

“Overreaction” of Asset Prices in General Equilibrium

S. Rao Aiyagari*

University of Rochester

and

Mark Gertler[†]

New York University and NBER

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We attempt to explain the overreaction of asset prices to movements in short-term interest rates, dividends, and asset supplies. The key element of our explanation is a margin constraint that traders face which limits their leverage to a fraction of the value of their assets. Traders may lever themselves, furthermore, either directly by borrowing short term or indirectly by engaging in futures and options trading, so that the scenario is relevant to contemporary financial markets. When some shock pushes asset prices to a low enough level at which the margin constraint binds, traders are forced to liquidate assets. This drives asset prices below what they would be with frictionless markets. Also, a shock which simply increases the likelihood that the margin constraint will bind can have a very similar effect on asset prices. We construct a general equilibrium model with margin constrained traders and derive some qualitative properties of asset prices. We present an analytical solution for a deterministic version of the model and a simple numerical computation of the stochastic version. *Journal of Economic Literature* Classification Numbers: G1, E0. © 1999 Academic Press

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1. INTRODUCTION

A long-standing challenge in finance and economics is to explain the volatility of asset prices. The baseline frictionless model (e.g., Lucas

* For Dilip and Meera.

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(1978)) has great difficulty accounting for the facts. Many formal studies have made this point in many different ways.¹

Coming from a rather different perspective, informal discussions of price volatility often emphasize distress selling by highly levered traders. A recent example involves the sharp jump in long-term interest rates during the spring and early summer of 1994 that occurred in the wake of a rather modest rise in short-term interest rates.² According to a number of observers, declining prices induced partly by the rise in short-term rates and partly by expectations of a further rise in short-term rates prompted many bond traders to unload part of their assets in order to avoid margin calls. The net effect was to magnify the drop in prices. Because the price decline reduced the capital of all of the major traders in the market, it is argued, new funds did not flow in instantly to return prices to their fundamental values. More generally, large selloffs by leveraged traders seem characteristic of periods of notable price contractions. This kind of behavior, for example, was prominent in both the 1929 and 1987 stock market crashes.³

Implicit in this informal story is the idea that asset trading involves specialization. In contrast to the standard complete markets framework, due to informational frictions and the like, only a relatively small group of individuals with the requisite expertise and financial resources are actively pursuing arbitrage at any point in time. Because these specialists may face frictions in the process of obtaining external finance—again, fundamentally due to informational frictions—periods of disruption in the smooth functioning of asset markets may be possible.

In this paper, we develop a general equilibrium model of asset pricing that attempts to capture this informal story. The model features traders who may occasionally face margin calls. Our goal is to explore the extent to which this kind of phenomenon can account for the volatility puzzle. In contrast to many of the models in the literature that develop pricing relationships within a representative agent setting, the model we build features traders who specialize in pursuing arbitrage profits. To finance their positions, traders obtain funds from nonspecialists via leveraged transactions. They may obtain funds either explicitly by issuing short-term debt, or implicitly by engaging in an equivalent futures or options transaction. In either case, margin requirements restrict the amount of leverage

¹ See Cochrane and Hansen (1992) for a survey of this literature.

² For a discussion of the connection between leveraging in the bond market and price volatility, see Bianco (1994). Campbell (1995) suggests that the behavior of highly leveraged traders was a possible explanation for this run up in long-term rates.

³ For descriptions of distress selling by leveraged traders (including portfolio insurers) during the 1987 crash, see Gammill and Marsh (1988), Greenwald and Stein (1988), and Leland and Rubenstein (1998).

they may use. In equilibrium, temporary periods of distress selling are possible.

Consistent with the evidence, our framework generates excess returns on risky assets that are larger and more volatile than in the benchmark frictionless model. The existence of the margin constraint increases the traders' effective degree of risk aversion, since they wish to avoid having to unload assets at discount prices. Furthermore, how “effectively” risk averse traders act depends on how close they are to violating their respective margin constraints. It is for this reason that excess returns, and hence prices, are more volatile than in the frictionless model. Furthermore, because the margin constraint is more likely to bind in market downturns than in upturns, the model generates price declines that are on average sharper than price increases, leading to negative skewness in ex post returns. This negative skewness in returns is also consistent with the data.⁴

Our paper bears some resemblance to several other approaches to the price volatility puzzle. One approach, for example, stresses the behavior of “noise” or “feedback” traders who buy when the price starts to rise and sell when the market price starts to fall.⁵ In this respect, their behavior is very similar to the “margin-constrained” traders in our framework. However, this literature simply postulates ad hoc decision rules for these individuals. In our model, traders optimize. The margin requirement links a trader's demand for risky securities to his net financial position, introducing a region where his demand for risky assets may be an increasing function of price. In this respect, the traders in our model behave somewhat like the collateral-constrained firms in the model by Kiyotaki and Moore (1993) of credit cycles, which reduce their investment as the market price of their collateral begins to fall.

Also relevant to our paper is the literature that stresses how stop loss trading rules, due, for example, to portfolio insurance considerations, can increase the volatility of asset prices (e.g., Grossman and Zhou (1994)). Because the margin constraint makes traders care about preserving the value of their capital, it generates behavior that is very similar to stop-loss trading. In addition, as we discuss below, one can interpret the traders in our model as portfolio insurers, since they are essentially in the business of absorbing the tails of the return distributions on risky assets.⁶

⁴ See, for example, Campbell *et al.* (1996).

⁵ See, for example, Cutler *et al.* (1990), and associated references.

⁶ A different approach to explaining asset price volatility has emphasized that special forms of habit formation that introduce slow-moving threshold levels of consumption can induce both the magnitude and the time variation in degree of risk aversion that seem to be needed (e.g., Kandel and Stambaugh (1991), Constantinides (1990), Abel (1990), and Campbell and Cochrane (1995)).

Finally, our framework is also similar in spirit to the work that emphasizes volatility stemming from limited participation (e.g., Allen and Gale (1994)). In this literature, typically, exogenous restrictions are placed on the ability of different groups of individuals to trade at any point in time. In our framework limited participation is also critical. It arises, however, from the combination of specialization in asset trading and frictions in the ability of traders to obtain funds.

The key aspect of our model is the link between a trader's net financial position and his demand for risky securities which arises through the margin constraint. For now we simply take the margin constraint as imposed by a regulatory agency. In fact the Federal Reserve does indeed impose and set margin requirements. Some exchanges, further, impose additional requirements. One could imagine, however, that independently of legal restrictions, agency problems could precipitate a need for capital by traders to partially collateralize the liabilities they issue to obtain speculative funds. We do not attempt to model this phenomenon nor do we attempt to rationalize the existence of margin constraints either as a market phenomenon or as reflecting optimal policy choice by a regulatory agency. In this paper we simply appeal to the fact that both margin constraints and traders' capital are pervasive features of securities markets.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 presents a preliminary descriptive analysis. Section 4 presents an analytical solution for a deterministic version of the model, and works through several comparative static experiments. We show that relative to the frictionless case, asset prices may "overreact" to either a permanent rise in the discount rate or a permanent cut in dividends. The degree of overreaction, further, is a simple function of several key model parameters. We also show that an increase in asset supplies can temporarily depress prices, something which does not ordinarily occur in a frictionless environment. Section 5 presents a numerical solution for a simple example in the stochastic case. As the simulation makes clear, the likelihood that the margin constraint may bind in the future will influence traders' behavior. Thus even if the margin constraint is directly binding, it may still affect equilibrium asset prices. As a consequence persistent departures between the market and fundamental prices are possible.

Section 6 discusses some modifications of the basic model. In practice, professional traders make widespread use of futures and options markets to finance their positions. We show that the leveraged positions that the traders in our model adopt may be interpreted as being the product of futures and options trading. The section also discusses the implications of having multiple risky assets. An outcome is that "spillover" effects of movements in asset prices may emerge. As a consequence, asset prices

may co-move by more than a frictionless model would predict. Concluding remarks are in section 7.

2. THE MODEL

Our framework is a variant of Lucas' (1978) asset pricing model. Since everyone is identical in the Lucas model, financial trade never arises in equilibrium. For our purposes, it is necessary to have an environment where one group of individuals specializes in asset trading and obtains funds from another group to finance their respective positions.⁷ We thus forgo the convenience of the representative agent construct and instead incorporate heterogeneity into the framework.

We introduce the necessary heterogeneity with an eye toward tractability. In addition to the representative household in the Lucas model, we add a second kind of representative agent called a “trader.” The trader has a comparative advantage in pursuing arbitrage profits. He can exchange risky stocks and bonds costlessly. While the household may hold these risky securities directly in its portfolio, it cannot trade them costlessly.

The trader operates a securities firm that is owned by the household and valued in a competitive market.⁸ He finances positions in risky securities, using the equity capital of the firm and by borrowing from the household. This borrowing takes the form of short-term riskless debt, which the household can trade costlessly. There are two key frictions in this process, however. First, margin requirements limit his use of leverage to some multiple of his capital. Second, the only way the trader can build his capital is by retaining earnings from trading profits. We assume that directly issuing new equity is prohibitively expensive. These two assumptions create a potential link between the value of the trader's existing capital and his gross holdings of risky assets.

