Notes on Estimating the Closed Form of the Hybrid New Phillips Curve

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Abstract

Galí and Gertler (1999) developed a hybrid variation of the New Keynesian Phillips curve that relates inflation to real marginal cost, expected future inflation and lagged inflation. Estimates of the model suggest that forward looking behavior is highly important; the coefficient on expected future inflation is large and highly significant. An alternative way to express the model is the closed form which relates inflation to a discounted stream of expected marginal cost and lagged inflation. Here we present estimates of the closed form and show that the results are very similar to those obtained from estimating the hybrid model directly. Hence the conclusions of GG and others regarding the importance of forward looking behavior are robust to estimating the model in closed form. Results to the contrary in the literature stem from not properly accounting for the mapping of parameters between the baseline hybrid version and the closed form.

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1 Introduction

Galí and Gertler (1999; henceforth, GG) propose a simple variation of Calvo's (1983) staggered contract model that allows for a fraction of firms to use a backward-looking rule of thumb to set prices. As in Calvo's original model, each firm has a probability of $1-\theta$ of being able to reset its price in any given period, independently of the time elapsed since its most recent price adjustment. GG's model assumes that of those able to adjust prices in a given period, only a fraction $1-\omega$ will choose prices optimally, i.e. on the basis of expected future marginal costs. A fraction ω, on the other hand, use instead a simple rule of thumb: they set price equal to the average of newly adjusted prices last period plus an adjustment for expected inflation, based on lagged inflation π_{t-1}. The mix of such backward looking elements , with the otherwise forward looking price setting leads to a simple hybrid variation of the New Keynesian Phillips curve. In particular, let mc_t be real marginal cost and β a subjective discount factor. Then the hybrid Phillips curve is given by

$$\pi_t = \lambda \; mc_t + \gamma_f \; E_t\{\pi_{t+1}\} + \gamma_b \; \pi_{t-1} \tag{1}$$

where

$$\begin{aligned} \lambda &= (1-\omega)(1-\theta)(1-\beta\theta)\phi^{-1} \\ \gamma_f &= \beta\theta\phi^{-1} \\ \gamma_b &= \omega\phi^{-1} \end{aligned}$$

with $\phi = \theta + \omega[1-\theta(1-\beta)]$.[1]

As GG show, the simple hybrid Phillips curve has the following closed form, conditional on the expected path of real marginal cost:

$$\pi_t = \delta_1 \pi_{t-1} + \left(\frac{\lambda}{\delta_2 \; \gamma_f} \right) \sum_{k=0}^{\infty} (\frac{1}{\delta_2})^k \; E_t\{mc_{t+k}\} \tag{2}$$

where δ_1 and δ_2 are, respectively, the stable and unstable roots of equation (1) and are given by:

$$\delta_1 = \frac{1-\sqrt{1-4\gamma_b\gamma_f}}{2 \; \gamma_f} \quad ; \quad \delta_2 = \frac{1+\sqrt{1-4\gamma_b\gamma_f}}{2 \; \gamma_f} \tag{3}$$

GG and Galí, Gertler and López-Salido (2001; henceforth, GGL) adopt an instrumental variables approach to estimate (1) using U.S. and European data, respectively. They find that this simple model provides a good description of inflation dynamics, with the forward

[1] The expression for λ arises in the case of constant returns to scale. Galí, Gertler, and López-Salido (2001) consider the case with decreasing returns to labor, which leads to a slight modification in λ.

looking component playing an important role. In particular, they show that: (i) baseline estimates of the fraction of backward looking firms range between 0.18 to 0.45 in the U.S., and between 0 and 0.35 in Europe, (ii) additional lags of inflation do not enter significantly; and (iii) conditional on the estimated coefficients of the hybrid model and on VAR-forecasts of future marginal costs, the right hand side of equation (2) (referred to as *fundamental* inflation) does a reasonably good job of tracking actual inflation.

An alternative approach is to estimate directly the closed form equation (2). There are some disadvantages of doing so (see section 5), but also some advantages. One advantage is simply to provide a robustness check on the estimates of the hybrid model. A second virtue, as noted by Rudd and Whelan (2001; henceforth, RW), is to provide a sharper test against the alternative of pure backward looking behavior. In particular, if the instrument set includes variables that directly cause inflation but are omitted from the hybrid model specification, the estimation may be biased in favor of finding a strong effect of expected future inflation on current inflation. GG and GGL address this issue by allowing for additional lags of inflation in the right hand side of (1) (in addition to using them as instruments), and then showing that these additional lags are not significant. This exercise provides a simple test of whether additional lags affect inflation independently of the information they contain about future inflation. Estimating the closed form provides a nice alternative test, in which the right hand side is given by a discounted stream of expected marginal cost, as opposed to just expected one-period ahead inflation.

