The Technology Cycle and Inequality

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Abstract

Trade in technology has risen in recent years, and intellectual property protection is now a major issue in trade negotiations. What would happen if markets in technology were perfect? Would poor countries be unable to afford the best technologies and if so, how would that affect their incomes? I analyze how technology would spread in a frictionless market and the income inequality that would arise. The estimated model generates a technology cycle of 68 years, but an inequality factor of only 2.3. Current resistance to worldwide protection of intellectual property thus seems unfounded.

1 Introduction

Intellectual property protection is now a big issue in the WTO negotiations for a new world trade deal, and developing countries are resisting stronger protection of those rights. The likely reason for why they resist IP protection is the positive association, documented recently by Comin and Hobijn (2004), between a country’s income and the average age of the technologies that its residents use. The fear is that frontier technologies will be too expensive for developing countries and that this will lower their incomes.

In fact, markets for technology are becoming more active; some measures of activity in these markets are:

- Licensing revenues.—As a percentage of R&D costs, royalty receipts (from abroad) in 2001 for patents, licenses, and copyrights were 64 (U.K.), 36 (Italy), 31 (Germany), 15 (U.S.), 11 (France) and 8 (Japan) (OECD 2004, tables 69-71).

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Serrano (2006) finds that 18 percent of patents granted to small inventors are traded at least once in their lives, and that the citations-weighted percentage is even higher. Large firms often sell their patents and enter into patent-sharing agreements.

- **International patenting.**—Eaton and Kortum (1999, Table 1) document that the U.S., the U.K., France, Germany and Japan patent abroad about one fifth of the patents that they take out domestically, probably the most valuable fifth.

- **Cross-border mergers.**—These have grown a lot in recent years and one function they perform is that of technology transfer. Target prices always reflect the intellectual property of the target and acquirer.

What would happen if markets for technology were frictionless? To answer this question, I set up a model in which technologies are associated with products, and in which new products are better than old products – a vintage-technology model. Each agents chooses a technology in a way that reflects his skill and the licensing costs of the available technologies. Skilled agents choose new technologies, unskilled agents choose old technologies. As it ages, a technologies moves down the skill chain, and is eventually abandoned. We may call this process the technology cycle. The length of that cycle is endogenous, as is the distribution of skills and the rate of growth.

**Results.**—The model explains observed diffusion lags, but not the bulk of the world’s income inequality. The extent of income inequality – the ratio of richest to poorest – depends in the model only on the strength of learning by doing in invention, in Arrow’s (1962) technical sense. Using information on incomes derived from invention, I estimate that learning parameter to be too small to account for most inequality. the reason is simple: Since technology improves at roughly one percent per year, a 63-year diffusion lag (which is what the data lead us to estimate) would imply a technology differential of $e^{.01 \cdot 63} = 1.88$. A very elastic response of the complementary factors would be needed for such a modest technology differential to produce a large effect. In the model, the elasticity of the complementary factors is unity, and the richest-to-poorest ratio consistent with observed diffusion lags is only 2.27. Thus there is little reason to fear that fully enforced rights on intellectual property would cause large development gaps.

The model relates to many others. The growth side combines a Lucas (1988) type of technology for investment in skills with an Arrow (1962) type of technology for invention. The assignment of skills to technologies is frictionless and continually changing as in Jovanovic (1998). Its technology cycle relates to those in Matsuyama (2002), who shows that income inequality is good for technological adoption, that each new product is bought first by the rich and then by the poor, and that diffusion lags depend positively on income dispersion, and to Antras (2005), who argues that as a product ages it needs less management and will then be produced offshore. Finally, while the allocative role of prices in markets for technology is as yet unexplored,
four other reasons have been offered for why agents do not all use the same, frontier technology: Technology-specific skills (Chari and Hopenhayn 1991), physical capital endowments (Basu and Weil 1999), incentives to free-ride (Eeckhout and Jovanovic 2002), and policy differences (Jones 1994).

Section 2 outlines the model’s implications, and reports the tests of those implications and the parameter estimates. Section 3 presents the full model, while Section 4 discusses its more tangential implications. Section 5 concludes the paper.

2 Cross-sectional implications and estimation

The paper reports only cross section tests, and it is easier to explain what the model does by beginning with the tests themselves. Only a small portion of the model underlies these tests, and we shall start out by explaining that portion alone. In the end, however, the reader will want to know why parameters cannot be chosen so as to raise the amount of inequality that technology diffusion lags can explain. The answer to that question will reside in the other parts of the model.

2.1 The assignment

An agent’s income, $Y$, depends on his skill, $s$ and the technology quality, $z$, that he uses:

$$Y = 2A_t(zs)^{1/2}.$$  

The parameter $A_t$ reflects aggregate demand and grows at the rate $g$. A menu of technologies is available, namely $z \in (0, z_{\text{max}}]$, and any $z$ in this set can be used upon payment of the license fee of $p(z)$. Then the net income of an $s$-skilled person is

$$\pi(s, z) = Y - p(z).$$

No markets for $s$ exist, so that a person is constrained to his own $s$, and the problem will be one of assigning technology to skill on a one-to-one basis. Why each person will wish to choose a different technology will be explained in Section 3.