The trader does not consume any resources. Instead his sole objective is to maximize the value of the securities firm to the household. To do so, he chooses a portfolio strategy and a retained earnings/dividend payout policy, subject to the margin and equity issue constraints. Furthermore, he takes as given exogenously the initial equity in the firm. By making the trader simply a manager of the securities firm, as opposed to a individual with distinct preferences, etc., we nest the Lucas model as a special case.

⁷ The notion of specialization in risky asset trading is consistent with the data. One can interpret the evidence in Blume and Zeldes (1994) as suggesting that less than 1% of households actively engage in stock trading.

⁸ We are implicitly assuming that the ownership claim to the securities' firm may be traded costlessly in a competitive market by the household. We elaborate in the next section.

Absent the margin and equity issue constraints, the model boils down to Lucas' frictionless representative agent paradigm. The same would be true if the household could trade securities costlessly. In this instance, the trader would have no role.

We now describe the environment in detail. The model economy is a single good endowment economy. For most of the paper, we consider the pricing of a single risky asset, interpretable as an equity, that is a claim on an exogenously given endowment stream. We focus on equity because it is the most general kind of claim. But the analysis extends easily to the case of long-term real bonds by setting the real dividend to a constant, and it extends easily to the case of long-term nominal bonds by having the real dividend vary inversely with inflation.

Since all households are identical, we express the model in terms of a representative household interacting with a representative trader and the government. The only role the government plays in our model is that it permits an analysis of the effects of changing asset supplies arising through government open market operations and/or changing lump-sum taxes. After describing the household, the trader, and the government, we conclude the section by characterizing the equilibrium conditions. For now, we assume that the trader obtains a leveraged position explicitly by using short-term debt to finance the acquisition of risky assets. Later we show how it is possible to reinterpret the trader's use of leverage as the product of futures and options trading.

Finally, we emphasize that our model will have nothing new to say about the determination of the short-term riskless rate. Indeed, the determination of the short-term rate is exactly the same as in the Lucas model. In the simulations we present later, we will in fact rig the endowment process to produce the desired movements in the short-term rate. In this respect, our framework is best thought of as a model of fluctuations in the prices of risky assets conditional on the behavior of short-term rates.

2.1. The Household

Let c_t , w_t , D_t , and T_t be the household's consumption, income, dividends received from the trader, and lump-sum taxes, all in period t ; let b_t and s_t be the household's holdings of short-term bonds and equity shares, respectively, all at the beginning of period t ; let R_t^f be the gross return on the (risk-free) short-term bond from t to $t + 1$, and let d_t and q_t be the period t dividend on the equity and the equity price, respectively. The household receives utility from consumption and disutility in the form of effort costs from buying or selling equity. Its objective is to choose sequences of consumption, equity holdings, and short-term bond holdings

to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U(c_t) - a_t q_t (s_{t+1} - s_t)^2 / 2 \right], \quad (1)$$

subject to the sequence of budget constraints,

$$(d_t + q_t) s_t + b_t + D_t + w_t - c_t - q_t s_{t+1} - b_{t+1} / R_t^f - T_t \geq 0. \quad (2)$$

Note that in expression (1), the disutility of effort arising from transactions in stocks is quadratic in the size of the household's equity trade.⁹ The time varying parameter a_t influences the magnitude of the adjustment cost. Later we will specify the behavior of a_t .

With this formulation we are trying to capture the idea that, because the household is not a specialist, it reacts slowly to arbitrage opportunities (reacting quickly is the trader's job). The quadratic cost specification is the most convenient way to motivate this slow adjustment. Think of the household as having a fixed amount of time to either trade securities or enjoy leisure (e.g., watch football or play with children). Since the household is not a specialist, large transactions absorb a large amount of time (due to, say, double checking, rearranging funds, etc.). With diminishing marginal utility of leisure this could lead to convex costs of trading, which may be approximated by a quadratic loss function.

An alternative scenario would be to introduce heterogeneous households that trade only at staggered intervals, i.e., each household rebalances its portfolio only several times a year.¹⁰ As nonspecialists, furthermore, households restrict their use of leverage to obtain securities. This kind of scenario could also introduce friction in the flow of funds from the household sector to profitable arbitrage opportunities. For now, we adopt the simplest approach—the quadratic cost specification.

2.2. The Trader

The trader may exchange securities costlessly. There are, however, some restrictions on his ability to obtain credit.

The trader's objective is to maximize the expected discounted value of dividend payouts to the household. (Recall that the household owns the trading firm.) The discount rate the trader uses reflects the household's intertemporal marginal rate of substitution. Specifically: let M_{t+i} ($= \beta^i U'(c_{t+i}) / U'(c_t)$) be the household's marginal rate of substitution of

⁹ Heaton and Lucas (1994) also adopt this specification of the transaction technology.

¹⁰ For a model with heterogeneous households that face proportional trading costs, see Aiyagari and Gertler (1991).

consumption between $t + i$ and t , and let b_t^* and s_t^* be the trader's holdings of short-term bonds and stocks, respectively, at the beginning of period t . We assume that the trader maximizes

$$E_t \sum_{i=0}^{\infty} M_{t+i} D_{t+i}, \quad (3)$$

where the dividend payout at t , D_t , equals the difference between the net assets the trader has at the beginning of period t and the net assets he has at the end of t , as given below¹¹:

$$D_t = [(d_t + q_t)s_t^* + b_t^*] - q_t s_{t+1}^* - b_{t+1}^*/R_t^f. \quad (4)$$

We define the trader's capital as his beginning-of-period net assets, $[(d_t + q_t)s_t^* + b_t^*]$. To extend his end-of-period equity holdings beyond the value of his capital, the trader must borrow (i.e., go short in short-term debt). We assume, however, that a margin requirement restricts his use of leverage. In particular, the trader must finance the fraction κ of his equity holdings directly with his own funds, which consists of his capital minus any dividends he pays the household during the current period. Specifically, the margin constraint is given by

$$[(d_t + q_t)s_t^* + b_t^*] - D_t - \kappa q_t s_{t+1}^* \geq 0, \quad (5)$$

where κ is the margin requirement. Equivalently, the amount of leverage the trader employs cannot exceed the fraction $1 - \kappa$ of his stock holdings. Notice that the margin constraint introduces a relationship between the value of the trader's capital and the value of his stock purchases.

Finally, for the margin constraint to have force it must be the case that the trader cannot instantly and costlessly obtain new capital by issuing new equity. Informational and commitment factors could motivate why, in practice, traders use leverage rather than new equity issues to pursue arbitrage opportunities at the margin. We make no attempt to model these factors explicitly, but we appeal to them by assuming that new equity issues are prohibitively expensive. In our model, this is tantamount to requiring that dividends payments must be nonnegative:

$$D_t \geq 0. \quad (6)$$

¹¹ To motivate the trader's objective function, we are implicitly allowing the household to costlessly trade the ownership claim to the securities firm on a competitive market. Let V_t denote the value of this claim. Then the household's first-order condition gives $(V_t - D_t)u'(c_t) = E_t\{V_{t+1}\beta u'(c_{t+1})\}$. Solving for V_t then gives the objective function described in Eq. (3). We justify the ability of the household to trade the ownership claim to the securities firm simply as a way to give the trader a plausible objective.

That is, the household cannot replenish the trader’s capital simply by paying in new funds. As in the real world, the trader must rely mainly on retained earnings to build his capital.

2.3. Government

The government levies lump-sum taxes on the household and conducts open market operations in short-term risk-free debt and the stock, subject to the following budget constraint:

$$T_t + b_t^g + (d_t + q_t)s_t^g - b_{t+1}^g/R_t^f - q_t s_{t+1}^g \geq 0, \quad (7)$$

where b_t^g and s_t^g denote the government’s beginning-of-period holdings of risk-free bonds and stocks, respectively. Note that we are assuming that government consumption is zero.

2.4. Equilibrium Conditions

The total number of equity shares is normalized to unity. Therefore, the equilibrium conditions in the asset markets are

$$b_t + b_t^* + b_t^g = 0, \quad (8a)$$

$$s_t + s_t^* = 1 - s_t^g \equiv \bar{s}_t. \quad (8b)$$

The equilibrium condition in the goods market is

$$c_t = w_t + d_t. \quad (9)$$

The income process w_t , the dividend process d_t , and the total supply of shares facing the household and the trader \bar{s}_t are given exogenously.¹²

3. DESCRIPTIVE ANALYSIS

In this section we develop some qualitative implications of the model. We first analyze the solution to the household’s and trader’s decision problems. We conclude by illustrating how the financial market frictions may influence the excess return on equity. This exercise will shed some light on how the frictions we have introduced ultimately influence the volatility of asset prices.

¹² It is possible to interpret the equity as a government consol (a long-term government bond) by replacing the 1 in (8b) with 0 and deleting d_t in (9). If d_t is constant, it could be thought of as a real consol, and if it is random, it could be thought of as a nominal consol subject to inflation risk.