In these notes we accordingly describe an instrumental variables procedure for estimating the closed form that is in the spirit of the approach we used to estimate the simple hybrid formulation. We present estimates of the closed form and show that they are very close to the estimates we obtained in our earlier work. We also describe how to extend the procedure to allow for additional lags of inflation and show that our results are robust to doing so.

2 Relation to Previous Literature

Our analysis is related to several other papers, in addition to our own previous work. In particular, Sbordone (1999) estimates the closed form specification for the limiting case of pure forward looking behavior, i.e., $\omega = 0$. In that case, the closed form becomes simply

$$\pi_t = \lambda \sum_{k=0}^{\infty} \beta^k \; E_t\{mc_{t+k}\} \tag{4}$$

Sbordone uses a simple minimum distance criterion to estimate the equation parameters and then shows that the fitted model does an excellent job of tracking the actual data. We instead estimate the hybrid model, thus allowing for a non-zero $\omega = 0$. Thus we are able to directly test the importance of forward versus backward looking behavior. We also differ by using an instrumental variables approach to estimation.

Rudd and Whelan (2001) (henceforth RW) estimate the pure forward looking closed form model appended given by equation (4), modified to allow for additional lags of inflation. In particular, for the case of one lag of inflation, the equation they consider is given by

$$\pi_t = \lambda \sum_{k=0}^{\infty} \beta^k E_t\{mc_{t+k}\} + \phi\ \pi_{t-1} \tag{5}$$

Conditional on a value for β, and the ex-post realizations of real marginal costs, they construct a time series for the truncated discounted sum variable $\pi_t^{RW} = \sum_{k=0}^{12} \beta^k\ mc_{t+k}$. Then they proceed to estimate,

$$\pi_t = \lambda\ \pi_t^{RW} + \phi\ \pi_{t-1} + \xi_t$$

where ξ_t is orthogonal to information available at time t and hence parameters λ and ϕ can be estimated using an instrumental variables procedure.[2]

While RW's approach does permit a test of the pure forward looking model (i.e., under that null, $\phi = 0$), it does not provide a legitimate test of the hybrid model. In particular, their empirical specification does not nest the hybrid in an explicit way. The reason is straightforward: as equation (1) makes clear, the discount factor for expected marginal cost in the closed form does NOT generally correspond to β; instead it is a function of γ_f and γ_b, i.e., the coefficients on expected future inflation and lagged inflation in the hybrid model. It is only equal to β in the limiting case of no backward looking behavior.[3]

In addition, it is critical to note that, in the closed form, the parameter ϕ on lagged inflation does not provide a simple measure of the degree of backward looking behavior. That is, ϕ should not be confused with γ_b, the coefficient on lagged inflation in the baseline hybrid specification. More specifically, (2) implies that ϕ should correspond to the eigenvalue δ_1 which, as (3) makes clear, is a nonlinear function of γ_f and γ_b. To illustrate the danger of interpreting ϕ as a measure of the relative importance of the backward looking component, consider the following numerical example. Suppose that $\beta = 1$, and $\gamma_f = \gamma_b = 0.5$, so that forward and backward looking behavior are equally important. It is easy to check that in this case $\phi = \delta_1 = 1$. It would clearly be incorrect, however, to suggest that an estimate of $\phi = 1$ implies pure backward looking behavior. All this suggests that one cannot assess the relative importance of forward versus backward looking behavior from the RW specification and that it is important to identify the parameters γ_f and γ_b directly.

Given these considerations, we estimate the appropriate closed form for the hybrid model, i.e. the one given by equation (1). Among other things, this involves estimating the discount factor directly, as opposed to calibrating it. Overall, our approach allows us to recover estimates of γ_f and γ_b, along with standard errors. As shown below, the resulting estimates are similar to the ones obtained in GG.

[2] For simplicity we are ignoring the component error term that results from the truncation of the discounted sum.

[3] In footnote 12 of their paper, RW seem to be aware of this issue, but proceed to ignore the problem altogether in their subsequent empirical analysis.