**Technology choice.**—Given his skill, the producer chooses that technology $z$ that maximizes $\pi(s, z)$. The first-order condition is

$$A_t \left( \frac{s}{z} \right)^{1/2} = p'(z).$$  

**Adding up constraint on the assignment.**—Let the measure of technology qualities exceeding $z$ be $F(z)$ and the measure of skills exceeding $s$ be $G(s)$. The model endogenizes these distributions later. Suppose that the measure of agents is unity so that the quality, $s_{\text{min}}$ of the lowest skill satisfies $G(s_{\text{min}}) = 1$. If skills are fully
employed and if the best technologies are used, a positive assignment $z = \psi(s)$ satisfies $F(\psi [s]) = G(s)$, i.e.,

$$\psi(s) = F^{-1} (G[s]).$$

(3)

The worst technology in use, $z_{\text{min}}$, satisfies

$$z_{\text{min}} = \psi(s_{\text{min}}) = F^{-1} (G[s_{\text{min}}]).$$

If the process of technological invention has gone on for a long time, there will be many more technologies than agents, and all those technologies with $z < z_{\text{min}}$ will have been abandoned and will rent at a zero price. By continuity of $p$, this means, that

$$p(z_{\text{min}}) = 0.$$  (4)

The case of log-uniform $z$ and $s$.—Suppose that $\ln z$ and $\ln s$ are uniformly distributed so that $F(z) = a(\ln z_{\text{max}} - \ln z)$ and $G(s) = b(\ln s_{\text{max}} - \ln s)$. This will be a necessary implication of constant growth. Then (3) implies the linear assignment

$$z = \psi(s) = \frac{b}{a}s,$$

(5)

and (2), which now reads $A_t \left( \frac{a}{b} \right)^{1/2} = p'(z)$, and (4) imply that

$$p(z) = A_t \left( \frac{a}{b} \right)^{1/2} (z - z_{\text{min}}).$$

(6)

2.2 Estimates

This subsection reports the model’s implications for three cross-section relations, and the parameter estimates based on those implications.

2.2.1 The relation between income and age of technology

Incomes grow at the rate $2g$, where $g$ is the common growth rate of all skills and the growth rate of the technology frontier. We shall use the estimate $2g = 0.0235$. Then $\hat{g} = 0.012$. The equilibrium cross-section relation between agent’s income, $Y$, and the age, $\tau$, of the technology that he uses at date $t$ is

$$Y \equiv e^{2gt} e^{-g\tau},$$

for $\tau \leq T$. Then the relation between relative income and the age of the technology used is linear:

$$\ln Y_{\text{max}} - \ln Y = g\tau.$$  (7)

1This is the midpoint of 2.37% (estimate of world real GNI per-capita growth from the World Development Indicators of the World Bank 1962-2005) and 2.33% (estimate of US real GDP per-capita growth from the Penn World Table 1951-2000).
Data from Comin and Hobijn (2004) support such linearity – see Figure 1.

Replacing “max” by “USA,” we have

\[ t_i - t_{USA} = -\frac{1}{\dot{g}} (\ln Y_{US} - \ln Y_i) \].

A plot of the two sides of (8) is in Figure 1. The slope is negative and significant. The regression line should pass through the point \((0, 0)\) which it does almost exactly. If countries were homogeneous in \(h\), the regression’s slope would, in theory, be \(-\frac{1}{\dot{g}}\). With \(\dot{g} = .012\) this would be 83.3. But countries are not homogeneous: Table 5 of Sala-i-Martin (2005) shows that within-country inequality is between 28% and 38% of inequality worldwide. Therefore the slope should have been \(-\left(\frac{3}{3}\right) 83.3 = -55.5\), and it is significantly smaller than that. This is another way of saying that there are other reasons for per-capita income differences.

We also can express (6) in terms of income, \(Y\). The cross-section relation, at date \(t\), between income and the amount \(p\) paid for technology by an agent with income \(Y\) is

\[ p = Ce^{2\dot{g}t} \left(1 - \frac{Y_{\text{min}}}{Y}\right), \]

where \(C\) is a constant. Thus the share of technology in income rises with income overall, starting at zero when \(Y = Y_{\text{min}}\), and ending up at a positive level (see Figure 5). This is consistent with the finding by Caselli and Coleman (2001) that the income elasticity of imports of technology exceeds unity.\(^2\)

\(^2\)The univariate results in their Table A.2 are the appropriate ones because in this model \(s\) varies
2.2.2 Estimating $gT$ using patent-income data

Consider a technology that was at the frontier at date $t$ and, hence, was of quality $z_{\text{max}}(t)$. At that point, its license fee was $p_t(z_{\text{max}}[t])$. Then let $p_{t+\tau}(z_{\text{max}}[t])$ denote its license fee $\tau$ periods later, i.e., at date $t + \tau$. Then (6) implies that

$$p_{t+\tau}(z_{\text{max}}[t]) = A_{t+\tau} \left( \frac{a}{b} \right)^{1/2} (z_{\text{max}}(t) - z_{\text{min}}(t + \tau)).$$

Its license fee at $t$ relative to its license fee at age zero is

$$\frac{p_{t+\tau}(z_{\text{max}}[t])}{p_t(z_{\text{max}}[t])} = \left( \frac{A_{t+\tau}}{A_t} \right) \frac{z_{\text{max}}(t) - z_{\text{min}}(t + \tau)}{z_{\text{max}}(t) - z_{\text{min}}(t)} = e^{g\tau} \frac{1 - e^{g(\tau-T)}}{1 - e^{-gT}}$$

On the RHS of (10), at first, the aggregate demand effect (i.e., the growth of $A$) is stronger than the obsolescence effect, and patent revenues rise until they peak at the point $t^* = T - \frac{\ln 2}{g}$. This peak will exist as long as $gT > \ln 2$. Since $\ln 2 = 0.7$, we get the estimator

$$gT = 0.7 + \hat{g}t^*.$$  

Once again $\hat{g} = 0.012$. As for $t^*$, we shall seek to measure it using the information in Figure 2. This is the empirical counterpart of (10), and we have the estimate $\hat{t}^* = 10$ with which (11) yields $gT = 0.82$.

