

## Assignment

1. Merely use the Deduction Theorem, Modus Ponens and the basic structural properties of  $\vdash$  to show that the following formulas are theorems:

$$\begin{aligned} & (A \supset (B \supset B)), \\ & (((A \supset A) \supset B) \supset B), \\ & (A \supset (B \supset C)) \supset ((A \supset (C \supset D)) \supset (A \supset (B \supset D))). \end{aligned}$$

2. Use the Deduction Theorem to show that  $\Delta \vdash A$  iff there are finitely many formulas  $A_1, A_2, \dots, A_n$  in  $\Delta$ ,  $n \geq 0$ , such that  $\vdash (A_1 \supset (A_2 \supset \dots (A_n \supset A) \dots))$ .

3. Let  $P = \{p_1, p_2, \dots\}$  be a (possibly infinite) group of people and suppose that certain finite non-empty subgroups  $Q_1, Q_2, \dots$  of  $P$  are 'incompatible'. Call a subgroup  $P'$  of  $P$  *harmonious* if it contains no incompatible subgroup.

(i) Show that there is a maximally harmonious subgroup of  $P$ . (Hint: follow the proof strategy of Lindenbaum's Lemma);

(ii) Show that there may be no *unique* maximally harmonious subgroup (Hint: suppose that the only incompatible subgroup is  $\{p_1, p_2\}$ );

(iii) Show that result under (i) still holds when the incompatible subgroups are allowed to be infinite but there are only finitely many of them. (Hint: look at a smallest subgroup of  $P$  that overlaps with each of the incompatible subgroups; and let the maximally harmonious subgroup be its complement.)

(iv) Show that the result under (i) fails when infinitely many infinite incompatible subgroups are allowed. (Hint: suppose that *all* infinite subgroups are incompatible).