We proceed by first re-estimating the baseline hybrid model to confirm the results in GG, as well as to facilitate comparison with estimation of the closed form. We then present estimates of the closed form of the hybrid model, first for the case of one lag of inflation and then for the case of two lags (the appendix outlines the approach for the general case.)

3 Estimates of the Hybrid Model

3.1 The Baseline Hybrid Model

We estimate the hybrid model (1) using the same approach as in and GG and GGL[4]. We follow GGL by using a smaller instrument set than in GG in order to minimize the potential estimation bias that is known to arise in small samples when there are too many over-identifying restrictions. Accordingly, we restrict the instrument set to four lags each of inflation, real marginal cost, detrended real output and nominal wage inflation. We also consider a smaller instrument set with four lags of inflation, but only two lags of the other variables. As in GG, we also consider two different measures of inflation: (i) the percent change in the GDP deflator and (ii) the percent change in the non-farm business (NFB) deflator.

Table 1 presents the estimates of the hybrid model for each measure of inflation. In each case we report results for the two instrument sets. The results are very similar to those obtained in GG. Marginal cost enters with the correct sign and is statistically significant. The coefficient on expected future inflation exceeds that of lagged inflation in each case. The results, further, are very similar across instruments sets. With the GDP deflator, γ^f is roughly 0.65, while γ^b is roughly 0.35. With the NFB deflator, γ^f is roughly 0.78, while γ^b is roughly 0.18.

3.2 The Hybrid Model with Additional Inflation Lags

The baseline hybrid model restricts the instruments (other than inflation lagged once) to affect inflation only via their predictive power for expected future inflation. Bias could emerge if the instruments also have a direct impact on inflation. GG and GGL recognize the issue and take the following approach to address it: If the backward looking model is really true, then the most likely source of bias involves the exclusion of the additional lags of inflation in the instrument set from appearing directly in the right side of equation (1). Accordingly, GG and GGL re-estimate the hybrid model allowing three additional lags of inflation of the right hand side.

We repeat the exercise here. Again the results are very similar to those obtained in both GG and GGL. Table 2 reports the results. For the GDP deflator, the additional lags

[4] See GGL for a complete description of our approach. Note that we add a constant term to the regression equation. As is standard in the literature, we assume that all firms index their prices to the steady state rate of inflation.

are not significant and the sum of their coefficients is not much different from zero. For the NFBD deflator, the coefficient on the third lag of inflation is significant, but the coefficient on the first lag is no longer significant. The sum of the coefficients on all four lags is about the same as in the case of one lag: about 0.20.

In sum, we conclude that our results are robust to this simple test of specification bias. The most basic implication of the backward looking model of inflation, namely that additional lags of inflation should have a direct impact on inflation, does not appear to be supported by the data. Of course, there may be other implications that we have not checked. However, unless one takes a very explicit stand on what are the implications of the backward looking model, it is very difficult to test.

4 Estimates of the Closed Form

4.1 The Closed Form of the Baseline Model

Let z_t denote either of our two instrument sets described above. Then, under rational expectations, equation (2) defines the following set of orthogonality conditions:

$$E\{(\pi_t - \delta_1 \pi_{t-1} - \overline{\lambda} \sum_{k=0}^{\infty} \delta_2^{-k} \; mc_{t+k}) \; \mathbf{z}_t\} = 0 \tag{6}$$

with $\overline{\lambda} = \frac{\lambda}{\delta_2 \gamma_f}$, and where (3) defines the mapping from the roots δ_1 and δ_2 to the parameters of hybrid model, γ_b and γ_f. Further, we follow Rudd and Whelan (2001) by using a truncated sum to approximate the infinite discounted sum of real marginal costs. However, as we stressed in the introduction, we differ by estimating the discount factor and exploiting the link between the hybrid model and its closed form to identify the key parameters γ_b and γ_f of the hybrid model.

We use sixteen leads of real marginal cost to construct the discounted stream of real marginal cost. The results are robust to using either twelve or twenty-four. Table 3 presents the results for both the GDP and NFB deflators, again using the two instrument sets in each case. The table reports estimates of the roots δ_1 and δ_2, along with estimates of the key parameters of interest, γ_b and γ_f. What is striking is that the estimates γ_b and γ_f are very close to those obtained from directly estimating the hybrid model given by (1). Indeed, for the GDP deflator, they are virtually identical (roughly 0.65 for γ_f and roughly 0.35 for γ_b). The estimates for the NFB deflator in this case, interestingly, are very similar to those obtained for the GDP deflator (in contrast to the previous where forward looking behavior was slightly more important for the NFB deflator). Finally, the slope coefficient on the discounted stream of expected future marginal cost is positive and highly significant in each instance.