3.1. Household Behavior

The household's intertemporal marginal rate of substitution pins down the risk-free rate in standard fashion, as Eq. (10) implies.

$$R_t^f = E_t\{M_{t+1}\}^{-1}. \quad (10)$$

The household does not face any costs of adjusting short-term debt. Therefore, the conventional pricing relation for the riskless rate holds. Note that since consumption is exogenous, the intertemporal marginal rate of substitution is exogenous.

To simplify the algebra that follows, we set the household's adjustment cost parameter for equity transactions, a_t (see Eq. 1), equal to $a \cdot u'(c_t)$, where the parameter a is a constant and c_t is now interpretable as aggregate consumption, which the household takes as given.¹³ In this instance, the household's demand for new equities depends positively on the gap between the value of the equity under frictionless markets, q_t^f , and the prevailing market price, q_t , in the following simple fashion:

$$s_{t+1} - s_t = [(q_t^f/q_t) - 1]/a, \quad (11)$$

where q_t^f is given by

$$q_t^f = E_t \sum_{j=1}^{\infty} M_{t+j} d_{t+j}. \quad (12)$$

Note that the relation for q_t^f is the conventional present value formula for the price of equity when markets are perfect. For convenience, we refer to q_t^f as the fundamental price.

The partial adjustment rule given by Eq. (11) is, of course, mainly the product of quadratic adjustment costs. These costs limit the speed at which the household can respond to a deviation of the market price from the fundamental price.¹⁴ Note that as the adjustment cost parameter a goes to

¹³ Convenience, as opposed to realism, provides the justification for this assumption. By controlling for the impact of shifts in the marginal utility of consumption goods on the relative utility costs from securities trading—an impact which is likely second order—we greatly simplify the household's first-order condition for equity holdings. Put differently, we are simply normalizing the costs of securities trading in terms of consumption goods.

¹⁴ In deriving (11), we made use of the fact that, in equilibrium, the sequence of discount factors (the M 's) is exogenous, since aggregate consumption is exogenous. We also abstract from growth in specifying our model. It is easy to incorporate growth if we assume that the utility function is isoelastic, say $U(c) = c^{1-\mu}$, $\mu > 0$, $\mu \neq 1$, or $U(c) = \log(c)$. Then we can permit the household's income w_t and the dividends on the stock d_t to exhibit trend growth at some rate $g > 0$. It can be verified that the value of equity transactions grows at the trend rate g , and the utility transactions costs in purchasing equity shares behave on average like $c_t^{1-\mu}$.

zero, q_t must converge to q_t^f . In this limiting case the household is as skilled as the trader at pursuing arbitrage opportunities.

3.2. Trader Behavior

Let $\Omega_t M_t$ and $\mu_t M_t$ be the nonnegative multipliers associated with the budget and margin constraints (4) and (5), respectively. Furthermore, let γ_t ($= \Omega_t + \mu_t$) be the dollar shadow value of a dollar of wealth to the trader. Finally, let R_{t+1}^e ($= (q_{t+1} + d_{t+1})/q_t$) be the gross return on equity. From the first-order conditions for the trader's problem we obtain

$$1 \leq \gamma_t, \quad \text{with } = \text{ if } D_t > 0, \quad (13a)$$

$$\gamma_t = E_t\{\gamma_{t+1} M_{t+1} \cdot R_t^f\} + \mu_t, \quad (13b)$$

$$\gamma_t = E_t\{\gamma_{t+1} M_{t+1} \cdot R_{t+1}^e\} + \kappa \mu_t. \quad (13c)$$

When the margin and dividend constraints do not bind, the asset pricing formulae implied by Eqs. (13) are the same as the relations that hold under frictionless markets. With γ_t equal to unity and μ_t equal to zero, Eqs. (12) and (13) relate the risk-free rate and the return to equity to the household's intertemporal marginal rate of substitution in a standard manner.

To understand what will occur when the financial constraints may impinge on the trader's behavior and how this might enhance the excess return on equity, it is useful to develop the following proposition.

PROPOSITION 1. *If (i) the margin constraint binds at t (i.e., $\mu_t > 0$), or if (ii) the margin constraint may bind in the future with positive probability (i.e., $\mu_{t+i} > 0$ for some $i \geq 1$), then the dividend constraint must bind at t (i.e., $\gamma_t > 1$).*

The proposition is straightforward. The trader may relax the margin constraint if he is able to obtain additional funds by reducing dividend payouts. If the margin constraint is truly binding, therefore, the trader must have reduced dividends to the minimum level of zero. The trader will also not pay dividends if there is some possibility he may be constrained in the future. In this event, he will retain all earnings in order to build up his capital as rapidly as possible.

The trader's capital thus plays a key role. If it falls below the level required to keep the margin constraint from ever binding, then the asset pricing formulae given by (13) will differ from the frictionless case. While the riskless interest rate remains unchanged (see Eq. 10), the expected return on equity differs. The latter becomes sensitive to whether the margin constraint is currently binding or is likely to bind in the future. Any

effect on the expected return to equity, of course, translates into an effect on the price.

3.3. The Excess Return for Equity

To gain some intuition on how the financial market frictions influence the price of equity, we combine (13b) and (13c) to obtain an expression for the risk premium:

$$\bar{R}_t^e - R_t^f = [-\text{COV}_t(\gamma_{t+1}M_{t+1}, R_{t+1}^e) + (1 - \kappa)\mu_t] / E_t\{\gamma_{t+1}M_{t+1}\}, \quad (14)$$

where $\bar{R}_t^e (= E_t\{R_{t+1}^e\})$ is the expected return to equity.

In the benchmark case where the margin constraint never binds (i.e., $\mu_t = 0$ and $\gamma_{t+1} = 1$), the expression for the risk premium reduces to its familiar formula under frictionless markets. The excess return depends simply on the covariance of the equity return with the households' marginal utility of consumption in $t + 1$ (the numerator of M_{t+1}). We refer to the excess return in this case as the "fundamental" risk premium.

If the margin constraint is currently binding ($\mu_t > 0$), the excess return rises above its fundamental value. In this instance, the trader is forced to shed some of his security holdings, driving the asset price below its fundamental value. The differential does not vanish instantly, since transaction costs preclude the household from quickly exploiting the arbitrage opportunity.

Importantly, the excess return may also exceed its fundamental value simply if there is some chance that the margin constraint could bind in the future. Note that the risk premium depends on the covariance of the equity return with γ_{t+1} , the trader's shadow value of wealth at period $t + 1$. This covariance between R_{t+1}^e and γ_{t+1} is likely to be negative for the following reason. As noted in Proposition 1, when the margin constraint binds at $t + 1$ or at some other future date, γ_{t+1} will exceed unity, and this is also the occasion on which the equity price at $t + 1$ drops below its fundamental value, thus lowering the ex post return R_{t+1}^e . It follows that the expectation of a binding margin constraint in the future can make the excess return exceed its fundamental value.

Intuitively, the trader wishes to avoid the margin constraint, since it forces him to unload his assets at a discount. He therefore values a security according to how its return covaries with the possibility of hitting this constraint. This is equivalent to valuing a security according to how its return covaries with the shadow value of wealth, γ_{t+1} , since the latter reflects the likelihood that the margin constraint will bind in a subsequent

period. For hedging purposes, the trader prefers securities that have a high ex post payoff in the event the margin constraint binds. Unfortunately, the single equity in this model has just the opposite property. Increased likelihood of the margin constraint binding is likely to coincide with a poor ex post payoff on the equity. This feature reduces the value of the equity to the trader.

In summary, the excess return on equity depends not only on whether the margin constraint is currently binding, but also on the probability it may bind in the future. To the extent these factors vary over time, so too will excess returns. In this way, the financial market frictions may enhance the volatility of excess returns and thereby enhance the volatility of asset prices.¹⁵

Up to this point, we have simply evaluated how the multipliers associated with the financial constraints may influence the excess return, taking these multipliers as given. In general equilibrium, of course, there is feedback. A fall in the asset price associated with a rise in the excess return reduces the trader’s capital, possibly tightening the financial constraints. To flesh out this idea, we need to solve the full model. Unfortunately, numerical methods are necessary to handle the general case with uncertainty. An analytical solution is available for the deterministic case, however. Working through this solution provides some useful intuition. Therefore, as a prelude to presenting a numerical solution for the stochastic case, we analyze a deterministic version of the model in the next section.

4. DETERMINISTIC CASE

We first present the analytical solution for the deterministic case with constant consumption and constant dividends. We then turn to some comparative static exercises to illustrate how the model may produce “overreaction” of asset prices. In doing so, we relate the quantitative importance of this overreaction to several key parameters.

