Assignment

1. Let PL^- be the same as PL but without the Axiom of Specification. Show that $\forall xPx \supset Px$ is not a theorem of PL^- . (Hint: consider extended valuations in which every universal formula is made true. Show by induction that each theorem of PL^- is true in every such extended valuation but that $\forall xPx \supset Px$ is not).

2. Say that an existential formula $\exists xA(x)$ is *implicitly witnessed* in a set of formulas Δ if there is a variable y for which $\exists xA(x) \supset A(y)$ is derivable from Δ. Show that every existential formula is implicitly witnessed in Δ if every existential theorem of PL is implicitly witnessed in Δ. (Hint: show that $\exists y(\exists xA(x) \supset A(y))$ is a theorem of PL as long as y does not occur free in $\exists xA(x)$).

3. Given an interpretation I, let $Th_I = \{A: I \models A\}$. Which of the following are true? Which false? Give your reasons.

(i) For any interpretation I, there are infinitely many formulas that are witnessed in Th_I;

(ii) For any model M whose domain is the set of natural numbers, there is an interpretation $I = (M, \delta)$ for which Th_I is fully witnessed.

(iii) For any model M, there is an interpretation $I = (M, \delta)$ for which Th_I is not fully witnessed.