Meta-Logic (S01): Assignment

1. Give a rigorous proof on the basis of the truth-definition that the following formulas are valid: (∀xPx v ~∀xPx); ∀x(Px v ~Px); ∀x(Px v ~∀xPx).

2. (i) A formula A (possibly containing free variables) is said to be *true under* a model M if M|=A[δ] for any assignment δ and to be *false under* a model M if M =| A[δ] for any assignment δ (I use '=|' for 'falsifies). Show that under this definition a formula cannot be both true and false but may be neither true nor false.

(ii) Suppose that we had defined a formula A to be *true under* a model M if M|=A[δ] for *some* assignment δ and to be *false under* a model M if M =| A[δ] for *some* assignment δ. Show that under this new definition a formula must be true or false but may be both truth and false.

3. Consider the following formulas:

T(ransitivity). ∀x∀y∀z(Rxy & Ryz ⊃ Rxz);
A(symmetry). ∀x∀y(Rxy ⊃ ~Ryx);
I(rreflexivity). ∀x~Rxx.

In the case of each formula, consider whether it is a consequence of the other two. Either show that it is or produce a counter-model to show that it is not.