We conclude that estimating the closed form yields results very close to those obtained from directly estimating the hybrid model given by (1).

In addition, the estimated closed form equation does a reasonably good job of describing the actual path of inflation. To demonstrate, we use the parameter estimates of the closed form to construct estimates of fundamental inflation, following GG. In particular, we use the parameter estimates along with VAR forecasts of the future path of real marginal cost to construct period by period estimates of current inflation implied by the closed form model given by equation (2). Figures 1 and 2 plot the paths of fundamental inflation against actual inflation for the GDP deflator and for the NFB deflator. In each instance, fundamental inflation tracks actual inflation reasonably well.

We conclude that our estimated hybrid model does a reasonably good job of characterizing actual inflation.

4.2 Estimates of the Closed Form with Two Inflation Lags

We next consider generalizing the model to allow for additional lags of inflation. Doing so is conceptually straightforward, but cumbersome because the mapping from the parameters of the hybrid model to the closed form becomes increasingly complex as the lag length increases. Here we provide estimates for the case of two lags. In the next section we describe how to generalize to the case of an arbitrary number of lags.

$$\pi_t = \lambda \; mc_t + \gamma_f \; E_t\{\pi_{t+1}\} + \gamma_{b1}\pi_{t-1} + \gamma_{b2}\pi_{t-2} \tag{7}$$

It is straightforward to motivate equation (7) as a variation of GG's hybrid model, where backward looking price setters use a combination of two lags of past inflation to forecast current inflation, instead of just one lag.

The closed form solution in this instance is given by

$$\pi_t = (\delta_1 + \delta_2)\pi_{t-1} + \delta_1\delta_2\pi_{t-2} + \overline{\lambda}\sum_{k=0}^{\infty}(\frac{1}{\delta_3})^k E_t\{mc_{t+k}\} \tag{8}$$

with $\overline{\lambda} = \frac{\lambda}{\delta_3\gamma_f}$, and where δ_1 and δ_2 are the two stable roots of equation (7) and δ_3 is the unstable root. The mapping to the parameters of the hybrid model given by

$$\frac{1}{\gamma_f} = \delta_1 + \delta_2 + \delta_3 \tag{9}$$

$$\frac{\gamma_{b1}}{\gamma_f} = (\delta_1 \cdot \delta_2) + (\delta_1 \cdot \delta_3) + (\delta_2 \cdot \delta_3)$$

$$\frac{\gamma_{b2}}{\gamma_f} = -(\delta_1 \cdot \delta_2 \cdot \delta_3)$$

We estimate the model proceeding along the same lines as the previous case. Table 4 presents the results. Notice that, overall, the estimates are very similar to the case with one lag. Indeed, the estimate of the coefficient on the second lag of inflation in the hybrid model, γ_{b2}, is very close to zero, while the estimates of γ_{b1} and γ_f are very close to their respective values obtained in the case of one lag.

Since the extension to two lags has only a very negligible effect on the results, it appears that the model with one lag does an adequate job of characterizing the data. These results are consistent with the findings of section 3.

5 Extensions

There are two problems that arise in estimating the closed form. First, it is difficult to estimate the two key primitive parameter of the model: the frequency of price adjustment, θ, and the fraction of backward looking price setters, ω . The slope coefficients γ_b and γ_f are nonlinear functions of these primitive parameters (see equation (1)). Estimating the closed form adds another layer of nonlinearity. Second, given the way the measure of the discounted stream of future marginal cost is constructed, there is a problem of generated regressors, which clouds the construction of standard errors.

Both these considerations suggest that it may be interesting to explore the goodness of fit approach to estimating the closed form of the hybrid model that Sbordone (1999) employs. Since she finds that the pure forward looking model works very well, however, we would expect that our findings will be quite robust to using her approach.

References

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[2] Galí, Jordi and Mark Gertler, 1999, "Inflation Dynamics: A Structural Econometric Approach," Journal of Monetary Economics, 44(2), October, 195-222.

[3] Galí, Jordi, Mark Gertler, and David López-Salido, 2001, "European Inflation Dynamics," European Economic Review, 45(7), June, 1237-1270.

