Sequential Equilibrium in Monotone Games: Theory-Based Analysis of Experimental Data*

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December 17, 2006

Abstract

A monotone game is an extensive-form game with complete information, simultaneous moves and an irreversibility structure on strategies. It captures a variety of situations in which players make partial commitments and allows us to characterize conditions under which equilibria result in socially desirable outcomes. This paper explores the relationship between equilibrium behavior in a class of monotone games, namely voluntary contribution games, and the behavior of human subjects in an experimental setting. We find that the qualitative features of equilibrium match the behavior observed in the laboratory. Both pure- and mixed-strategy equilibria and several key features of the symmetric Markov perfect equilibrium are replicated in the data.

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[*] This research was supported by the Center for Experimental Social Sciences (C.E.S.S.) and the C. V. Starr Center for Applied Economics at New York University. We thank Tom Palfrey for detailed comments and suggestions. We also thank an associate editor and two anonymous referees for their comments. The paper has benefited from suggestions by the participants of seminars at several universities. For financial support, Gale acknowledges National Science Foundation for support under Grant No. SBR-0095109 and the C. V. Starr Center for Applied Economics at New York University, and Kariv thanks UC Berkeley for support under a COR Grant. Kariv is grateful for the hospitality of the School of Social Science in the Institute for Advances Studies.

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We estimate a model of Quantal Response Equilibrium (McKelvey and Palfrey 1995, 1998) and find that it does a good job of explaining the data.

JEL Classification Numbers: D82, D83, C92.
Key Words: monotone games, equilibrium refinements, pure-strategy equilibrium, mixed-strategy equilibrium, symmetric equilibrium, Markov perfect equilibrium, quantal response equilibrium, experiment.

1 Introduction

A major concern of game theorists is to understand the conditions under which self-interested behavior gives rise to cooperative outcomes. To cite one famous example, the Folk Theorem for infinitely repeated games demonstrates that cooperation can be sustained in long run relationships; however, the Folk Theorem is only partly successful as a theory of cooperative behavior. It guarantees the existence of a large class of equilibria, some of which are efficient and many more of which have unattractive welfare properties. One response is to introduce more structure into the game. Aumann and Sorin (1989), Lagunoff and Matsui (1997), and Gale (1995, 2001) have all shown different ways of adding sufficient structure to guarantee efficient equilibrium outcomes in repeated games.

Here we follow the approach suggested by Gale (1995, 2001) in his study of *monotone games*. Formally, a monotone game is a repeated game in which players are constrained to choose stage-game strategies that are non-decreasing over time. This restriction converts a repeated game into a stochastic game in which the state is the profile of stage-game strategies from the previous period. Because strategies are non-decreasing, a player is committing himself to maintain a given level of activity whenever he changes his strategy. Alternatively, one can think of a monotone game as a dynamic implementation of a static game in which players gradually commit themselves to a final strategy. Gale (1995) demonstrated that, in the monotone game based on an infinitely repeated coordination game, all sequential equilibria are efficient in the limit as the length of the time period converges to zero.

In this paper, we explore the relationship between equilibrium behavior in a class of monotone games and the behavior of human subjects in an experimental setting. The class of games we choose to focus on are naturally interpreted as *voluntary contribution games*. Suppose there is an indivisible public project with cost K and N players, each of whom has an endowment of E tokens. The players make irreversible contributions to the project at a sequence of dates $t = 1, \ldots, T$. At the end of T periods, the public project is carried out if and only if the sum of the contributions is large enough to meet the cost of the project. Each player assigns the value A to the project, so his utility if the project is completed is equal to A *plus* his endowment *minus* his contribution. If the project is not completed, his payoff equals his endowment *minus* his contribution. Although it is natural to interpret this class of games in terms of voluntary contributions to a public good, other interpretations are possible.

Two features of monotone games are essential for sustaining cooperation. The first is the requirement that strategies be non-decreasing over time, the defining feature of a monotone game. The second is the assumption of sequential rationality. In general, games that are infinitely repeated and/or involve simultaneous moves, cannot be solved by backward induction, but the monotonicity assumption makes it possible to use backward induction on the payoff-relevant state of the game, rather than the stage of the game. The irreversibility of contributions and the backward induction logic allow players to coordinate their actions and support cooperative outcomes.