The data, provided by and described in Giummo (2005), contain information on 1172 inventions originally patented in Germany in 1977-1982 and their patents all with $z$ and cannot be held constant as $z$ varies.
renewed for the maximum 20 subsequent years. Each was also patented in the U.S., so that we may think of these inventions as perhaps having been licensed internationally, and having been among the most valuable inventions of their day. German law requires that firms estimate incomes derived from their employees’ inventions for each year until the patent expires, and this allows us to infer income derived as a function of patent age. Figure 2 indicates that \( \hat{t}^* = 10 \) which, when substituted into (11), implies that \( \hat{T} = 0.7\frac{0.7}{0.012} + 10 = 68.3. \)

While \( \hat{T} \) is large, direct measures of the speed of diffusion of technology suggest that for major technologies this estimate is reasonable. Many people in the world still use animal power for plowing, e.g., even though the tractor was commercialized by 1910. Many still do not have access to electricity, a technology that was widely implemented 100 years ago; using the Comin and Hobijn (2004) data, in Figure 3 we show the long diffusion lags for electricity by region.

2.2.3 Income inequality

The price function in eq. (6) has four parameters of which \( A \) and \( z_{\text{min}} \) change over time; In particular,

\[
A_t = A_0 e^{gt}, \quad z_{\text{max}}(t) = z_{\text{max},0} e^{gt}, \quad \text{and} \quad z_{\text{min}}(t) = z_{\text{min},0} e^{gt}.
\]

As \( s_{\text{min}} \) and \( s_{\text{max}} \) also grows at the rate \( g \), the ratio \( a/b \) is also constant. Let \( T \) be the age of the oldest technology in use. Then (5) and (11) imply that

\[
\frac{z_{\text{max}}}{z_{\text{min}}} = \frac{s_{\text{max}}}{s_{\text{min}}} = e^{gT} = e^{0.82} = 2.27.
\]
This estimate is roughly what one would have expected: Since technology improves at 1.2 percent per year, a 63-year diffusion lag would imply a technology differential of $e^{(0.012)63} = 2.13$. True, the coefficient of technology in (1) is 0.5, but this is undone by the perfect positive assignment (5).

Since the size of the firm is fixed, could we followed Kremer (1993) who also has a fixed firm size, and written (1) as

$$\text{output} = 2A_t(zs)^n$$

for $n > 1/2$? This would produce more inequality in response to a given variation in $z$ and $s$. But this is where the supply side of $z$ and $s$ begins to matter and where we have to consult the rest of the model. For $n > 1/2$, $p(z)$ would be convex in $z$ (see Kremer’s equation [10], e.g.) and $\pi(s, \psi[s])$ would become convex in $s$. Given the linear technology that the model uses for generating growth in $s$ (Section 3.2) and $z$ (Section 3.3), second order conditions would fail. Moreover, for $n$ sufficiently large, $p$ would monotonically decline in $\tau$, contrary to what Figure 2 shows.

## 3 The Model

Trade is frictionless. The following four world markets exist:

- Final goods – perfectly competitive
- Intermediate goods – monopolistically competitive producers
- Labor markets – perfectly competitive
- Technologies – perfectly competitive

We thus have a single world economy; we shall map agents into countries when we discuss world inequality, but we shall then offer no theory of why similarly-endowed agents tend to cluster together.

**Final-goods.**—Final goods producers are competitive. There is just one final good in the world that is produced using intermediate goods only. The world output of the final good is

$$y = \left( \int_0^\infty x_i^{1/2} di \right)^2,$$

where $x_i$ is the quantity of the $i$'th intermediate good. Let $P_i$ be the price of good $i$ in units of the final good. For factor shares to be constant in this model as the economy grows, we shall need the elasticity of substitution in production to equal 2. The final-goods producers problem is

$$\max_{(x_i)_{i\in I}} \left\{ y - \int_0^\infty P_i x_i di \right\}$$

with the first-order condition

$$y^{1/2} x_i^{-1/2} - P_i = 0. \quad (13)$$
Demand is elastic and total revenue, \( P_i x_i = y^{1/2} x_i^{-1/2} \), always rises with output.

**Intermediate goods.**—We assume a one-to-one relation between goods and technologies. With his skill, \( s \), an intermediate-goods producer can make

\[ x = zs \tag{14} \]

units of good \( z \). \(^3\) The producer cannot augment \( s \) by hiring on the outside market. For fixed \( z \), returns to \( s \) are constant. A technology’s \( z \) never changes, it is a vintage model of technology. Let \( p(z) \) be the period license fee for making good \( z \). This yields a profit of

\[ P x - p(z) = y^{1/2} z^{1/2} s^{1/2} - p(z), \]

which is the basis for eq’s (1) with \( 2A_t = y_t^{1/2} \). The objective of the intermediate-goods producers is to maximize this quantity by selecting the technology, \( z \), to license.