4.1. Analytical Solution

Suppose $w_t = w$, $d_t = d$ and $\bar{s}_t = \bar{s}$. Then: $c_t = c = w + d$;

$$M_{t+1} = \beta U'(c)/U'(c) = \beta, \quad R_t^f = R^f = \beta^{-1}, \quad \text{and} \quad q_t^f = q^f,$$

¹⁵ Given that $\bar{R}_t^e = E_t\{(d_{t+1} + q_{t+1})/q_t\}$, we can express q_t as the following discounted stream: $q_t = E_t \sum_{j=1}^{\infty} d_{t+j} / \prod_{i=0}^{j-1} (\bar{R}_{t+i}^e)$, where the sequence of discount factors $\{\bar{R}_{t+i}^e\}_{i=0}^{\infty}$ is given by (14). Any variation in this sequence thus induces variation in q_t .

given by

$$q^f = E_t \sum_{i=1}^{\infty} \beta^i d = [\beta/(1 - \beta)]d. \quad (15)$$

Proposition 2 characterizes the possible outcomes for the deterministic case with constant consumption and dividends.

PROPOSITION 2 (Deterministic Case).

Case 1. If

$$(d + q^f)s_i^* + b_i^* \geq \kappa q^f s_i^*, \quad (16)$$

then the margin constraint does not bind at any t , $q_{t+i} = q^f$, $s_{t+i}^* = s_t^*$ for all $i \geq 0$.

Case 2. If

$$(d + q^f)s_i^* + b_i^* < \kappa q^f s_i^*, \quad (17)$$

then the margin constraint binds at t only, $q_t < q^f$, $s_{t+1}^* < s_t^*$, and $q_{t+i} = q^f$, $s_{t+i}^* = s_{t+1}^*$, for all $i \geq 1$.

4.1a. *Case 1*

We begin with Case 1. Note first that when $q_t = q^f$, the household is neither buying nor selling equity (see Eq. 11). Thus, the trader must simply be maintaining his equity position. If (16) obtains, then at the frictionless price the trader has sufficient net assets to maintain his equity holding, pay nonnegative dividends, and satisfy the margin constraint.

Note next that the trader can use the dividend earnings on his net holdings of equity (which are least $d\kappa q^f s_i^*$) to draw down his debt from t to $t + 1$. That is, the trader can feasibly maintain $b_{t+1}^* \geq b_t^*$. This implies that the trader can feasibly keep his capital from shrinking, i.e., keep $(d + q^f)s_t^* + b_{t+1}^* > (d + q^f)s_t^* + b_t^*$. It follows that if the margin constraint is satisfied in t , it will also be satisfied in $t + 1$. By similar reasoning, the trader can satisfy the margin constraint indefinitely into the future. Since the margin constraint will never bind, q_t will remain fixed at its fundamental value q^f .

4.1b. *Case 2*

Now consider the more interesting case, Case 2, where the margin constraint binds at t . In this instance, the trader's net assets evaluated at the fundamental price q^f are not sufficient to permit him to jointly satisfy the margin constraint, pay dividends, and maintain his current holdings of

equities (see (17)). Accordingly, he must reduce dividends to zero and shed securities to the point where he is just able to satisfy the margin constraint:

$$(d + q_t)s_t^* + b_t^* = \kappa q_t s_{t+1}^*, \quad (18)$$

where $s_{t+1}^* < s_t^*$. Since it is costly for the household to instantly absorb the securities, q_t will drop below q^f (as we will make clear shortly).

By inverting (18), we obtain a demand curve for the trader's equity holdings in the regime where the margin constraint is binding:

$$s_{t+1}^* = [(d + q_t)s_t^* + b_t^*] / \kappa q_t. \quad (19)$$

In this case the trader's demand for equities is upward sloping in price. To see this, first note that the term $(b_t^* + ds_t^*)$ is negative. For the margin constraint to bind, the trader must be unable to cover his debt obligation simply with his dividend earnings. If $(b_t^* + ds_t^*)$ is negative, then s_{t+1}^* varies positively with q_t .

Intuitively, in this regime a percentage change in the price of equity leads to a greater percentage change in the trader's capital. That is,

$$(\Delta q_t)s_t^* / [(q_t + d)s_t^* + b_t^*] > \Delta q_t / q_t. \quad (20)$$

By using leverage to finance equity purchases, the trader absorbs a disproportionate share of the fluctuation in the asset price. A decline in the equity price, therefore, will lower his net asset holdings proportionately more than his gross asset holdings. He is then forced to shed assets to meet the margin requirement.

The supply curve facing the trader is simply the total fixed supply of shares facing the household and the trader minus the quantity demanded by the household:

$$s_{t+1}^* = \bar{s} - s_{t+1}. \quad (21)$$

Inserting the household's decision rule for s_{t+1} (Eq. 11) into (21) yields

$$s_{t+1}^* = s_t^* - [(q^f / q_t) - 1] / a. \quad (22)$$

The supply curve of shares facing the trader is upward sloping in price. As the price converges toward its fundamental value, households become less willing to absorb shares, leaving more for the trader. In the absence of adjustment costs (as a goes to zero), the supply curve becomes perfectly elastic.

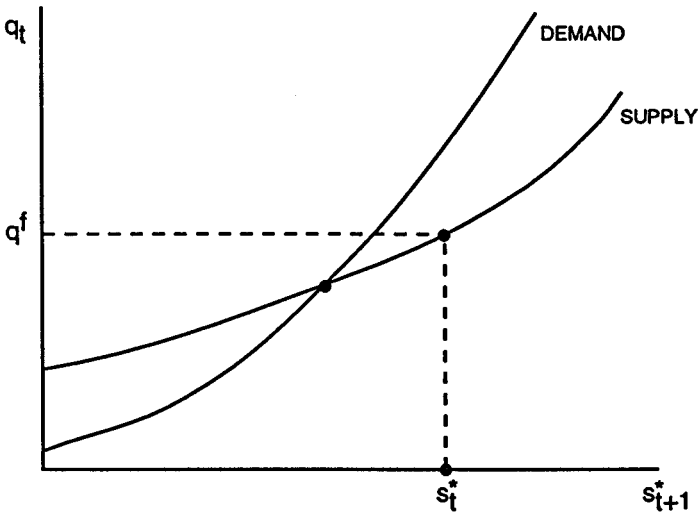


FIG. 1. Binding margin constraint in period T .

Figure 1 illustrates the equilibrium. The supply curve (defined by Eq. 22) is positioned so that $q_t = q^f$ when $s_{t+1}^* = s_t^*$. When $s_{t+1}^* = s_t^*$, both the trader and the household are maintaining their current equity positions. The only way the household will do this is if $q_t = q^f$. The figure also portrays the upward sloping demand curve (defined by (19)). When the margin constraint is binding, the demand curve is to the left of the supply curve at $q_t = q^f$. In this instance, when $q_t = q^f$, the maximum feasible number of shares the trader can hold is less than s_t^* . Given the position of the demand curve, the equilibrium lies at a point with q_t below q^f and s_{t+1}^* below s_t^* .

q_t lies further below q^f the more inelastic is the supply curve facing the trader. When the margin constraint binds, the trader must unload shares to the household. If it is very costly for the household to quickly absorb these shares, the price must drop sharply. Conversely, the gap between the fundamental and the market price is greater, the more elastic is the demand curve. A low margin requirement makes demand elastic. With a low (and binding) margin requirement, a small drop in the trader's capital forces a large drop in assets.

For an equilibrium to exist it must be the case that the supply curve lies above the demand curve at the point where $s_{t+1}^* = 0$. This condition is

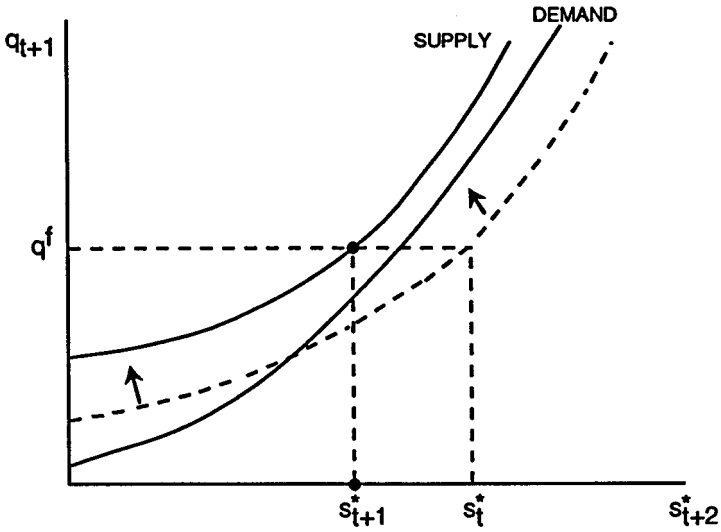


FIG. 2. Adjustment in $T + 1$ to binding margin constraint in T .

satisfied when

$$q_t^f / (as_t^* + 1) > -(b_t^* + ds_t^*) / s_t^*. \quad (23)$$

This condition boils down to the requirement that the supply curve not be too inelastic and the demand curve not be too elastic.¹⁶ Otherwise, the market simply collapses when the margin constraint binds.