[4] Rudd, Jeremy and Karl Whelan, 2001, "New Tests of the New Keynesian Phillips Curve," mimeo, Federal Reserve Board

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A The General Case with $\mathbf{N-1}$ lags

Consider the following generalization of the hybrid model to the case with $n-1$ lags. It is possible to express the hybrid model as the following n-th order difference equation:

$$\pi_t = \lambda mc_t + \gamma_f \pi_{t+1} + \sum_{i=1}^{n-1} \gamma_{b,i}\pi_{t-i},$$

with $\sum\limits_{i=1}^{n-1} \gamma_{b,i} = \gamma_b$. Lag one period to obtain:

$$\pi_{t-1} = \lambda mc_{t-1} + \gamma_f \pi_t + \sum_{i=1}^{n-1} \gamma_{b,i}\pi_{t-1-i}.$$

Solve for π_t

$$\pi_t = \frac{1}{\gamma_f}\pi_{t-1} - \sum_{i=1}^{n-1} \frac{\gamma_{b,i}}{\gamma_f}\pi_{t-1-i} - \frac{\lambda}{\gamma_f} mc_{t-1}.$$

Use the lag operator

$$\pi_t \left(1 - \frac{1}{\gamma_f} L + \sum_{i=1}^{n-1} \frac{\gamma_{b,i}}{\gamma_f} L^{i+1}\right) = -\frac{\lambda}{\gamma_f} mc_{t-1}.$$

The term $1 - \frac{1}{\gamma_f}L + \sum\limits_{i=1}^{n-1} \frac{\gamma_{b,i}}{\gamma_f} L^{i+1}$ can be written as $\prod_{i=1}^{n}(1-\delta_i L)$ with $\delta_1, \delta_2, \ldots, \delta_{n-1} < 1$ and with $\delta_n > 1$. Then

$$\pi_t \prod\nolimits_{i=1}^{n}(1-\delta_i L) = -\frac{\lambda}{\gamma_f} mc_{t-1}.$$

Rearrange to obtain

$$\pi_t (1-\delta_1 L)(1-\delta_2 L)\cdots(1-\delta_{n-1}L)\left(1 - \frac{1}{\delta_n}F\right) = \frac{\lambda}{\delta_n \gamma_f} mc_t.$$

Then

$$\pi_t (1-\delta_1 L)(1-\delta_2 L)\cdots(1-\delta_{n-1}L) = \frac{\lambda}{\delta_n \gamma_f} \sum_{i=0}^{\infty}\left(\frac{1}{\delta_n}\right)^i mc_{t+i}.$$

The LHS is

$$\begin{aligned} \pi_t (1-\delta_1 L)(1-\delta_2 L)\cdots(1-\delta_{n-1}L) &= \pi_t \left[1 - \sigma_1 L + \sigma_2 L^2 - \sigma_3 L^3 + \ldots + \sigma_{n-1}(-L)^{n-1}\right] \\ &= 1 + \sum_{i=1}^{n-1} (-L)^i \sigma_i, \end{aligned}$$

where $\sigma_1, \sigma_2, \ldots, \sigma_{n-1}$ are the **homogenous symmetric polynomials** in $\delta_1, \delta_2, \ldots, \delta_{n-1}$. The homogenous symmetric polynomial of degree i in $\delta_1, \delta_2, \ldots, \delta_{n-1}$ is the sum of all products of i elements obtained from $\delta_1, \delta_2, \ldots, \delta_{n-1}$, for $i = 1, \ldots, n-1$:

$$\begin{aligned}
\sigma_1 &= \delta_1 + \delta_2 + \ldots + \delta_{n-1}, \\
\sigma_2 &= \delta_1\delta_2 + \delta_1\delta_3 + \ldots + \delta_1\delta_{n-1} + \delta_2\delta_3 + \ldots \delta_2\delta_{n-1} + \ldots + \delta_{n-2}\delta_{n-1}, \\
\sigma_3 &= \delta_1\delta_2\delta_3 + \delta_1\delta_2\delta_4 + \ldots + \delta_{n-3}\delta_{n-2}\delta_{n-1}, \\
&\vdots \\
\sigma_{n-1} &= \delta_1\delta_2 \cdots \delta_{n-1}.
\end{aligned} \tag{10}$$