**The supply of inventions.**—A product’s \( z \) is constant over its lifetime. New products, with higher \( z \)’s are invented at a constant rate, to be determined later. Each is retired at age \( T \) which, for now, is also given. The age distribution of goods is then uniform on the interval \([0, T]\). Assume that the frontier advances at a constant rate \( g \), so that the highest-quality good hitherto invented is given by

\[ z_{\text{max}}(t) = e^{gt}. \]

For now, \( g \) too is given.

**The stationary distribution of \( z \) conditional on \( g \) and \( T \).**—This distribution shifts over time but it always has the same shape. We shall describe its state at \( t = 0 \). Let \( \tau \) denote a technology’s age at \( t = 0 \); that technology’s quality then is \( z_{\tau} = e^{-g\tau} \), and the worst technology in use is of quality \( e^{-gT} \). Each agent makes a different good. Therefore, the number of goods equals the number of agents. That is, since \( \ln z = -g\tau \), and since \( \tau \) is uniform on \([0, T]\), the density of \( z \) is

\[ \phi(z) = \left( \frac{1}{gT} \right) \frac{1}{z}, \quad \text{for } z \in [e^{-gT}, 1]. \tag{15} \]

Then if the distribution shifts to the right at the rate \( g \), then for \( t \geq 0 \), \( \ln z_t \) is uniform on \([g(t - T), gt]\); the interval is of fixed length and shifts to the right at the rate \( g \). Moreover, \( \phi_t(z) = \left( \frac{1}{gT} \right) \frac{1}{z} \) for \( z \in [e^{g(t-T)}, e^{gt}] \). That is why in Section 2 we assumed \( z \) was log uniform.

### 3.1 The market for licenses

Intermediate-goods firms are run by single agents, i.e., by sole proprietors. There is a unit measure of such agents, the only agents in this model. In contrast to Krugman

\(^3\)From now on we shall refer to a good not by its index \( i \), but by its efficiency, \( z \).
any agent can make any product, and in contrast to Eaton and Kortum (1999) technology diffusion is endogenous. It is determined in the market for licenses.

An agent’s skill will be denoted by \( s \), and \( s \) will differ among agents. The only expense is payments for technology: To make product \( z \) at a given date, a firm must pay its per-period license fee \( p(z) \). To begin with, let us assume that there is exactly one producer per product. We shall derive the prices at which all markets clear, and then verify that at those prices no one has the incentive to enter a market as a second producer.

The pricing of technology is competitive. Each technology has an infinite and fully enforced patent.\(^4\) On the demand side, users of the technology compete with other potential users, and inventors compete with other inventors with similar-quality technologies. We start, then, with a one-to-one assignment with side payments – the “transferable-utility” case. Taking the distributions of \( z \) and \( s \) as given, let us find the market-clearing license-fee function \( p_t(z) \). For now, we shall continue to take \( g \) and \( T \) as given.

The technology-adoption decision: Taking his general-skill-level \( s \) as given, an intermediate-goods monopolist then solves

\[
\pi(s) = \max_z \left\{ y^{1/2} z^{1/2} s^{1/2} \right\} - p(z) \right\}
\]

Revenue rises with output and the firm always produces at full capacity given \( s \). The first-order condition reads

\[
\frac{1}{2} \left( \frac{sy}{z} \right)^{1/2} = p'(z) = 0. \tag{16}
\]

The above is a first-order differential equation in \( p \) with world output, \( y \), taken as fixed at a moment in time, and with \( s \) varying with \( z \) according to the equilibrium assignment. The corner condition (4) again must hold and for the same reason.

The market-clearing assignment and license fee \( p(z) \).—For any \( \theta > 0 \), the assignment

\[
z = \theta s \tag{17}
\]

is an equilibrium if

\[
p(z) = \frac{1}{2} \left( \frac{y}{\theta} \right)^{1/2} (z - z_{\text{min}}) \tag{18}
\]

This can be verified by substituting from (17) and (18) into (16) and (4). In (18) the terms \( z_{\text{min}} \) and \( y \) will be changing over time. In particular, as the technology ages, \( p(z) \) responds to two opposing forces: A positive aggregate-demand effect through the growth of \( y \), and a negative obsolescence effect via the decline in \( z \). We have

\(^4\)Patents in the U.S are protected for 18 years, 20 years in Europe. Discounting at the equity rate of about 7 percent, earnings that an inventor would, if he could, draw from year 21 and beyond would probably be a negligible fraction of the present value of a patent so that imposing an 18- or 20-year limit should have little impact on the equilibrium outcome.
Figure 4: The movement of \( n_t(s) \) over time

a one-parameter family of solutions for the assignment and for \( p \), indexed by the parameter \( \theta \). We shall proceed on the assumption that \( \theta \) is exogenous, and later we shall solve for it.

Technology-market clearing.—Let \( n(s) \) be the date-zero density of \( s \). License-market clearing at \( t = 0 \) requires that for all \( z \in [e^{-gT}, 1] \),

\[
\int_{z}^{1} \phi(v)dv = \int_{z/\theta}^{1/\theta} n(s)ds.
\]

If (19) is to hold, then the distribution of \( s \) must also be log uniform. That is, for any positive \((g, T, \theta)\), (18) and (17) constitute an assignment equilibrium when the distributions \( z \) and \( s \) are given by (15) and by

\[
n(s) = \left( \frac{1}{gT} \right) \frac{1}{s}, \quad \text{for } s \in \left[ \frac{1}{\theta} e^{-gT}, \frac{1}{\theta} \right].
\]

as illustrated in Figure 4. Thus \( s_{\text{max}}(0) = \frac{1}{\theta} \) and \( s_{\text{min}}(0) = \frac{1}{\theta} e^{-gT} \). The functional form of the density is the same for all \( t \), only the domain changes; at date \( t \), the domain is \([\frac{1}{\theta} e^{g(t-T)}, \frac{1}{\theta} e^{gt}]\), as shown in Figure 4. This is why in Section 2 we assumed that \( s \) was log uniform.

