Mock Final

Answer four questions.

1. Let SL + Δ be the result of adding the formulas of Δ as axioms to the system SL. Show the following (you may presuppose that the system SL is complete):
   (i) SL + Δ is sound iff each formula of Δ is a theorem of SL;
   (ii) SL + Δ is sufficient for any set of formulas Δ;
   (iii) SL + Δ is complete (i.e. sound and sufficient) iff each formula of Δ is a theorem of SL;
   (iv) every formula is a theorem of SL + Δ iff Δ is inconsistent.

2. (i) Given a formula A, let Ay/x be the result of substituting y for every occurrence of x in A.
   (i) Under what conditions is A the same formula as Ay/x? Justify your answer.
   (ii) Under what conditions is Ay/x the same formula as Ax/y? Again, justify your answer.

3. (i) What is a model for PL? An interpretation? When is a formula of PL valid?
   (ii) Suppose that truth and falsehood in an interpretation are defined in the usual way but that negation is subject to the following clause:
   \[ I \models \neg A \iff I \models A.\]
   Show that no formula is then valid. (Hint: consider an interpretation in which each atomic formula is false).

4. Using TFC as the sole derived rule, show that the following are derivable in PL: Px \(\Rightarrow\exists xPx;\)
   \(\forall xPx \Rightarrow \exists xPx; (\forall xPx \& \exists xQx) \Rightarrow \exists x(Px \& Qx); (\forall x(Px \Rightarrow Qx) \& \exists xPx) \Rightarrow \exists x(Px \& Qx).\)

5. For each of the following formulas, either produce a counter-model or show that it is valid:
   \(\forall x \exists yRxy \Rightarrow \exists y\forall xRxy; \exists y\forall xRxy \Rightarrow \forall x \exists yRxy; \forall x \exists y\forall zRxyz \Rightarrow \exists y\forall x\forall zRxyz; \exists y\forall x\forall zRxyz \Rightarrow \forall x \exists y\forall zRxyz.\)

6. Given an extended valuation \(\alpha\) for PL, what is the corresponding canonical interpretation \(I_\alpha\)? Let us say that \(I_\alpha\) agrees with \(\alpha\) if \(I_\alpha\models A\) whenever \(\alpha\models A\). Show that:
   (i) it is not true that, for every extended valuation \(\alpha\), \(I_\alpha\) agrees with \(\alpha\);
   (ii) it is not true that, for any interpretation \(I\), there is an extended valuation \(\alpha\) for which \(I = I_\alpha\);
   (iii) it is true that, for any interpretation \(I\) whose domain is the set of variables and whose assignment maps each variable into itself, there is an extended valuation \(\alpha\) for which \(I = I_\alpha\).