After period t , the price returns to its fundamental value. Figure 2 illustrates the outcome. The supply curve at $t + 1$ moves in to the point where, at $q_{t+1} = q^f$, $s_{t+2}^* = s_{t+1}^*$. This reflects the shuffling of equities from the trader to the household. The demand curve (or, more accurately, the relation between s_{t+2}^* and q_{t+1} defined by the updated version of Eq. 19) either does not change position between t and $t + 1$ or shifts to the right. This occurs because the trader can use the earnings on his net holdings of securities at t to either maintain or reduce his debt. Since the trader's debt does not rise from t to $t + 1$, the demand curve does not move inward. Therefore, at $t + 1$, he has sufficient capital to maintain his equity holdings and satisfy the margin requirement. Since the margin

¹⁶ Condition (23) also guarantees that the solution for q_t in this case is unique.

requirement is relaxed, the equity price can float to its fundamental value q^f .¹⁷

4.2. Comparative Statics

We consider the impact of three kinds of shocks. The first is a permanent increase in the discount rate $(1 - \beta)/\beta$ (equivalently, a permanent rise in the net riskless rate $R^f - 1$). The second is a permanent reduction in dividends. The last is a permanent increase in the supply of the risky asset facing the household and the trader.

Before proceeding we emphasize that in the deterministic setting, the various shocks we are considering will produce a discrepancy between the market and fundamental prices that last for only one period. It follows from Proposition 2 that one period after the shock the price will revert to its fundamental value. In the next section we consider a numerical solution for the stochastic case, where the economy may continually be buffeted by new shocks and where, accordingly, persistent departures of the price from its fundamental value are possible.

4.2a. *A Permanent Increase in the Discount Rate*

Define $r = R^f - 1$ as the net riskless interest, which in equilibrium equals the discount rate $(1 - \beta)/\beta$. Suppose that there is a permanent increase in the discount rate that raises the net riskless rate from r_1 to r_2 . The fundamental price accordingly drops from $q_1^f = d/r_1$ to $q_2^f = d/r_2$. Suppose, further, that the margin constraint is slack prior to the shock, but that the shock forces the constraint into the region where the margin constraint kicks in. Figure 3 illustrates the impact. The supply curve shifts outward. The drop in the fundamental value makes the households prefer to sell more shares to the trader, for any given price q_t . Since the shock moves the supply curve to a point where the margin constraint binds, the

¹⁷ As noted earlier, we fix the initial portfolios of the household and the trader arbitrarily and do not permit them to determine this division in some optimal fashion. To justify this, consider a particular household–trader pair among the large number of such pairs with identical initial portfolios. Obviously, market prices would not be influenced by rearranging the initial portfolios of this particular pair. Suppose that we rearrange the initial portfolios of this pair in such a way that $(d_0 s_0 + b_0)$ is constant, i.e., any change in the initial stock holdings of the household and the consequent change in dividends to be received is compensated for by a corresponding change in its initial short-term bond holdings. If the market price is below the frictionless price, then this household will still want to buy exactly the same number of shares as before the portfolio rearrangement—see Eq. (11). As a result, the household will suffer exactly the same adjustment costs as before. Furthermore, the household will consume exactly the same amount as before, since the budget constraint (2) is unaffected. Consequently, there will be no effect on the household’s welfare, and the household–trader pair will be indifferent to any rearrangement of its portfolio.

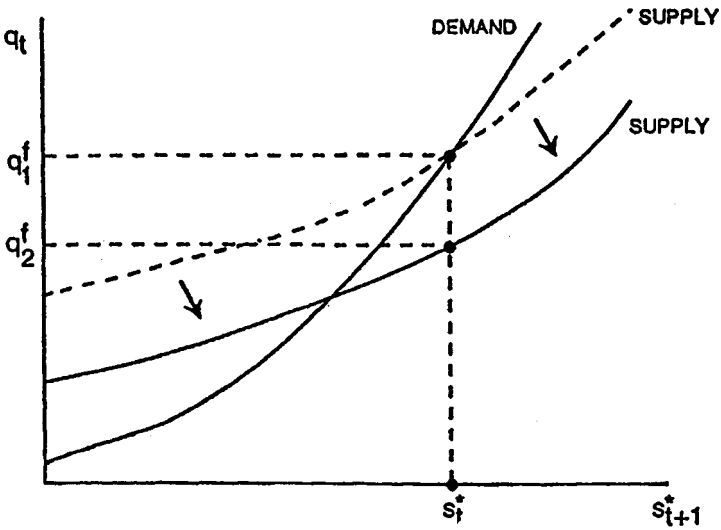


FIG. 3. Impact of a permanent rise in the discount rate.

price drops below the new fundamental price q_2^f . The asset price thus overreacts to the shock in the sense that the drop in the price exceeds the drop in the fundamental price.

We may quantify the overreaction by using the demand and supply curves to compute the elasticity of the asset price with respect to the permanent interest rate shock, η_{qr} ¹⁸:

$$\eta_{qr} = -1/[1 - (\eta^d/\eta^s)] < -1, \tag{24}$$

where η^d and η^s are, respectively, the demand and supply elasticities with respect to price, given by

$$\eta^d = (1 - \kappa)/\kappa; \quad \eta^s = 1/as_t^*.$$

At any stable equilibrium the (upward sloping) demand curve must be less elastic than the supply curve, implying that $\eta_s > \eta_d$. Therefore, η_{qr} is less than -1 . For benchmark purposes, note that the corresponding elasticity in the frictionless case is exactly -1 . ($(\partial q^f/q^f)/(\partial r/r) = -1$, since $q^f = d/r$.)

¹⁸ In deriving (24), we take advantage of the fact that the stock price initially equals its frictionless value q^f .

Equation (24) indicates that the degree of overreaction varies positively with the ratio η^d/η^s . A high adjustment cost parameter a for the household makes the household's demand for stocks inelastic and, thereby, makes the supply curve facing the trader inelastic. With an inelastic supply curve (low η^s), the price drops sharply as the trader tries to unload shares in the wake of the shock. A low margin requirement makes the demand curve more elastic because the trader must sell a larger quantity of shares to satisfy his margin requirement. With an elastic demand curve (high η^d), the price falls sharply, since the trader must unload a larger quantity of shares.

As an example, suppose that the margin requirement is 20%, implying that η^d equals 4. Suppose, further, that $\eta^s = 5$. That is, if the price declines 1%, then the trader can sell 5% of his shares to the household in the current period. In this case $\eta_{qr} = -5$, implying that the drop in the market price will be five times the drop in the fundamental price. With $\eta^s = 10$, η_{qr} becomes -1.66 , still a substantial degree of overreaction.

4.2b. *A Permanent Drop in Dividends*

The impact of a permanent decline in dividends on the asset price is, for the most part, similar to the impact of a rise in the discount rate. The fundamental price drops, shifting the supply curve to the right. If the drop in the fundamental price moves the trader into the region where the margin constraint binds, then the price dips below the fundamental, as in the previous case. There is, however, an additional reinforcing effect. The drop in current dividends reduces the trader's capital, which tightens the margin constraint. As a consequence, the demand curve shifts inward, enhancing the price decline. Figure 4 illustrates.

In the frictionless case, the dividend elasticity of the price is unity (i.e., -1 times the interest rate elasticity, since $q^f = d/r$). When the margin constraint binds, this elasticity, η_{qd} , is given by

$$\eta_{qd} = [1 + (\eta_{kd}/\eta^s)]/[1 - (\eta^d/\eta^s)] > 1, \quad (25)$$

where η_{kd} is the percentage change in the trader's capital due to the percentage change in dividends, given by

$$\eta_{kd} = (R^f - 1)/\kappa.$$

Since η_{kd} is positive, η_{qd} exceeds η_{qr} in absolute value (see Eq. (24)). Intuitively, the dividend cut affects both the fundamental price and the trader's current capital, while the rise in the interest rate affects only the former.

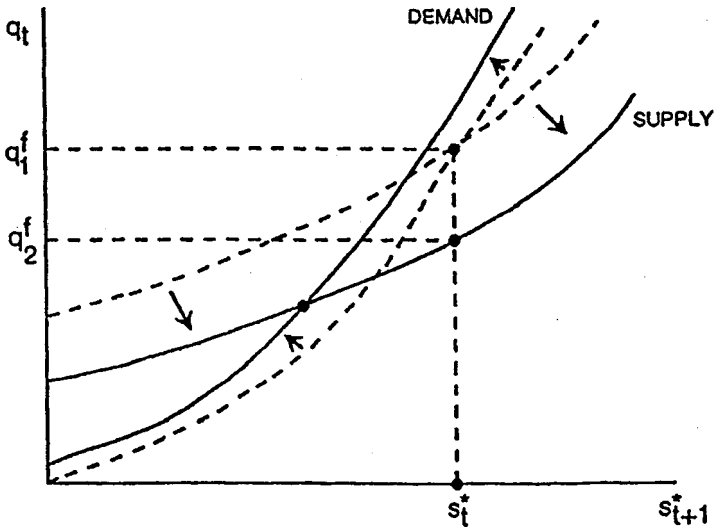


FIG. 4. Impact of a permanent cut in the dividend rate.