Although there are several nice results that can be proved for these games, there are important questions that theory alone cannot answer. First, even such a simple game has many sequential equilibria, so the theory does not make unambiguous predictions. It is an empirical question which of these equilibria, if any, most closely approximates the observed behavior. The set of equilibria can be reduced somewhat by imposing refinements, such as sequential equilibrium, Markov equilibrium, symmetric equilibrium, or pure-strategy equilibrium. Although these refinements are standard in game theory, whether any of these refinements is reasonable in practice is an empirical question.

The conclusions from our theoretical/empirical exercise can be summarized as follows. First, the theory does a good job of accounting for the broad features of the data. While no one would expect the theory to fit perfectly, there are many points at which the qualitative features of equilibrium match the data surprisingly well. Although the multiplicity of equilibria means the predictive value of the theory is weak, we find the equilibrium properties very useful for interpreting the data.

Secondly, as suggested by the theory, there is a very high level of provision of the public good. This stands in stark contrast to experimental results based on static models of public good provision, in which the Nash equilibrium prediction is for zero provision. To check that the high level of provision is due to sequential rationality in a dynamic framework, we contrast these results with the outcome for a one-shot game which is in other respects identical to the dynamic game. We find that, under certain circumstances, provision falls close to zero in the static game.

Thirdly, we find evidence of both pure- and mixed-strategy equilibria. This is interesting because the mixed-strategy equilibria have different properties from the pure-strategy equilibria. Obviously, mixed-strategy equilibria allow for the possibility of providing the public good with (positive) probability less than one, unlike the pure-strategy equilibria, in which the probability is either zero or one. Less obvious is the fact that mixed strategies support equilibria with zero provision for a strictly larger set of parameters than the pure-strategy equilibria.

Fourthly, although the theory cannot provide a good guess about which of the multiple equilibria occur, we find some evidence that subjects use Markov strategies (strategies that depend only on payoff-relevant variables) and several key features of the symmetric Markov perfect equilibrium (SMPE) are replicated in the data. Although, in this model, even the SMPE are not necessarily unique, the set of SMPE gives us much stronger predictions than we could derive from the set of all sequential equilibria.

Finally, although the theoretical model matches the data, qualitatively and quantitatively, at several points, estimating a structural model provides a more systematic test of the theory. We estimate a model of Quantal Response Equilibrium (QRE) as advocated by McKelvey and Palfrey (1995, 1998). We find that the QRE model fits the data and matches the large-scale features of the SMPE. This provides important support for the use of QRE to interpret experimental data.

The rest of the paper is organized as follows. The next section describes the theoretical model and presents some theoretical results that guided our experimental design. Section 3 summarizes questions that can be addressed using the experimental data. Section 4 describes the experimental procedures. Section 5 describes the experimental results and Section 6 provides the QRE analysis. Section 7 concludes by discussing the results and relating them to the literature. Proofs are gathered in Section 8.

2 The game

The monotone game we study can be interpreted as a *voluntary contribution game*. The game is formally described using the following notation. There are N players, indexed $i = 1, ..., N$. Each player is endowed with a finite number E of indivisible tokens. The game is divided into T periods, indexed $t = 1, ..., T$. In each period, the players simultaneously choose how many tokens to contribute to the provision of the public good. The cost of the public good is K tokens and the public good is provided if and only if the total number of tokens contributed is at least K. If the public good is provided, each player receives A tokens in addition to the number of tokens retained from his endowment. The players have perfect information in the sense that, in each period, they know the full history of the game up to that period. Since contributions are irreversible, the player's contribution is monotonically non-decreasing over time.

Each game is defined by five parameters (A, E, K, N, T). These parameters are positive integers except for A, which is a non-negative real number. Each of the parameters influences the set of equilibria of the game in a distinct way and, as we explain later, the comparative static properties of the equilibrium set correspondence motivate our experimental design. We assume that the aggregate endowment is at least as the cost of the public good $(NE \geq K)$ so that provision of the public good is always feasible; and we assume that the aggregate value of the good is greater than the cost $(NA > K)$ so that provision is always efficient. These assumptions are maintained throughout the rest of this section.