Let us recap. If technology is to improve at the rate \( g \) with each technology being retired at age \( T \), and if the assignment of \( z \) is to be proportional with a proportionality constant \( \theta \), then the cross-section distributions of \( z \) and \( s \) must at each date be log uniform, and must both move to the right at speed \( g \).
Total Revenue = \( y^{1/2} \theta^{1/2} s \)

**Figure 5: The Breakdown of Income into Licensing Fees and Profits**

Net income, \( \pi(s) \), is linear in \( s \),

\[
\pi(s) = \frac{1}{2} (\theta y)^{1/2} (s_{\min} + s),
\]

and output, \((\theta y)^{1/2} s\), and license fees, \( p(\theta s) = \frac{1}{2} (\theta y)^{1/2} (s - s_{\min})\), are also linear in \( s \). Figure 5 illustrates the situation. The cross-section (marginal) return to skill is \( \frac{1}{2} (\theta y)^{1/2} \), the slope of the blue line. The constancy of this return stems from the positive association between \( s \) and \( z \)—a leverage effect that rising skill has in raising \( z \). Without the accompanying rise in \( z \), the marginal returns to \( s \) would diminish.

Taking \( \theta, g \) and \( T \) as given, Figure 6A illustrates the relation between the two distributions and their movement over time. We have assumed that \( z_{\min}(0) = e^{-gT} \) and that \( z_{\max}(0) = 1 \). The date-zero distribution of \( z \) must then be on the interval \([e^{-gT}, 1]\) which is marked by the heavy line segment on the vertical axis. Since \( z = \theta s \), this means that \( s_{\min}(0) = \frac{1}{\theta} e^{-gT} \) and that \( s_{\max}(0) = \frac{1}{\theta} \). The date-zero distribution of \( s \) must then be on the interval \([\frac{1}{\theta} e^{-gT}, \frac{1}{\theta}]\), and this is marked by the heavy line segment on the horizontal axis. We then shift to date \( t \), when both distributions have been scaled up by a factor of \( e^{gt} \).

The technology cycle.—The technology or, equivalently, product cycle arises because of the different ways in which the distributions of \( s \) and \( z \) shift. As we shall shortly see, each agent’s \( s \) grows at the same rate \( g \) and the distribution of \( s \) therefore exhibits no rank reversals. On the other hand, the distribution of \( z \) shifts entirely through replacement, and each good has a \( z \) that is fixed over time. Put differently, the technology cycle arises because \( s \) grows entirely on the intensive margin while
Figure 6A: Assignment at two distinct dates

Assignment at date t
Assignment at date zero

Figure 6B: The Technology Cycle

Frontier technology comes in at $t_0$
it ages

$\ln z_{\text{max}}(t) = gt$

$\ln z_{\text{min}}(t) = g(t-T)$
it dies at $t_0 + T$
z grows entirely on the extensive margin. Thus the assignment \( z = \theta s \) can hold at each \( t \) only if products move down the skill distribution. Figure 6B describes how the distribution of \( z \) grows. At any date \( t \), the support of the distribution of \( \ln z \) is \( [g(t - T), gt] \). The upper and lower bounds of \( \ln z \) are also drawn. Now consider the technology that is introduced at date \( t_0 \). Its efficiency is \( \ln z_{\text{max}}(t_0) = gt_0 \), where it remains for the duration of the technology’s lifetime, which ends at date \( t_0 + T \). The technology’s efficiency rank declines continuously over this period. As its rank declines, so does the relative quality of its match. The absolute quality of its match remains unchanged at \( s_{\text{max}}(t_0) \) which, at date \( t_0 \), is the highest skill around but which, by date \( t_0 + T \), is the lowest skill. This movement of a given \( z \) down the skill distribution is what we shall understand to be the technology cycle.

The sustainable monopoly condition.—We assumed monopoly in each product. No firm should want to enter as a second firm in someone else’s market. Suppose firm \( s_0 \) were to invade firm \( s \)’s market. It could do so only if it paid the license fee \( p(\theta s) \). Industry output would then be \( z(s + s_0) = \theta s(s + s_0) \). For monopoly to be sustainable, firm \( s_0 \)’s payoff from doing so must be less than its payoff in its own market:

\[
y^{1/2}(\theta s [s + s_0])^{-1/2} zs_0 - p(\theta s) \leq \pi(s_0). \tag{22}
\]

The Appendix shows that (22) holds for all \((s, s_0) > 0 \). This establishes that the one-to-one assignment is indeed an equilibrium.

So far, \( \theta, g, \) and \( T \) were taken as given. The next subsection derives \( g \).

### 3.2 Accumulation of skill

We shall now study the accumulation of human capital of an agent who lives in a world in which other agents accumulate \( s \) at the rate \( g \) and in which the distribution of \( z \) shifts to the right also at the rate \( g \). From (14) and (17) \( y \) will grow at the rate \( 2g \). Each agent will take the rate of interest, \( r \), and the path of \( y_t = y_0 e^{2gt} \) as given.