4.2c. *A Permanent Increase in the Asset Supply*

Now suppose that there is a permanent increase in the supply of the risky asset facing the household and the trader from \bar{s}_1 at time t to \bar{s}_2 at time $t + 1$. This change may be thought of as arising as a result of a once-and-for-all open market sale of stock by the government in exchange for short-term bonds or a reduction in lump-sum taxes. Note that there is no change in the aggregate resource constraint. In the frictionless environment, this kind of change in asset supplies has no impact on relative prices, since the Modigliani-Miller theorem applies in this setting. With the financial constraints present, however, a rise in asset supplies can temporarily depress the price.

Since the total supply of the asset is shifting between t and $t + 1$, the supply curve facing the trader is now given by

$$s_{t+1}^* = s_t^* + \bar{s}_2 - \bar{s}_1 - [q^f/q_t - 1]/a. \tag{26}$$

The trader must absorb the difference between the growth in the total supply over the period, $\bar{s}_2 - \bar{s}_1$, and the quantity that the household absorbs, $[q^f/q_t - 1]/a$. (Compare Eqs. (22) and (26).) The demand curve remains the same as before, given by Eq. (19).

Now consider the impact of a permanent rise in the asset supply at $t + 1$. To absorb the increase, the trader must issue short-term debt to the

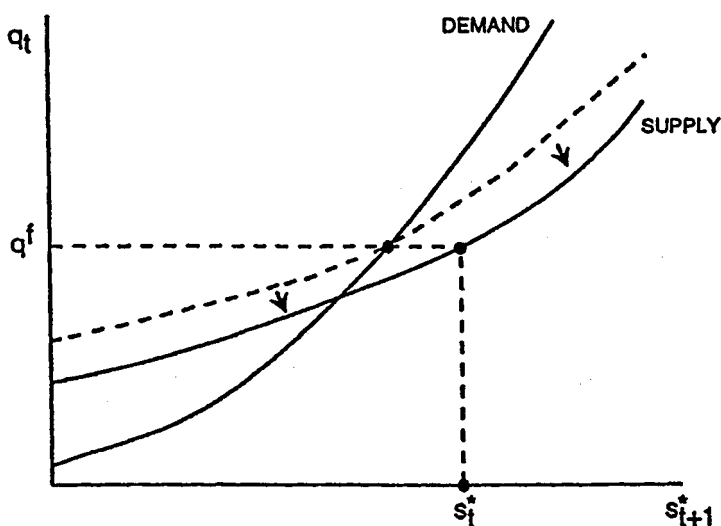


FIG. 5. Impact of a rise in the asset supply.

household (or engage in an equivalent future's market transaction). If he is able to do so at the fundamental price q^f without violating the margin constraint, then the rise in the supply of risky assets has no impact on the market price. If he is unable to do so, then the market price must drop below the fundamental price to encourage the household to increase its holdings. Figure 5 illustrates this outcome. The rise in the supply of the asset shifts the supply curve to right, reducing the price below the fundamental when the margin constraint binds.¹⁹

An expression for the elasticity of the price with respect to a rise in supply, $\eta_{q\bar{s}}$, is given by

$$\eta_{q\bar{s}} = -\eta_{s^* \bar{s}} / [\eta^s - \eta^d] \quad (27)$$

where $\eta_{s^* \bar{s}} = \bar{s}/s^*$ is the percentage by which the trader must increase his asset holdings to absorb a 1% increase in the total supply outstanding, and where η^s and η^d remain the price elasticities of supply and demand, given by Eq. (24). The larger the increase in supply relative to the trader's existing holdings, the more the price must drop.

¹⁹ This outcome is consistent with anecdotal evidence which suggests that large Treasury refinancings tend to depress bond prices. See, for example, the report by Dave Kansas in *The Wall Street Journal* (February 7, 1995), who notes that, "Ahead of the \$40 billion refunding, bond prices fell as traders made room for the new supply."

5. THE STOCHASTIC CASE: SOME NUMERICAL RESULTS

We now report some numerical solutions for the stochastic model.²⁰ Before we begin, it is important to note that, within the model, the trader has no incentive to permanently maintain a leveraged position. He accordingly acts to reduce his debt to zero over time, either by selling shares or by retaining earnings. The stationary long-run equilibrium of the economy, therefore, is the frictionless equilibrium.

Our simulation thus explores how the trading restrictions we have imposed affect asset prices during the transition to the stationary equilibrium. Our goal, accordingly, is not to try to match data, but is rather to provide some qualitative feel for how the model works.

In the real world, of course, traders systematically maintain highly leveraged positions; and, hence, the volatility our model describes need not disappear over time. In the conclusion we discuss one way to modify the model to give the trader the incentive to stay levered, so that the financial constraints may influence asset price volatility in the steady state.

Our numerical exercise proceeds as follows. For simplicity, we consider the pricing of a long-term bond subject to real interest rate risk. To do so, we assume that the dividend is fixed, so that the security is interpretable as a consol. To generate real interest rate risk, we assume that there is random movement in the household's endowment. This random movement in the endowment in turn generates random movements in the household's consumption and, accordingly, its intertemporal marginal rate of substitution. In this way, interest rate risk emerges.

We assume that the endowment process (and hence consumption) obeys a two-state Markov chain. It follows, of course, that the real interest will similarly obey a two-state process. We consider an experiment where the economy begins in the good state (low interest rate state) in period zero and moves to the bad state (high interest rate state) in the first period. Thereafter, the interest rate moves randomly between states. Furthermore, we start the trader with a portfolio (i.e., holdings of stocks and short-term debt) with which he has sufficient leverage to make the financial constraints impinge on his behavior.

²⁰ To compute an equilibrium we proceed as follows. The two endogenous states are the division of the risky asset between the trader and the household and the division of riskless asset. We first discretize the state space and then use the household's first-order conditions to obtain solutions for the riskless rate and the stock price conditional on the state at t and $t + 1$. We next compute the solution to the trader's problem via value function iteration and obtain transition rules for the adjustment of his portfolio, again conditional on the state at t and $t + 1$. We keep iterating until the portfolio transition rules that evolve from the trader's problem coincide with the initial sequence of states on which we conditioned. Overall, the nonlinearity induced by the margin constraint makes the computation quite difficult.

We set the supply of the risky asset facing the household and the trader to unity, i.e., the government's holdings are constant at zero. We take the annual discount rate to be 0.04 and the annual dividend (paid smoothly through the year) to equal 4% of average consumption. We take the margin constraint as 0.25, which is the actual maintenance requirement on stocks purchased on margin debt. This number is also close to the effective margin constraint on futures and options trading (see Section 7), which is a number somewhere between 0.15 and 0.2. To further ease the computation, we picked the model period to be a quarter (we will consider a shorter period in subsequent work). Finally, we calibrated the Markov chain to generate variation in the real interest rate that is twice that observed in the data.²¹ This is a crude way to compensate for the fact that in the real world long-term bond prices are sensitive to short-term nominal interest rates, since they are nominally denominated. Nominal short-term interest rate variation greatly exceeds short-term real rate variation.

Figure 6 reports the impact of the experiment on the behavior of the market price q_t relative to the fundamental price q_t^* . The economy begins period zero in the good state (low interest rate), but because the trader is highly leveraged, the market price is below the fundamental price (1.95 versus 2.02). In period one it moves to the bad state (high interest rate). The market price drops to 1.78, well below the fundamental price of 1.9. The drop in the market price between periods is roughly 50% greater than the drop in the fundamental price. The gap between the market price and the fundamental price persists for several more periods. Thus, in contrast to the deterministic case, some persistence is possible.

Figure 7 shows the behavior of the trader's portfolio. To avoid the margin constraint, the trader sharply reduces his stock holdings—from 0.75 in period 0 to just under 0.35 by period 5—which enables him to sharply curtail his debt—from -1.2 in period zero to about -0.4 in period 5. Thereafter, he slowly reduces his debt to zero, by gradually reducing his stock holdings and by retaining earnings.

Of course, because of the transaction costs on the household's stock purchases, the stock price falls as the trader unloads his shares. Figure 8 provides an idea of the magnitudes of the costs we are assuming. In the simulations, we set the adjustment cost parameter a equal to $1/3$. This implies, according to Fig. 8, that during the peak period of selling by the trader, the household's marginal adjustment cost rose to 7% of period consumption. This works out to about \$300, which is not high, considering that the household is absorbing about 40% of the shares on the market.

²¹ That is, we picked a two-state endowment process that generated variation in the real interest rate equal to twice that observed in the data.