We next develop a number of theoretical results about the game. All proofs are gathered in Section 8.

2.1 Coordination

The one-shot game $(T = 1)$ provides a useful benchmark for our subsequent analysis of the dynamic games. Our first objective is to compare the provision of the public good between static and dynamic games. To avoid trivial cases, we assume that $\min\{A, E\} < K$. This condition ensures that it is either infeasible $(E < K)$ or individually irrational $(A < K)$ for a single player to provide the public good.

Proposition 1 (one-shot) *Suppose that* $T = 1$ *and* $\min\{A, E\} < K$. *Then there exists a Nash equilibrium with no provision. Conversely, under the maintained assumptions, there exists at least one Nash equilibrium in which the good is provided with probability one.*

In the one-shot game, non-provision of the public good is explained as a *coordination failure*. If a player thinks that no one else will contribute, it is not optimal for him to contribute. Conversely, if he thinks that his contribution is both necessary and sufficient for provision, then he will be happy to contribute. Hence, provision of the good can be supported, in spite of the free-rider problem, because each contributing player is pivotal (Bagnoli and Lipman (1989)).

Turning to dynamic games, the sharpest result is obtained for the case of pure-strategy equilibria. We assume that the value of the public good is greater than any player's endowment ($A > E$) and the length of the game is sufficiently long ($T \geq K$). The condition $T \geq K$ highlights the central role of backward induction in the analysis of the monotone game. It takes time to ensure cooperation and without enough time there may exist a non-provision equilibrium. The condition $A > E$ is needed to avoid a "self-punishing" strategy. Proposition 6 below shows what happens when endowments are "too high."

Proposition 2 (pure strategy) *Suppose that $A > E$ and $T \geq K$. Then, under the maintained assumptions, in any pure strategy sequential equilibrium of the game, the public good is provided with probability one.*

The logic of the proof can be illustrated by an example. Suppose that there are three players ($N = 3$) and each player has an endowment of one token ($E = 1$). There are two periods ($T = 2$), the cost of the public good is two ($K = 2$), and the value of the public good is greater than the endowment ($A > 1$). In any pure strategy equilibrium, the probability of provision is either one or zero, so it is enough to show that the zero-provision equilibrium is not sequential. Suppose, contrary to the claim in the proposition, that there exists a pure strategy sequential equilibrium with zero provision. Then every player's payoff is simply the value of his endowment $E = 1$. If one player contributes a token at period 1, one of the remaining players can earn at least $A > 1$ by contributing his endowment at period 2. Thus, the good must be provided at period 2 if one player contributes at period 1. Anticipating this response, it is clearly optimal for someone to contribute a token at period 1.

Although pure-strategy equilibria give us a very clean result, they do not tell the whole story. To get a more robust result, we should take account of mixed strategies. Mixed strategies are relevant for several reasons. The experimental data will show that they are empirically relevant. They are also theoretically relevant because they expand the set of parameters for which there exists a no-provision equilibrium. The preceding example, where $N = 3$, $A > E = 1$, and $T = K = 2$, can be used to illustrate this possibility. If one player contributes a token in period 1, the continuation game at period 2 consists of two active players, only one of whom needs to contribute a token in order to provide the good. The continuation game possesses a symmetric mixed-strategy equilibrium where each of the two active players contributes in period 2 with probability $0 < \lambda < 1$. A necessary and sufficient condition for λ to be an equilibrium strategy is that each of the two uncommitted player be indifferent between contributing and not contributing. Simple calculation shows that indifference requires

$$\lambda = (A - 1)/A.$$

In this mixed-strategy equilibrium, the good is provided unless neither of the two players contributes, that is, the good is provided with probability $1 - (1 - \lambda)^2$. Then, if the player who contributes in period 1 anticipates his opponents will play the symmetric mixed-strategy equilibrium at period 2, it is rational for him to contribute if

$$\left[1 - (1 - \lambda)^2\right] A \geq 1$$

or $A^2 - A \geq 1$. The critical value of A is thus $A^* \approx 1.618$. For any $A < A^*$ it is not worthwhile for a player to move first if he anticipates that the other two will play the mixed-strategy equilibrium, whereas in the pure-strategy case it is sufficient to assume $A > 1$. Thus, the use of mixed strategies in the continuation game can discourage an initial contribution and support an equilibrium with no provision. In the two-period game $A > A^*$ is necessary and sufficient for provision of the good with positive probability. In fact, this condition is sufficient for positive provision for games of any length $T \geq K$. We summarize this discussion in the following proposition.