Intermediate-goods manufacturers own their human capital and decide how to accumulate it over time. Each has a unit of time that he divides between production (\( u_P \)), research (\( u_R \)), and human-capital investment (\( u_I \)):

\[
u_P + u_R + u_I = 1. \tag{23}
\]

An agent’s skill supply to his own business then is

\[
s = u_P h.
\]

Human capital investment uses only time, as in Lucas (1988):

\[
h = \eta u_I h. \tag{24}
\]

Wealth maximization: We shall now solve the accumulation problem of someone who is forced to set \( u_{R,t} = 0 \) for all \( t \). The solution will be the same as for people
who can set $u_{R,t} > 0$ because the research wage per unit of $h$ will be the same as the return of $h$ in production. Let $u_t \equiv u_{P,t} + u_{R,t}$. The expression in (21) pertains to period zero, but $s_{\text{min}}$ grows at the rate $g$. An agent that at date $t$ supplies skill $s_t = u_t h_t$ will receive an income

$$\pi_t(u_t h_t) = \frac{1}{2} (\theta y_t)^{1/2} \left( e^{gt} s_{\text{min}} + u_t h_t \right).$$

He maximizes $\int_0^\infty e^{-rt} \pi_t(u_t h_t) dt$, but he cannot influence the term $\frac{1}{2} (\theta y_t)^{1/2} e^{gt} s_{\text{min}}$. Since $y_t^{1/2} = y_0^{1/2} e^{gt}$, he picks $u_t$ to maximize $\frac{1}{2} (\theta y_0)^{1/2} \int_0^\infty e^{-(r-g)t} u_t h_t dt$, which is equivalent to the problem

$$\max_{(u_t, h_t)_0} \int_0^\infty e^{-(r-g)t} u_t h_t dt, \text{ s.t. } \dot{h}_t = \eta (1 - u_t) h_t,$$

with $h_0$ given. The Hamiltonian is

$$H = e^{-(r-g)t} u h + \bar{\mu} \eta (1 - u) h,$$

Let $\mu = e^{-(r-g)t}\bar{\mu}$ be the current value multiplier so that the current-value Hamiltonian is just $u h + \mu \eta (1 - u) h$. We shall only analyze constant-growth paths. Evaluated at a point at which $\dot{\mu} = 0$, the FOC’s are

$$1 - \mu \eta = 0,$$

and

$$\mu \eta (1 - u) + u = (r - g) \mu.$$

Eliminating $\mu$ we have

$$r = \eta + g.$$  \hfill (25)

This is an arbitrage condition equating the rate of interest to the rate of return to investing in $h$.

**Saving.**—Utility is homothetic, and we need to worry only about the world’s per capita consumption, $c_t$. Given his wealth, the agent maximizes his lifetime utility:

$$\int_0^\infty e^{-\rho t} \frac{c_t^{1-\sigma}}{1-\sigma} dt.$$  

If $c$ is to grow at the same rate as $y$, i.e., at the rate $2g$, we must have:

$$2g = \frac{r - \rho}{\sigma}.$$  

Together with (25) this implies that $(2\sigma - 1) g = \eta - \rho$

$$g = \frac{\eta - \rho}{2\sigma - 1}.$$  \hfill (26)

For the consumer’s problem to be defined we need that $e^{-\rho t} c_t^{1-\sigma} \to 0$ as $t \to \infty$ which, as long as $\eta > \rho$, requires that $\sigma > 1$.

All agents must choose the same $u_P$ and, if $s$ is to grow at the same rate as $h$, they must choose the same $u_R$ and, hence, the same $u_R$.  

15
3.3 Research

Now that we have $g$, we determine $\theta$ and $T$ next.

The quality of products.—Following Arrow (1962) and assume that the quality of an invention depends on the cumulative number, $N$, of previous inventions, i.e., on the invention’s “serial number” as Arrow would put it. His eq. [8] would amount to $z_{\text{max}} = N^\alpha$, but we shall assume the slightly different functional form

$$z_{\text{max}} = e^{\alpha N}.$$  

We may think of it as the result of learning in the research sector, learning by producing inventions.\(^5\) The growth in $z_{\text{max}}$ per period is

$$g_t = \alpha \dot{N}_t.  \quad (27)$$

Birth of technologies.—The number of new products is proportional to the quantity of human capital employed in research. Let $H_t$ be the aggregate human capital employed in research. The flow of new products is

$$\dot{N}_t = \frac{\lambda}{z_{\text{max}}(t)} H_t.  \quad (28)$$

The presence of $z_{\text{max}}$ in the denominator implies a “fishing out” external effect in the discovery of new products; the discovery of the first product takes fewer resources than the discovery of the second, and so on.

