Assignment

1. Merely use the Deduction Theorem, Modus Ponens and the basic structural properties of $\vdash$ to show that the following formulas are theorems:

   $(A \supset (B \supset B))$,

   $(((A \supset A) \supset B) \supset B)$,

   $(A \supset (B \supset C)) \supset ((A \supset (C \supset D)) \supset (A \supset (B \supset D)))$.

2. Use the Deduction Theorem to show that $\Delta \vdash A$ iff there are finitely many formulas $A_1, A_2, \ldots, A_n$ in $\Delta$, $n \geq 0$, such that $\vdash (A_1 \supset (A_2 \supset \ldots (A_n \supset A) \ldots))$.

3. Let $P = \{p_1, p_2, \ldots\}$ be a (possibly infinite) group of people and suppose that certain finite non-empty subgroups $Q_1, Q_2, \ldots$ of $P$ are ‘incompatible’. Call a subgroup $P'$ of $P$ harmonious if it contains no incompatible subgroup.

   (i) Show that there is a maximally harmonious subgroup of $P$. (Hint: follow the proof strategy of Lindenbaum’s Lemma);

   (ii) Show that there may be no unique maximally harmonious subgroup (Hint: suppose that the only incompatible subgroup is $\{p_1, p_2\}$);

   (iii) Show that result under (i) still holds when the incompatible subgroups are allowed to be infinite but there are only finitely many of them. (Hint: look at a smallest subgroup of $P$ that overlaps with each of the incompatible subgroups; and let the maximally harmonious subgroup be its complement.)

   (iv) Show that the result under (i) fails when infinitely many infinite incompatible subgroups are allowed. (Hint: suppose that all infinite subgroups are incompatible).