Proposition 3 (mixed strategy) *Suppose the parameters of the game are* $N = 3$, $E = 1$, $T \geq K = 2$. *If* $1 < A < A^*$, *where* $A^* \approx 1.618$, *then there exists a mixed strategy equilibrium in which the good is provided with probability zero.*

The irreversibility of the player's contributions, which is the essential element of a monotone game, has a critical impact on the play of the game. The relationship between past and future contributions can be quite complex, as in any dynamic game, but in this case there is a systematic tendency for the irreversibility of contributions to promote cooperation in the provision of the public good. More precisely, as long as the value of the public good is sufficiently high, rational behavior always leads to provision of the public good with positive probability. The next result generalizes Proposition 3.

Proposition 4 (provision) *For any positive integers* E, K, N, *and* T *satisfying* $K < \min\{NE, T\}$ *there exists a number* $A^*(E, K, N, T)$ *such that, for any* $A \geq A^*(E, K, N, T)$, *the probability that the public good is provided is positive in any sequential equilibrium of the game defined by* (A, E, K, N, T).

2.2 Free-riding

Under-provision of the public good depends on two factors, coordination failure and the free-rider problem. Coordination failure occurs most readily in the one-shot game ($T = 1$), where players would all be better off if the good were provided, but are unable to escape from the bad equilibrium. In a dynamic game, the irreversibility of contributions and the backward induction logic allow players to coordinate their actions. The free-rider problem arises because each player would like someone else to contribute on his behalf. The severity of the free-rider problem depends on the extent to which it is possible to make unequal contributions in equilibrium. The games in which $K = NE$ provide a useful benchmark because each player must contribute his entire endowment in order for the public good to be provided. There is no possibility of taking a free ride on the contributions of other players.

To illustrate, suppose that $N = 3$, $A > E = 1$, and $T \geq K = 3$. In any sequential equilibrium of this game, each player can guarantee himself a payoff equal to $A > 1$ tokens. To see this, suppose that two players have already contributed in period $T - 1$. Then the remaining player will contribute in period T and each of the players receives A tokens. Now suppose that one player has already contributed by period $T - 2$. Then by the previous argument, either of the two other players can guarantee himself a payoff of A tokens by contributing in period $T - 1$. Clearly, any of the three players can guarantee himself A tokens by contributing in period $T-2$.

The following result generalizes the preceding argument. Note that it does not rule out the use of mixed strategies, even along the equilibrium path.

Proposition 5 (no-free-riding) *Suppose that $K = NE$, $A > E$ and $T \geq K$. Then the good is provided with probability one in any sequential equilibrium of the game.*

Taking the condition $K = NE$ as defining the absence of free riding, the free-rider problem must be worse in some sense when the total endowment exceeds this level. If the total endowment is too large, non-provision is consistent with sequential equilibrium, as the following result shows. It provides a kind of converse to Proposition 2, where there is an upper bound on the size of the endowment $E < A$.

Proposition 6 (free-riding) *Suppose that $E > A$ and $T \geq K$. Then under the maintained assumptions, there exists a pure strategy sequential* *equilibrium of the game in which the public good is provided with probability zero.*

The essential ingredient in the construction of this equilibrium is the self-punishing strategy employed by Gale (2001). Whoever contributes first condemns himself to making a contribution so large that it outweighs his benefit from the public good. Once the first contribution has been made, it becomes a sunk cost and the player cannot stop himself from making further individually rational contributions. The other players are, of course, only too happy to wait until after he has punished himself for deviating. Anticipating the ultimate outcome, the player will never make that first, fatal contribution. To illustrate, note that if $E = K = 2$ and $1 < A < 2$, it is clearly not worthwhile for a single player to contribute two tokens because the value of the good is only $A < 2$. On the other hand, if a player is foolish enough to contribute one token, it is rational for him to contribute a second token later, since $A > 1$ and the first token is now a sunk cost. Hence, if a player contributes one token, he is condemned to contribute the second later in the game and that will make him worse off.