Birth = death of technologies.—Each technology lives for $T$ periods, and the death rate, $1/T$ must equal their birth rate, $\dot{N}$:

$$\dot{N} = \frac{1}{T}. \quad (29)$$

Research Labor Supply = Demand.—Each agent must be indifferent between devoting the marginal unit of time to his business and working as a research worker. By (21), the profit foregone from a unit reduction in $s$ is $\frac{1}{2} \left( \theta \bar{y} \right)^{1/2} = \frac{1}{2} \theta^{1/2} e^{gt}$. Hence the research wage must equal

$$w_t = \frac{\theta^{1/2}}{2} e^{gt}. \quad (30)$$

\(^5\) An alternative interpretation of $\alpha$ is possible: In the quality-ladder literature, $\alpha$ would be the ladder-step size; e.g., Grossman and Helpman (1991, p. 560) and Aghion and Howitt (1992, p. 328). In Howitt (2000, Section IV), e.g., the frontier grows at a rate assumed to be proportional to the overall flow of innovations; this factor of proportionality is the spillover coefficient analogous to $\alpha$. The product window is fixed in these models, whereas here it moves to the right over time.
Since $h = \frac{s}{u_P}$, (20) $n(s) = \left(\frac{1}{gT}\right) \frac{1}{s}$ implies that

$$H = \frac{u_R}{u_P} \int_{s_{min}(t)}^{s_{max}(t)} s n_t(s) ds = e^{gT} \frac{u_R}{u_P} \frac{1 - e^{-gT}}{gT}.$$ 

Substituting for $H$ into the RHS of (28), the growth of $H$ exactly offsets the fishing-out externality and the flow of new ideas is constant:

$$\dot{N} = \frac{\lambda u_R}{\theta u_P} \frac{1 - e^{-gT}}{gT}.$$ (31)

**Free-entry condition.**—The value of an invention is the discounted flow of license fees. The period-$t$ frontier technology is $z_{max}(t) = e^{gt}$. Using (18), the license fee of that technology $\tau$ periods hence is

$$p_{t+\tau}(e^{g\tau}) = e^{2gt} \frac{\theta^{-1/2}}{2} e^{g\tau} \left(1 - e^{g(\tau - T)}\right).$$ (32)

for $\tau \in [0, T]$. The present value of the license income from a frontier technology is

$$V_t = \int_0^T e^{-rt} p_{t+\tau}(e^{g\tau}) d\tau = e^{2gt} \frac{\theta^{-1/2}}{2} \left(1 - e^{-(r-g)T} - e^{-gT - e^{-(r-2g)T}} \frac{1 - e^{-(r-2g)T}}{r-2g}\right).$$

The RHS of (28) implies that the free-entry condition is

$$w_t = \frac{\lambda}{z_{max}(t)} V_t.$$ (33)

Upon dividing (33) equality by $\lambda$ and using (30) and (25), we get

$$\theta \frac{\lambda}{\eta} = \frac{1 - e^{-\eta T}}{\eta} - e^{-gT} \frac{1 - e^{-(\eta - g)T}}{\eta - g}.$$ (34)

Thus $\theta$ is determined by the free-entry condition (33). Since $T = \alpha/g$ and since $g$ is given by (26), the RHS of (34) does not depend on $\lambda$. Thus $\theta$ is proportional to $\lambda$. The intuition is that demand for technology, as summarized by $\theta$ per unit of skill, must equal the supply, which comes forth at the speed $\lambda$.

**Stationary equilibrium.**—It consists of 8 real numbers $g, T, \theta, u_P, u_R, u_I, w$ and $\dot{N}$ that solve (23), (24), (26), (27), (29), (30), (31), and (34).

Existence of the equilibrium follows by using $g$ from (26), $u_I = g/\eta$ via (24). Then $u_P$ is a function only of $u_R$ alone via (23), and (34) gives us $\theta$ uniquely in terms of $T$. The remaining equations are linear and easily shown to have a unique solution. We have thus shown that there is a steady state with inequality, but we did not prove the stability of that steady state. Even if it is stable, if $T = 68.3$, the transition dynamics are likely to be quite long.
4 Other properties of the model

We shall now elaborate on the discussion in Section 2, and list other properties of the model.

Output growth.—Output of final goods and intermediate goods grows faster than the sole measured input \( h \); that is,

\[ y_t = e^{2gt}, \tag{35} \]

where \( g \) is given by (26). From (32) and (35), we get (10).

The learning-by-doing parameter alone determines inequality.—Final-goods production uses no labor, earns zero profits and generates no income. All income derives from profits earned from producing intermediate goods and from wages earned in research. Let us assume, however, that firm ownership is fully diversified so that each agent holds a fraction of the world portfolio proportional to his or her wealth which, in turn, is proportional to his or her \( h \). In that case an agent’s income from all sources is proportional to \( h \). Moreover, all agents choose the same \( u_i \)’s so that income differentials coincide with wealth differentials. Taking the ratio of richest to poorest,

\[ \frac{Y_{\max}}{Y_{\min}} = \frac{h_{\max}}{h_{\min}} = e^\alpha. \tag{36} \]

Thus \( \alpha \) alone determines inequality, because \( \alpha \) alone governs the dispersion in technological quality among the measure 1 of the most recent vintages, thereby dictating the dispersion of \( h \) that will arise in steady state.

Markups.—We value the cost of the entrepreneur’s time at the market wage \( w \). Using (18) and (30) the total cost is \( ws + p(\theta s) = \frac{1}{2} (\theta y)^{1/2} s + \frac{1}{2} (\theta y)^{1/2} (s - s_{\min}) = (\theta y)^{1/2} (s - \frac{1}{2}s_{\min}) \). The marginal cost concerns producing an additional unit of the same good an additional unit of which, according to (14) and (17), requires \( \frac{1}{s} = \frac{1}{\theta y} \) additional units of skill, which we again price at \( w \) to obtain a marginal cost of \( \theta y)^{1/2} \frac{1}{s} \). From (13), \( P = y^{1/2} x_i^{-1/2} = y^{1/2} (zs)^{-1/2} = y^{1/2} (zs)^{-1/2} = (\theta y)^{1/2} s^{-1} \). Therefore \( (P-MC)/MC = 1 \) on all goods which matches Hall’s (1988, Table 4) median estimate of 1.096 well, but is higher than Roberts and Supina’s (2000, Table 1) median-industry estimates which range from 0.347 in 1977 to 0.893 in 1987.