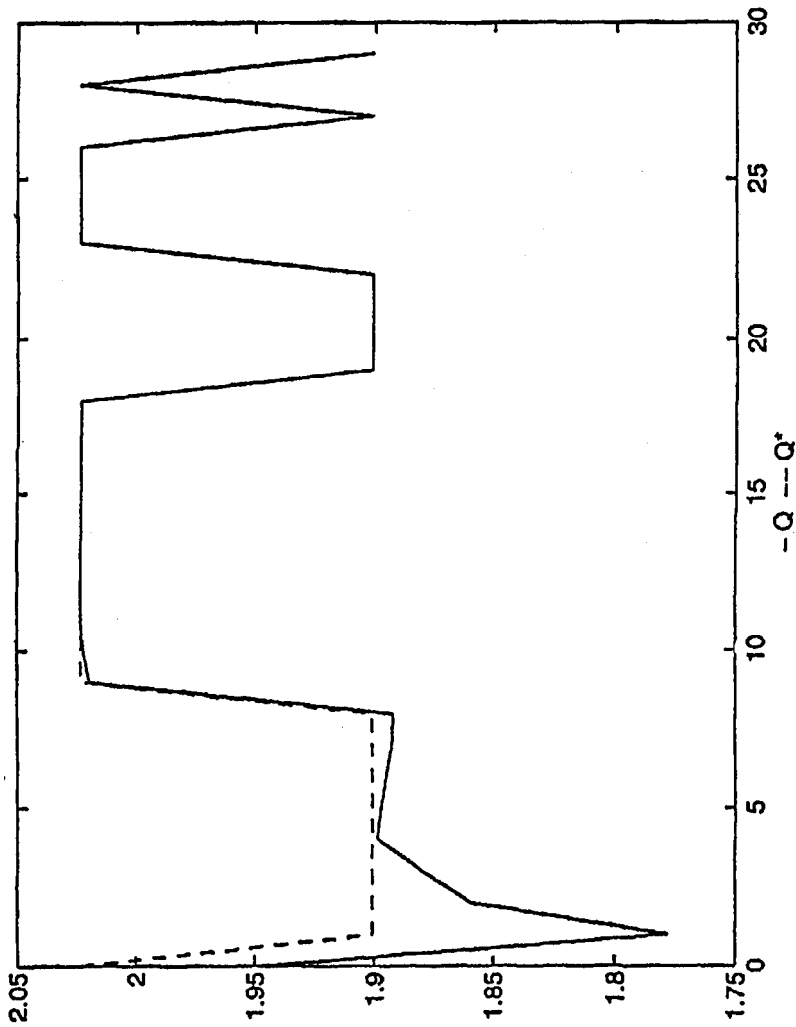


FIG. 6. Market price $Q(t)$ vs. fundamental price $Q^*(t)$.

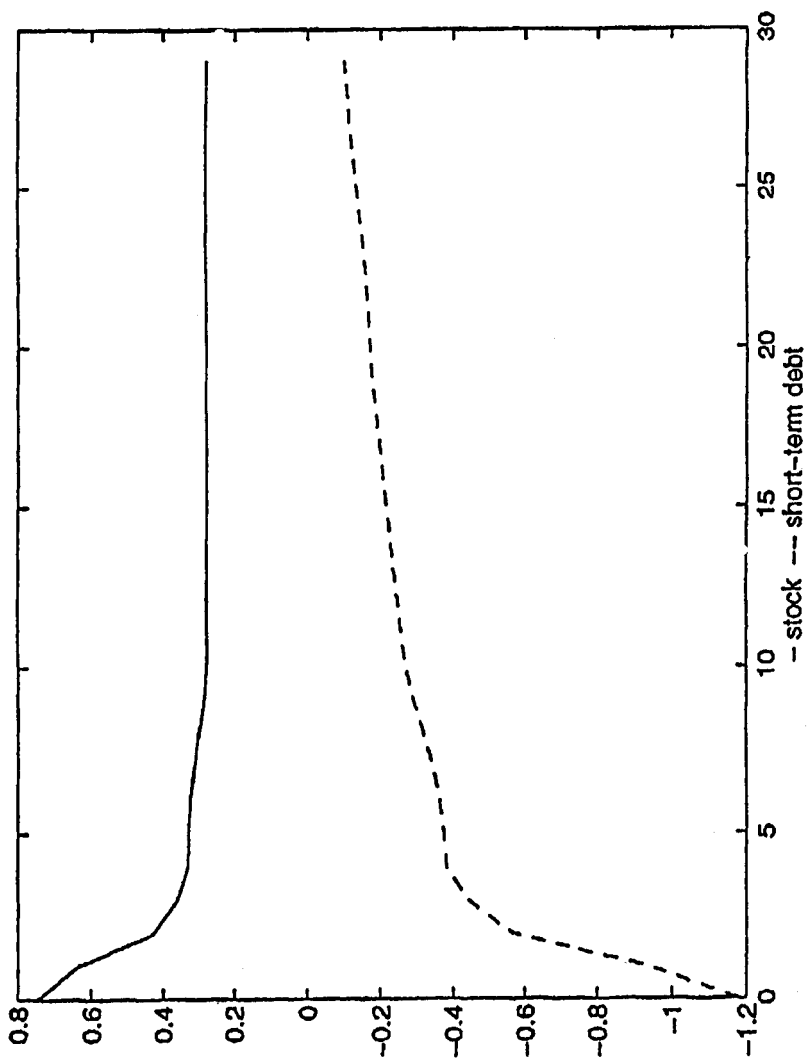


FIG. 7. Trader's stock and short-term debt.

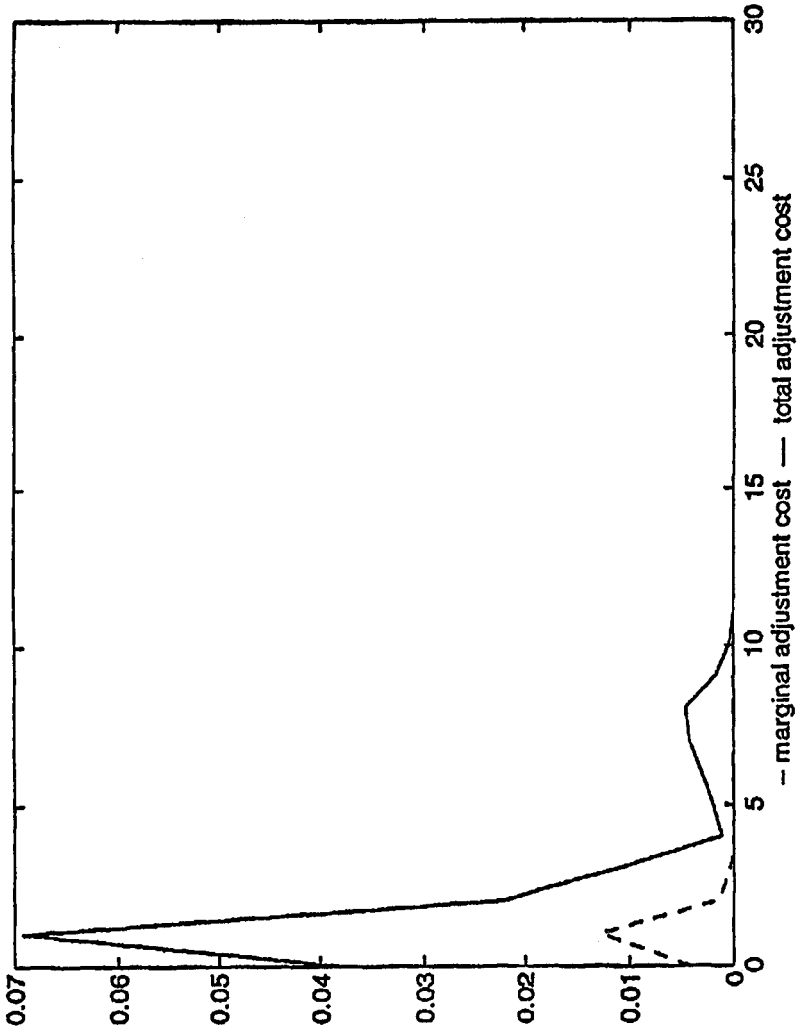


FIG. 8 Marginal adjustment and total adjustment costs.

Shortening the model period from perhaps a quarter to a month is likely to lower the costs further. Finally, because of the quadratic costs specification, the total cost was actually well below the marginal cost, as the figure illustrates.

6. EXTENSIONS OF THE BASIC FRAMEWORK

We now consider two extensions of the model. The first introduces futures and options trading as a way for the trader to obtain leverage. The second allows for multiple risky assets.

6.1. Futures and Options Trading versus Direct Leveraging by the Trader

Instead of issuing debt to obtain funds, the trader may implicitly borrow by engaging in futures and options trading. In this section we show how it is possible to reinterpret the trader's use of leverage in our model in terms of these types of transactions. We first discuss futures trading and then briefly discuss options trading.²²

Consider a futures contract between the trader and the household that works as follows. At time t the trader agrees to buy a share of equity from the household at time $t + 1$, at the price f_t .²³ Furthermore, the trader obtains ownership prior to the dividend yield at $t + 1$.

