

## Set Theory (F01): Exercises

### Exercises on the Properties of Relations (Optional)

1. A relation is an *equivalence* if it is reflexive, transitive and symmetric.

(i) Show that these properties are independent, i.e. there is a relation having any two of these properties without having the third;

(ii) How many equivalence relations are there on a domain with one element? A domain with two elements? A domain with three elements?

(iii) Show, given any domain, that there is an equivalence relation on that domain which is included in all other equivalence relations on that domain and that there is an equivalence relation on that domain which contains every equivalence relation on that domain.

(iv) Show (this is harder) that any subset of equivalence relations on a given domain has a lub and glb.

2. A relation is an *order* if it is reflexive, transitive, antisymmetric and connected (on the domain of the relation).

(i) show that these properties are independent of one another.

(ii) how many orders are there on a domain with one element? Two elements? Three elements?

(iii) Show that no order on a given domain is included in any other order on that domain.

Hence show, given any domain with more than one element, that there is no order on that domain which contains all other orders on that domain or that is included in all other orders on that domain.