Average and marginal production costs.—Average cost of a person with skill \( s \) is

\[ \frac{w s + p(\theta s)}{z s} = \frac{(\theta y)^{1/2} (s - \frac{1}{2}s_{\min})}{\theta y s^2}. \]

Then

\[ \frac{AC}{MC} \bigg|_s = \frac{(\theta y)^{1/2} (s - \frac{1}{2}s_{\min})}{\theta y s^2 \left( \frac{1}{2} \left( \frac{y}{y} \right)^{1/2} s^{-1} \right)} = 2 - \frac{s_{\min}}{s} \leq 2 - e^{-\alpha}. \]

The RHS is the ratio pertaining to the richest agent, which we shall take as pertaining
to the U.S., in which case the model estimates it as

\[
\frac{\text{AC}}{\text{MC}}_{\text{U.S.}} = 2 - e^{-0.775} = 1.54
\]

which is larger than what the micro data show.\(^6\)

*Creative destruction.*—Products are phased out and the product window marches to the right. Combining (27) with (29) gives a reduced-form relation between two endogenous variables \(g\) and \(T\),

\[
g = \frac{\alpha}{T}, \quad (37)
\]

which emphasizes the creative-destruction aspect of the model: Higher growth demands faster replacement of technologies.

*Pattern of trade.*—There are no countries in the model, but suppose, now, that there are rich and poor countries, so that skill is unevenly distributed in geographical space. If we assume that the final good is produced in all countries, then the producers of the final good would, in each country, demand all intermediate goods. But new intermediate goods are produced only in rich countries, and old intermediate goods are produced in poor countries. Therefore the rich export new products and import old products. The poor import new products and export old ones.

5 Conclusion

In light of the rising trade in technology and of the prominence of intellectual property protection in trade negotiations, we asked what would happen if markets in technology were frictionless. We derived the technology assignment and the diffusion lags that would arise. We studied incentives for skill accumulation and product innovation. The estimated model generated a long a technology cycle, but not much income inequality. Thus the model suggests that the widespread resistance to worldwide protection of intellectual property is unfounded.

\(^6\)Because \(p(z)\) has a constant term, licensing costs are not proportional to \(z\) and, in contrast to the standard Cobb-Douglas case, the elasticity of cost with respect to output (i.e., MC/AC) is not constant. Ignoring the difference, however, we refer to Cobb-Douglas-based RTS estimates for manufacturing industries which typically range between 0.95 and 1.15. See Tybout and Westbrook (1996) for a summary of the literature.
References


6 Appendix

*Demonstration of (22).—*I assume that if a firm does switch, that firm and all other firms involved must stick to their equilibrium values of $u_I$ and $u_R$. Cancelling $y^{1/2}$ from both sides of (22), it reads

$$(\theta s [s + s_0])^{-1/2} (\theta s) s_0 - \theta^{1/2} (s - s_{\text{min}}) \leq \frac{1}{2} \theta^{1/2} (s_{\text{min}} + s_0),$$

i.e., 

$$(\theta s [s + s_0])^{-1/2} (\theta s) s_0 \leq \frac{1}{2} \theta^{1/2} (s_{\text{min}} + s_0),$$

i.e., $s^{1/2} (s + s_0)^{-1/2} s_0 \leq \frac{1}{2} (s + s_0)$. Dividing by $s$, (22) becomes $s^{-1/2} (s + s_0)^{-1/2} s_0 \leq \frac{1}{2} \left(1 + \frac{s_0}{s}\right).$ Now $s^{-1/2} (s + s_0)^{-1/2} = s^{-1} \left(1 + \frac{s_0}{s}\right)^{-1/2}$. Therefore letting $w \equiv \frac{s_0}{s}$, (22) simplifies to,

$$\frac{1}{2} (1 + w) - (1 + w)^{-1/2} w \geq 0$$

(38)
Now the LHS of (38) can be written as $(1 + w) \left( \frac{1}{2} - (1 + w)^{-3/2} \right)$. The term $\frac{w}{(1+w)^{3/2}}$ goes to zero as $w$ gets small or gets large, and it attains a unique maximum when $w$ solves

$$\frac{1}{(1+w)^{3/2}} = \frac{3}{2} \frac{w}{(1+w)^{5/2}},$$

i.e., $2(1+w) = 3w$, i.e., $w = 2$, that maximum being $\frac{2}{\sqrt{27}} = 0.385 < \frac{1}{2}$. Hence (22) holds.

Calculating the diffusion lags reported in Figure 1.—Comin and Hobijn (2004) have compiled data that cover 20 advanced countries and eleven technologies over the past two hundred years.\(^7\) The variable $t_i$ was defined to be the average of the dates that the eleven technologies spread to ten percent of country $i$’s population. Figure 7 illustrates how the $t_i$ were calculated. Ten percent is low enough that nine of eleven technologies have reached it in all the countries covered. The eleven technologies are private cars, radios, phones, television, personal computers, aviation passengers, telegraph, newspapers, mail, mobile phones, and rail.

\(^7\)See the "Historical Cross-Country Technological Adoption: Dataset" at www.nber.org/data/