

Mock Midterm (Logic S01)

Answer three questions.

1. (i) What is a formula of SL?
(ii) Let $nc(A)$ be the number of occurrences of 'v' in A and $ns(A)$ the number of occurrences of sentence-letters in A. Show by formula induction that $ns(A) = nc(A) + 1$.

2. (i) What is an assignment? What are the conditions for a statement (A&B) to be true or false under a given assignment α ?
(ii) Let A^d be the result of prefixing each sentence letter in A with \sim and of replacing each occurrence of 'v' in A with '&'. Show by formula induction that $\alpha \models A$ iff not $\alpha \models A^d$. (From this it follows that A is a tautology iff $\sim A^d$ is a tautology).

3. For each of the following statements, say whether it is true or false and give reasons for your answer:
 - (i) Every substitution-instance of a satisfiable formula is satisfiable;
 - (ii) Every substitution-instance of a tautology is a tautology;
 - (iii) Any two formulas have a substitution-instance in common;
 - (iv) Any two formulas are substitution-instances of a common formula.

4. (i) What are the axioms and rules of the system SL?
(ii) Show (without using any derived rules) that $((A \supset \sim A) \supset A)$ is a theorem of SL.

5. (i) Provide a statement of strong completeness, of weak completeness and of compactness.
(ii) Show that strong completeness is a consequence of weak completeness and compactness.