This futures transaction is equivalent to the trader acquiring equity by issuing riskless debt. The household receives a sure gross return of f_t/q_t on each share that it sells forward. By arbitrage, the gross return to the household from selling a share of its equity holdings forward must equal the return from holding riskless debt. That is, by arbitrage, $f_t/q_t = R_t^f$. Conversely, the gross return to the trader from buying the equity forward is the period $t + 1$ payoff from ownership ($d_{t+1} + q_{t+1}$), divided by the period price of the futures contract, f_t/R_t^f . But since $f_t/R_t^f = q_t$, the gross rate of return to the trader on the futures transaction in equity is exactly the same as the gross rate of return from holding the equity directly.

²² Futures markets do not exist for individual stocks, but they do exist for stock market index funds, which are mutual funds that replicate the performance of the market index. Options trades are available for individual stocks, however. And through options trading it is possible to replicate the payoffs on a simple futures contract.

²³ It is not necessary to have the household directly trade on the futures market. Instead, think of the household as investing in a mutual fund which issues riskless liabilities. The mutual fund, in turn, obtains "portfolio" insurance on its assets by selling all of its equity on the futures market.

Let x_t be the household's $t - 1$ forward sales of its equity holdings for delivery at t . Then, the household's budget constraint becomes

$$(d_t + q_t)s_t + R_{t-1}^f q_{t-1} x_t + b_t + D_t + w_t - c_t - q_t(s_{t+1} + x_{t+1}) - b_{t+1}/R_t^f \geq 0, \quad (28)$$

where, by arbitrage, $R_{t-1}^f q_{t-1} = f_{t-1}$. Next, let z_t be the trader's corresponding forward purchases of equity, and, let κ^f be the margin requirement on forward purchases of equity. Then the trader's budget and margin constraints become, respectively,

$$[(d_t + q_t)(s_t^* + z_t) + b_t^* - R_{t-1}^f q_{t-1} z_t] - D_t - q_t s_{t+1}^* - b_{t+1}^*/R_t^f = 0 \quad (29)$$

$$[(d_t + q_t)(s_t^* + z_t) + b_t^* - R_{t-1}^f q_{t-1} z_t] - D_t \geq \kappa q_t s_{t+1}^* + \kappa^f q_t z_{t+1}. \quad (30)$$

Finally, in equilibrium, sales of futures contracts must equal purchases:

$$x_t = z_t. \quad (31)$$

If the two margin requirements κ and κ^f are identical, then the addition of the future's market in equity has no impact. The price of equity and the riskless rate are unaffected. This is true, since the payoffs to the two parties from the futures transactions are the same as if the trader had financed the equity acquisition by issuing riskless debt to the household. If the margin requirements differ, then the less restricted type of transaction will drive out the more restricted one. If futures trading has lower margin requirements, for example, then the trader will use this financing vehicle exclusively.

Options trading permits a finer partitioning of the risk on the equity. However, it is also a way for the trader to finance equity acquisitions. The trader can use the options market to replicate the futures transaction we have just described. He can do so by buying at t a call option on the household's equity at $t + 1$ at the price o_t and simultaneously at t selling the household a put option for $t + 1$ at the same price. This trade guarantees the household a sure return, and it implies that the trader receives the difference between the ex post payoff on the equity and this sure return. This options trade is therefore equivalent to the future trade.

In summary, our model requires that the trader raise his exposure by using leverage to finance equity acquisitions. However, it makes no differ-

ence whether he obtains the leverage explicitly or instead obtains it implicitly via futures and options trading.

6.2. Multiple Risky Assets

With multiple risky assets, spillover or contagion effects may emerge in the sense that the prices of different assets may co-move more than they would in a frictionless framework. In addition, the relative volatilities of the prices of different assets will depend on their respective margin requirements.

To flesh out these ideas, suppose that there are now J risky assets. Each asset generates an exogenously given random dividend. The supply of each may vary exogenously over time. Assume, further, that the household faces quadratic costs of trading each asset that involve disutility of effort, in analogy to the case of a single risky asset. Let q_t^j be the price of risky asset j , let s_t^j be the households' holdings, and let a^j be the associated adjustment cost parameter. The household's demand for risky asset j is then given by

$$s_{t+1}^j - s_t^j = [(q_t^{fj}/q_t^j) - 1]/a^j, \quad (32)$$

where q_t^{fj} is the "fundamental price," given by

$$q_t^{fj} = E_t \sum_{\tau=1}^{\infty} M_{t+\tau} \cdot d_{t+\tau}^j \quad (33)$$

Equations (32) and (33) are, of course, straightforward generalizations of the household's decision rule in the case of a single risky asset (see Eqs. (11) and (12)).

The trader must now satisfy a margin requirement that applies to his holdings of J types of risky assets:

$$\sum_{j=1}^J (d_t^j + q_t^j) s_t^{*j} + b_t^* - D_t \geq \sum_{j=1}^J \kappa^j q_t^j s_{t+1}^{*j}, \quad (34)$$

where s_t^{*j} is trader's holdings of asset j , and κ^j is the margin requirement for asset j . As before, the trader's objective is to maximize the discounted stream of dividend payouts to the households, subject to the prevailing margin requirement and to a nonnegativity restriction on dividends. In analogy to the household's problem, the decision rule for each risky asset j

that emerges is parallel to the case of the single risky asset:

$$\gamma_t = E_t\{\gamma_{t+1}M_{t+1} \cdot R_{t+1}^{ej}\} + \kappa^j\mu_t, \quad (35)$$

where R_{t+1}^{ej} ($= (q_{t+1}^j + d_t^j)/q_t$) is the gross return on risky asset j . As in the single risky asset case, μ_t is the multiplier on the margin constraint, and γ_t is the shadow value of wealth (see Eq. (13)).

It is now apparent how spillover effects may arise. Whether the margin constraint binds depends on the total value of the trader's stock holdings. Suppose there is a shock that reduces the fundamental price of one of the assets, causing the margin constraint to bind. The trader will in general sell some of his holdings of each type of asset. The price of each asset will therefore drop below its respective fundamental value. To see this formally, observe that the excess return on each asset j rises as the margin constraint tightens (i.e., μ_t rises). Although the shock hits only one asset, the effect spills over to the values of the others.

Furthermore, how the shock influences the relative returns on two risky assets depends on the respective margin constraints. It follows from (35) that for risky assets j and k , the following condition must hold:

$$\begin{aligned} & [E_t\{(\gamma_{t+1}/\gamma_t)M_{t+1} \cdot R_{t+1}^{ej}\} - 1]/\kappa^j \\ & = [E_t\{(\gamma_{t+1}/\gamma_t)M_{t+1} \cdot R_{t+1}^{ek}\} - 1]/\kappa^k. \end{aligned} \quad (36)$$

Roughly speaking, Eq. (36) implies that when the margin constraint binds, the expected return on each asset is positively related to its respective margin requirement. To see this explicitly, consider the deterministic case with constant consumption, dividends, and asset supplies where the constraint binds only currently. Then $M_{t+1} = \beta$, $\gamma_{t+1} = 1$, and $q_{t+1}^j = q^{jf}$. It follows that $\kappa^j > \kappa^k \Rightarrow R_{t+1}^{ej} > R_{t+1}^{ek} \Rightarrow q_t^j/q_t^{jf} < q_t^k/q_t^{kf}$. Thus, relative to the fundamental price, the price of the asset with the higher margin requirement must drop more. The reason for this is that, everything else being equal, the trader can most efficiently meet a margin constraint by shedding assets that have high margin requirements. For example, selling \$100 worth of some asset subject to a margin requirement of 0.5 and using the proceeds to reduce debt creates a slack of \$50 in the margin constraint. If the margin requirement is only 0.1, then such a sale creates a slack of only \$10 in the margin constraint. So assets with a high margin requirement have greater value in terms of relaxing the margin requirement. In equilibrium, their prices must drop more, since the trader is trying to equate the loss from selling each type of asset plus its shadow value in terms of relaxing the margin constraint.

7. CONCLUDING REMARKS

We have developed a dynamic general equilibrium model of asset pricing in which specialized traders must satisfy margin requirements. The margin requirement leads to enhanced volatility of prices. Our analytical results and some simulations indicate that it is possible to have substantial departures of the market price from the corresponding price under frictionless markets.

The main shortcoming of the model is that traders have no incentive to maintain a leveraged position over time. They eventually move to 100% equity positions, either by retaining earnings or by shedding risky assets. Thus, over time the margin constraint no longer impedes behavior and, as a consequence, asset prices converge to their frictionless values.

Actual traders do seem to maintain highly leveraged positions, however, either directly or indirectly through futures and options trading, as we have discussed. It thus seems worthwhile to extend the model to account for this phenomenon. One possibility is that this kind of leverage provides outside lenders with a way to discipline traders, somewhat along the lines that Calomiris and Kahn [6] use to explain why commercial banks issue demandable debt.²⁴ In this kind of environment, then, the financial frictions we have described that give rise to asset price volatility may persist over time.

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²⁴ Commercial banks and traders have similar liability structures in that both rely on equity capital and extensive use of short-term debt, where the latter is defined to include exposure from futures and options trading. Thus, it does seem natural to search for common explanations for capital structure.

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