The solution then looks like

$$\begin{aligned}
\pi_t &= \sigma_1\pi_{t-1} - \sigma_2\pi_{t-2} + \ldots + \sigma_{n-1}(-)^n\pi_{t-n+1} + \frac{\lambda}{\delta_n\gamma_f}\sum_{i=0}^{\infty}\left(\frac{1}{\delta_n}\right)^i s_{t+i} \\
&= \sum_{i=1}^{n-1}(-)^{i+1}\sigma_i\pi_{t-i} + \frac{\lambda}{\delta_n\gamma_f}\sum_{i=0}^{\infty}\left(\frac{1}{\delta_n}\right)^i s_{t+i}
\end{aligned}$$

One can estimate the above equation and then get an estimate for the values of σ_1, $\sigma_2, \ldots$, σ_{n-1}. Having estimated the values of $\sigma_1, \sigma_2, \ldots, \sigma_{n-1}$, one can recover $\delta_1, \delta_2, \ldots, \delta_{n-1}$ from (10), in other words, from the $n-1$ equations defining the homogenous symmetric polynomials in $\delta_1, \delta_2, \ldots, \delta_{n-1}$.

Noting that

$$1 - \frac{1}{\gamma_f}L + \sum_{i=1}^{n-1}\frac{\gamma_{b,i}}{\gamma_f}L^{i+1} = 1 + \sum_{i=1}^{n}(-L)^i\sigma_i,$$

and having estimated the roots $\delta_1, \delta_2, \ldots, \delta_n$, one can get $\gamma_{b,1}, \ldots,$ $\gamma_{b,n}$ and γ_f from the following equalities

$$\begin{aligned}
\frac{1}{\gamma_f} &= \sigma_1, \\
\frac{\gamma_{b,1}}{\gamma_f} &= \sigma_2, \\
\frac{\gamma_{b,2}}{\gamma_f} &= -\sigma_3 \\
&\vdots \\
\frac{\gamma_{b,n-1}}{\gamma_f} &= (-1)^n\sigma_n.
\end{aligned}$$

In general

$$\frac{1}{\gamma_f} = \sigma_1,$$

and

$$\frac{\gamma_{b,i-1}}{\gamma_f} = (-1)^i \, \sigma_i,$$

for $i = 2, \ldots, n$.

Table 1

Hybrid Model

	Parameters		
	γ_b	γ_f	λ
GDP Deflator			
(1)	0.373 (0.032)	0.599 (0.022)	0.015 (0.006)
(2)	0.355 (0.039)	0.627 (0.023)	0.014 (0.006)
NFB Deflator			
(1)	0.181 (0.057)	0.792 (0.024)	0.017 (0.008)
(2)	0.169 (0.071)	0.803 (0.031)	0.019 (0.009)

Note: Sample Period: 1960-1998. Standard errors in parentheses The column D corresponds to the associated sticky prices duration. The row (1) corresponds to the first instrument set: inflation, detrended output, real marginal cost and wage inflation from t-1 to t-4; and the row (2) corresponds to estimates using as instruments:output gap, real marginal costs and wage inflation from t-1 to t-2, and inflation from t-1 to t-4.

Table 2

Hybrid Model: Further Inflation Lags

	Parameters						
	Structural			Further Inflation Lags			
	γ_b	γ_f	λ	ϕ_2	ϕ_3	ϕ_4	$\sum \phi_i$
GDP Deflator							
(1)	0.371 (0.055)	0.594 (0.038)	0.013 (0.006)	-0.105 (0.060)	0.056 (0.075)	0.070 (0.051)	0.021 (0.046)
(2)	0.372 (0.070)	0.649 (0.047)	0.011 (0.006)	-0.101 (0.076)	-0.029 (0.105)	0.111 (0.062)	-0.019 (0.055)
NFB Deflator							
(1)	0.069 (0.059)	0.774 (0.084)	0.016 (0.008)	0.000 (0.122)	0.218 (0.050)	-0.056 (0.065)	0.161 (0.084)
(2)	0.083 (0.069)	0.786 (0.102)	0.012 (0.010)	-0.046 (0.154)	0.207 (0.065)	-0.018 (0.093)	0.143 (0.097)

Table 3

Hybrid Model: Closed Form

	Parameters				
	Structural			*Roots*	
	γ_b	γ_f	λ	δ_1	δ_2^{-1}
GDP Deflator					
(1)	0.379 (0.023)	0.610 (0.015)	0.033 (0.008)	0.598 (0.045)	0.962 (0.032)
(2)	0.378 (0.033)	0.612 (0.021)	0.033 (0.011)	0.597 (0.060)	0.966 (0.041)
NFB Deflator					
(1)	0.371 (0.027)	0.617 (0.014)	0.041 (0.007)	0.576 (0.054)	0.957 (0.024)
(2)	0.363 (0.032)	0.628 (0.018)	0.038 (0.011)	0.558 (0.062)	0.966 (0.034)

