to suppose that in one or both of (LEM) and (T), the commitment holds only for sentences of a certain sort (perhaps “determinate, fact-stating sentences”); that it applies to other sentences (e.g. sentences that are crucially vague or sentences that invoke normative elements or some other elements that are deemed not straightforwardly factual) only under the pretense that they are “determinate” and “fact-stating”. Likewise, it is not absurd to suppose that our commitment does not extend to Liar sentences and other such “pathological” constructions. Whether these suppositions are reasonable is a matter I will have something to say about in later chapters, but my present point is that they are not totally absurd. So the exact breadth of the commitment we are making when we advocate (LEM) or (T) may be a matter of dispute; indeed, the notion of commitment is probably itself vague enough for this question to have no determinate answer. Even so, there is little doubt that if we introduce a new theoretical term in physics, then our commitment to (LEM) and (T) extends to sentences containing this new term.

It isn’t obvious that this point about the “automatic extensibility” of schemas has much bearing on the power of theories containing them: one natural view to take, certainly, is that the content of a theory expressed using (a bunch of single axioms and) a schema is simply that of (the single axioms plus) those instances of the schema that are in the current language; on this view,
the indefinite extensibility of the schema merely shows that our attitude is more than just the acceptance of this theory, it includes also a commitment to extend the theory when we extend the language. This view is certainly defensible; on the other hand, it is not absurd to imagine that we might somehow build this added commitment into the theory itself.

The idea of doing so has been suggested by Sol Feferman, in the context of number theory and set theory. (He traces the idea to earlier but somewhat less explicit sources.) Ordinary number theory contains a few single axioms plus an infinite collection of induction axioms, namely all axioms of the form

$$P(0) \land \forall x[P(x) \rightarrow P(x+1)] \rightarrow \forall xP(x)$$

(allowing free parameters in the instances of ‘P(x)’) that are in the language of the theory. Here the ‘P(x)’ is not part of the language of the theory, it is simply a device to indicate meta-linguistically what the axioms of the theory are. Feferman proposes a “schematic number theory” in which a schematic predicate letter ‘P’ is introduced into the language of the theory, with the above principle adopted for it; we then can reason in the theory with this schematic letter. Of course, we then need to be able to use the results of such schematic reasoning to draw conclusions expressed in ordinary vocabulary (vocabulary other than schematic letters); for this purpose we need a rule of substitution that allows us to obtain from any “schematic sentence” any ordinary sentence that results by uniformly substituting ordinary formulas for the schematic letters.

To what extent this succeeds in formalizing the commitment to extend a theory when we add new vocabulary to the language may not be entirely clear: one could, after all, accept this formalism but maintain that the rule of substitution applied only to the original language; one could say that the extension of the rule of substitution to an enlargement of the new language yields an expansion of the original theory, and one could even suppose that an advocate of the schematic theory in the original language was in no way committed to accepting the schematic theory in the enlarged language. But be this as it may, the idea of allowing reasoning with schematic letters is liberating: it allows us to do things we could not do before.

This is especially so, I think, in the context of truth theory, where we employ a schema with a somewhat different character than those in number theory and set theory: it is different in that a schematic letter is used both inside and outside quotes within the same sentence. Exploiting schematic reasoning involving such schemas requires a new rule of inference in addition to the rule of substitution; here I’m going beyond Feferman.

For the new rule I initially suggest the following: If \( \Theta(\theta_1,\ldots,\theta_n) \) is a schema in which all occurrences of the schematic letters \( p_1,\ldots,p_n \) are directly surrounded by quotes and not in the scope of further quotes, then if one has inferred

$$\Theta(\theta_1,\ldots,\theta_n)$$

one is licensed to conclude

For any sentences \( x_1,\ldots,x_n \), \( \Theta(x_1,\ldots,x_n) \).

Call this Rule Gen.
To illustrate the application of this rule, suppose we now consider (T) not simply as the infinite collection of its instances, but as a piece of the language that we are allowed to reason with. (To avoid issues about the paradoxes, we will understand the truth rule to apply only to sentences of L; where L is the object language, which I assume to be one for which we have a Tarskian theory.) And suppose we are faced with important generalizations like the (G) of Section 3 which, you’ll recall, were simply not provable from (T) on the ordinary understanding of that. I reproduce it:

(G) For every sentence, it is not the case that both it and its negation are true.

