Mock Final

Answer four questions.

1. Let SL + Δ be the result of adding the formulas of Δ as axioms to the system SL. Show the following (you may presuppose that the system SL is complete):
(i) SL + Δ is sound iff each formula of Δ is a theorem of SL;
(ii) SL + Δ is sufficient for any set of formulas Δ;
(iii) SL + Δ is complete (i..e. sound and sufficient) iff each formula of Δ is a theorem of SL;
(iv) every formula is a theorem of SL + Δ iff Δ is inconsistent.

2. (i) Given a formula A, let Ay//x be the result of substituting y for *every* occurrence of x in A.
 (i) Under what conditions is A the same formula as Ay//x? Justify your answer.
 (ii) Under what conditions is Ay//x the same formula as Ax//y? Again, justify your answer.

3. (i) What is a model for PL? An interpretation? When is a formula of PL valid?
 (ii) Suppose that truth and falsehood in an interpretation are defined in the usual way but that negation is subject to the following clause:
 $I \models {\sim}A$ iff $I \models A$.
Show that no formula is then valid. (Hint: consider an interpretation in which each atomic formula is false).

4. Using TFC as the sole derived rule, show that the following are derivable in PL: $Px \supset \exists xPx$; $\forall xPx \supset \exists xPx$; $(\forall xPx \,\&\, \exists xQx) \supset \exists x(Px \,\&\, Qx)$; $(\forall x(Px \supset Qx) \,\&\, \exists xPx) \supset \exists x(Px \,\&\, Qx)$.

5. For each of the following formulas, either produce a counter-model or show that it is valid: $\forall x\exists yRxy \supset \exists y\forall xRxy$; $\exists y\forall xRxy \supset \forall x\exists yRxy$; $\forall x\exists y\forall zRxyz \supset \exists y\forall x\forall zRxyz$; $\exists y\forall x\forall zRxyz \supset \forall x\exists y\forall zRxyz$.

6. Given an extended valuation α for PL, what is the corresponding canonical interpretation I_α? Let us say that I_α *agrees with* α if $I_\alpha \models A$ whenever $\alpha \models A$. Show that:
(i) it is not true that, for every extended valuation α, I_α agrees with α;
(ii) it is not true that, for any interpretation I, there is an extended valuation α for which $I = I_\alpha$;
(iii) it *is* true that, for any interpretation I whose domain is the set of variables and whose assignment maps each variable into itself, there is an extended valuation α for which $I = I_\alpha$.