Table 4

Two Inflation Lags: Closed Form

	Structural Parameters			
	γ_{b_1}	γ_{b_2}	γ_f	λ
GDP Deflator				
(1)	0.359 (0.055)	-0.049 (0.035)	0.629 (0.028)	0.032 (0.007)
(2)	0.353 (0.066)	-0.047 (0.034)	0.635 (0.042)	0.033 (0.011)
NFB Deflator				
(1)	0.304 (0.220)	-0.033 (0.201)	0.693 (0.040)	0.023 (0.009)
(2)	0.291 (0.057)	-0.029 (0.037)	0.680 (0.048)	0.062 (0.019)

Figure 1: Inflation (GDP Deflator): Actual vs. Fundamental

(nonfarm business labor share)

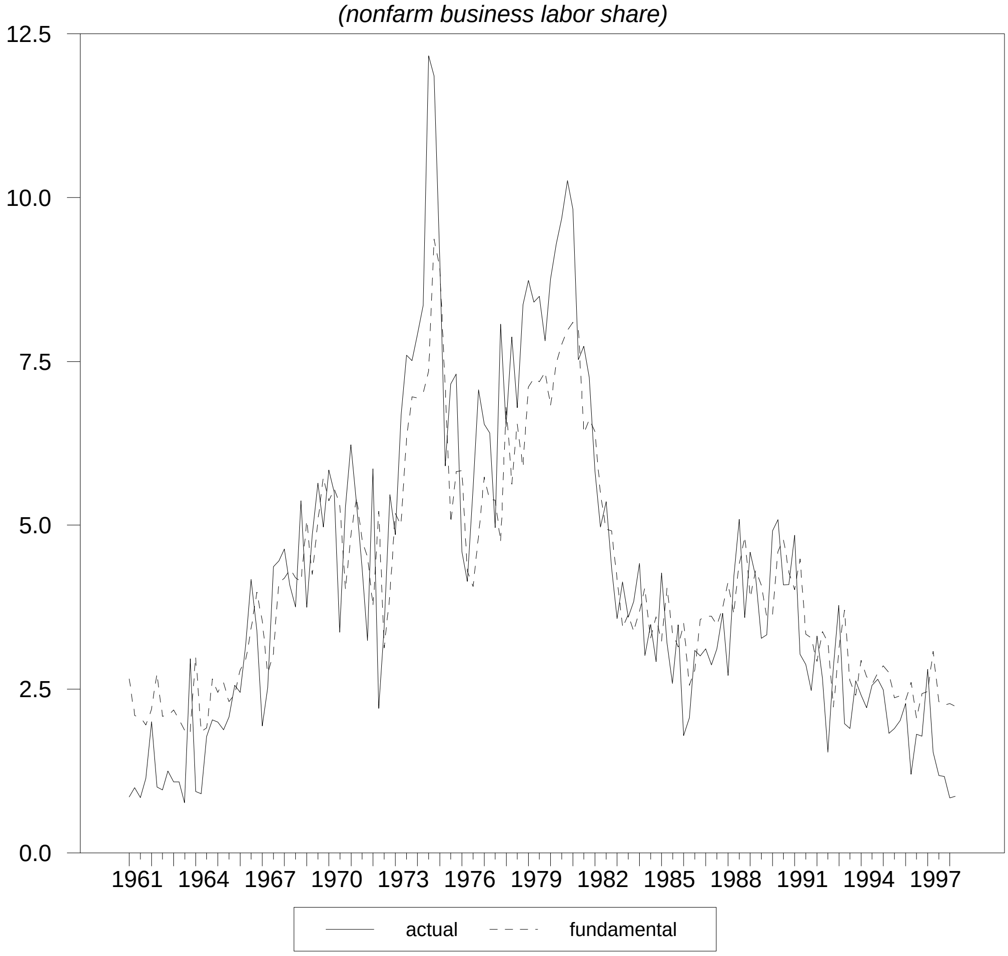

Figure 2: Inflation (NFB Deflator): Actual vs. Fundamental

(nonfarm business labor share)