I claim that such generalizations are provable from the new understanding of schemas, using Rule Gen. For instance, to prove (G) we reason as follows:

Step One: If ‘p’ is true then p; if ‘not p’ is true then not p; so if both ‘p’ and ‘not p’ are true then p and not-p, which is classically contradictory. Therefore it is not the case that both ‘p’ is true and ‘not p’ is true.

Step Two: ‘not p’ is the negation of ‘p’; so by the result of Step One, it is not the case that both ‘p’ is true and the negation of ‘p’ is true.

Step Three: By the result of Step Two and Rule Gen, we get (G).

We see, then, that the Tarski Schema (T) becomes a vastly more powerful theory in the context of schematic reasoning, taken to include Rule Gen. If one thinks of (T) in this way, it is no longer completely evident that the inductive truth theory presented in Section 3 has a genuine advantage over it.

The heart of the inductive theory in Section 3 was, of course, its compositional principles, principles like

The disjunction of A and B is true if and only if either A is true or B is true.

(For the moment I drop the relativity to an assignment function, i.e. I restrict to the case where A and B are sentences.) Those principles too embody important generalizations, ones we wouldn’t want to give up. So if the current suggestion is viable, those compositional principles had better also be derivable from “schematic (T)”. But they are:

Step One: ‘p’ is true if and only if p, and ‘q’ is true if and only if q; so (‘p’ is true or ‘q’ is true) if and only if p or q. But in addition, ‘p or q’ is true if and only if p or q. Putting these together, we have that ‘p or q’ is true if and only if ‘p’ is true or ‘q’ is true.

Step Two: ‘p or q’ is the disjunction of ‘p’ and ‘q’; so by the result of Step One, the disjunction of ‘p’ and ‘q’ is true if and only if ‘p’ is true or ‘q’ is true.

Step Three: So by Rule Gen, the disjunction of any two sentences is true if and only if one of those sentences is true.

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1 We will see later on in the book that there is a way to maintain (T) even as applied to the full metalanguage, but it requires a weakening of the underlying logic.
We have achieved what we needed.

The treatment of this compositional rule was simplified a bit by restricting it to the case where A and B are sentences; when A and B are allowed to contain free variables, we must generalize the rule by evaluating relative to an assignment function. But it turns out that one can get the generalized compositional rule by essentially the above route, if one starts out with a correspondingly generalized form of Schema (T). The generalized Schema (T) must contain schematic letters not for sentences, but for formulas with arbitrary numbers of free variables; what we essentially do is mimic the reasoning above, using a version of Rule Gen appropriate to schematic letters of this sort. I will skip the details here: the Bibliographical notes for the chapter contains reference to a paper that discusses the matter in more detail. That paper also discusses how to obtain the compositional rule for atomic sentences, and various other subtleties which would be out of place to discuss here.

It turns out that when one spells out the schematic reasoning approach with full attention to these subtleties, it is a bit messy: one needs several different kinds of schematic letters, with a separate rule of generalization for each. Because of this, the inductive approach is more useful than the schematic reasoning approach for certain kinds of logical investigations. Nonetheless, I think there is no serious doubt that this idea of schematic reasoning based on the truth schema is workable, and that it has the full power of the inductive approach discussed in Section 3.

I will also mention a conjecture, for which I will give even less detail: that just as compositional principles about truth can be derived by schematic reasoning that takes off from Schema (T), so too can compositional principles about meaning be derived by schematic reasoning that takes off from the following schema:


The main modification that would be required for this is a further expansion in the rule of generalization, one taking you from schemas that can contain clauses of the form ‘that p’ where ‘p’ is a schematic letter, as well as schematic letters that appear directly inside quotes. In the corresponding generalization, one has a quantifier over propositions in place of each schematic letter that appears directly inside the scope of a ‘that’, and a quantifier over sentences in place of each schematic letter that appears directly inside quotes; and if the same schematic letter is used in both ways in the same sentence then the corresponding quantifiers over sentences and propositions are restricted by the condition that the sentence expresses the proposition.

What philosophical significance do schematic truth theory and schematic meaning theory have? There has been a tendency in some philosophical quarters to worship compositional principles as metaphysically fundamental to the theory of truth and/or the theory of meaning. If what I’ve said is correct, then one could equally well argue that what is really fundamental isn’t the compositional principles, but rather the basic schemas (T) and (M).

