Set Theory (F01): Alternative Assignment

1. Provide detailed proofs of each of the following (i.e., specify the relevant function and establish that it is a one-one correspondence of the required sort):

   (i) $\varnothing \times A \sim \varnothing$;

   (ii) $\{a\} \times A \sim A$, for any a;

   (iii) $A \times B \sim B \times A$; and

   (iv) $A \times (B \times C) \sim (A \times B) \times C$.

Hence show that $0.\kappa = 0$, $1.\kappa = \kappa$, $\kappa.\lambda = \lambda.\kappa$ and $(\kappa.\lambda),\mu = (\kappa.\lambda).\mu$ for any cardinal numbers κ, λ, and μ.

2. Provide detailed proofs of the following:

   (i) $A^{\varnothing} \sim \{\varnothing\}$ (hint: even the empty set is a function!);

   (ii) $A^{\{b\}} \sim A$, for any b; and

   (iii) $A^{\{b, c\}} \sim A \times A$, for any distinct a and b.

Hence show that $\kappa^0 = 1$, $\kappa^1 = \kappa$ and $\kappa^2 = \kappa \cdot \kappa$ for any cardinal number κ.

3. Suppose that B and C are disjoint sets. Provide detailed proofs of the following:

   (i) $A \times (B \cup C) \sim (A \times B) \cup (A \times C)$;

   (ii) $A^{B \cup C} \sim A^B \times A^C$.

Hence show that $\kappa.(\lambda+\mu) = \kappa.\lambda + \kappa.\mu$ and $\kappa^{\lambda+\mu} = \kappa^\lambda.\kappa^\mu$.