Mock Midterm (Logic S01)

Answer three questions.

1. (i) What is a formula of SL?
   (ii) Let $\text{nc}(A)$ be the number of occurrences of ‘v’ in A and $\text{ns}(A)$ the number of occurrences of sentence-letters in A. Show by formula induction that $\text{ns}(A) = \text{nc}(A) + 1$.

2. (i) What is an assignment? What are the conditions for a statement $(A \& B)$ to be true or false under a given assignment $\alpha$?
   (ii) Let $A^d$ be the result of prefixing each sentence letter in A with ~ and of replacing each occurrence of ‘v’ in A with ‘&’. Show by formula induction that $\alpha = A$ iff not $\alpha = A^d$. (From this it follows that A is a tautology iff $\sim A^d$ is a tautology).

3. For each of the following statements, say whether it is true or false and give reasons for your answer:
   (i) Every substitution-instance of a satisfiable formula is satisfiable;
   (ii) Every substitution-instance of a tautology is a tautology;
   (iii) Any two formulas have a substitution-instance in common;
   (iv) Any two formulas are substitution-instances of a common formula.

4. (i) What are the axioms and rules of the system SL?
   (ii) Show (without using any derived rules) that $(A \vdash \sim A) \vdash A$ is a theorem of SL.

5. (i) Provide a statement of strong completeness, of weak completeness and of compactness.
   (ii) Show that strong completeness is a consequence of weak completeness and compactness.