If forced to choose between these positions I might slightly prefer the second, but my real view is that both positions are suspect, for both rest on the suspect idea that one way of presenting the theory of truth or meaning is “metaphysically fundamental”. I’d be suspicious of the claim that the schematic presentation of the theory of truth for the sort of object language we’ve been considering is superior to the compositional approach in Section 3: I see nothing in
any way problematic or misleading in the compositional approach. (Indeed, I’ve already remarked that the compositional approach is far easier to work with mathematically, at least in the case of the structurally simple languages that we’ve considered and that are of central importance in formal logic; that makes the compositional approach far more useful than the schematic in many contexts, for instance, for research into the semantic paradoxes.) On the other hand, it does seem to me philosophically important to see that the compositional approach is not inevitable: to see that the virtues that can be obtained by taking compositional principles as fundamental can also be obtained in another way. And for what it’s worth, the schematic presentation seems more directly based on the way in which we customarily learn to use the word ‘true’.

But there is another point to be made, which may in a sense tend to elevate the schematic-based approach over the compositional. It is that while compositional principles for logical notions are easy to attain, there are other notions for which they are not so easy. A famous example is attitude sentences: Jones believes that p, hopes that p, etc. A vast industry has grown up trying to get one’s compositional truth theories and/or compositional meaning theories to extend to attitude sentences. Could it be that this idea is misguided? We should of course accept instances of schemas (T) and (M) that involve attitude sentences: presumably even the most militant compositionalist doesn’t believe that we should suspend judgement on the claim

‘Jones believes that snow is white’ is true if and only if Jones believes that snow is white

until such time as we have succeeded in coming up with a compositional truth theory from which it follows. But I’m attracted to the more radical thought that in the case of the attitude sentences there is simply no need for the compositional theory.

The idea is this: the fact that we have a compositional truth rule for disjunction rests on the argument above, and the fact that we have a compositional meaning rule for disjunction rests on an analogous argument. But the argument essentially turned (in Step One) on a substitutivity principle for ‘or’: the principle that from

\[ p \text{ if and only if } p^* \]
\[ q \text{ if and only if } q^* \]

one can infer

\[ p \text{ or } q \text{ if and only if } p^* \text{ or } q^*. \]

This is a fact about the logic of ‘or’. The analogous assumption fails for ‘believes that’; for that reason, the schema

\[ (B) \quad 'S \text{ believes that } p' \text{ is true if and only if } S \text{ believes that } ('p' \text{ is true}) \]

\[ \]

Such a suggestion has been made before, in Schiffer 1987; but his concern was largely with meaning theory rather than truth theory, and he did not concern himself with how to give a non-compositional theory of truth (a theory that systematizes the important generalizations about truth that we all believe).
is suspect. (Even if we restrict to agents S who speak English, they may have peculiar beliefs about truth.) Moreover (1) in order for the composition clause to be usable in a full compositional semantics, we’d also need other applications of substitutivity that are likewise dubious; e.g., we’d need that S believes that ‘p or q’ is true if and only if S believes that ‘p’ is true or S believes that ‘q’ is true. (2) Even if one were to accept Schema (B), it isn’t obvious that one could turn it into a useful generalization by Rule Gen. In part this is because Rule Gen seems to depend for its plausibility on the assumption that we can unproblematically quantify into the context \( \Theta \); the appropriateness of Rule Gen in the ‘or’ case thus turned on the fact that it is possible to unproblematically quantify into ‘or’ contexts, a fact that has no analog in the case of ‘believes that’ or ‘hopes that’.

This suggests that it may simply be misguided to look for compositional truth or meaning principles for attitude constructions.

It also suggests that the fact that compositional principles of truth or meaning are straightforward for some constructions but not for others is not fundamentally a fact about the application of the notion of truth or meaning to different constructions, but is simply a fact about the underlying logic of those constructions. Facts about the logic of these constructions explains the facts about how the notions of truth and meaning that apply to them, rather than the other way around. To me, there seems to be an important difference between explaining a phenomenon in terms of truth or meaning and explaining it in terms of logic. It won’t seem like much of a difference to anyone convinced that one can only explain logic in terms of truth or meaning; but that is a view that I will consider and reject in chapter Three.