3 From theory to design

This section provides a bridge between the theory and the experimental design. First, we define a series of simple parametric examples that serve as the treatments in the experiments described in the next section. Then we use the theoretical properties of these examples to identify questions that can be explored using the experimental data.

3.1 Games

The four games we consider consist of a baseline game and three variants that are derived from the baseline by changing one parameter in each case. Throughout, we keep the number of players constant ($N = 3$) and consider two values of the public good, one *high* ($A = 3$) and one *low* ($A = 1.5$). The dynamic games come in two versions, a *long* version ($T = 5$) and a *short* version ($T = 2$). In some cases, the predictions of the theory differ and that provides one set of testable hypotheses. In addition, varying the value of the good (A) and the length of the game (T) provides a test of the robustness of our results.

The **baseline** game uses the parameters $A = 1.5, 3, E = 1, K = 2, T = 2, 5$. The game has a variety of sequential equilibria. The pure-strategy equilibria all involve provision of the good with probability one (Proposition 2), whereas the mixed-strategy equilibria allow for a positive probability that the good is not provided (Proposition 3). In the high-value treatments ($A = 3$), all sequential equilibria are characterized by a positive probability of the provision of the public good, whereas in the low-value treatments ($A = 1.5$) there exists a mixed-strategy sequential equilibrium in which the good is provided with probability zero (Proposition 3).

The **high-cost** game is identical to the baseline game except that the cost of the public good has been increased to $K = 3$, so provision of the good requires every player to contribute ($K = NE$). In this game, there are two factors that affect the rate of provision of the public good. On the one hand, because every player must contribute in order to provide the public good, there is no possibility of taking a free ride here. On the other hand, the high cost of providing the good does allow for the existence of a non-provision equilibrium when the horizon is short ($T = 2$), in contrast to Proposition 5.

The **high-endowment** game is the same as the baseline game except that the endowment is increased to $E = 2$. By increasing the individual endowments, we increase the potential asymmetry of contributions to the provision of the public good and hence the potential for free riding. For this reason, there exists an equilibrium in which provision is zero when the value of the good is low ($A = 1.5$) (Proposition 6).

Finally, the **one-shot** game is the same as the baseline game except that $T = 1$. The one-shot game possesses a pure-strategy equilibrium in which the good is not provided (Proposition 1). Every game, static or dynamic, possesses a pure-strategy sequential equilibrium in which the good is provided, but provision of the good must be positive in any sequential equilibrium if $T \geq K$ and certain other conditions are satisfied. Thus, if we observe positive provision in the dynamic games with $T \geq K$, but not in the one-shot game with $T = 1$, we can take that as a sign that backward induction is responsible for the difference.

The various games are summarized in the table below. The right hand column lists the propositions that characterize the most relevant theoretical properties of each game.

Game	A	E	K	T	Theoretical Properties
Baseline	$1.5, 3$	1	2	$2, 5$	Proposition 2, 3
High-cost	$1.5, 3$	1	3	$2, 5$	Proposition 5
High-endowment	$1.5, 3$	2	2	$2, 5$	Proposition 6
One-shot	$1.5, 3$	1	2	1	Proposition 1

3.2 Research questions

We showed that, under certain conditions, dynamic games do not possess equilibria with zero provision (Proposition 4), whereas the static game always possesses a non-provision equilibrium. The first thing we want to check is whether provision rates are higher in dynamic games and, if so, why.

Question 1 (time) *Is provision rate higher in the baseline game than in the static game, and, for each dynamic game, is provision higher in the long version ($T = 5$) than in the short version ($T = 2$), holding the value of the good (A) constant?*

The theory does not make any predictions about the effect of the value of the good (A), except in relation to endowment (E), but it is an obvious question to ask.

Question 2 (value) *For each game, is provision higher in the high-value version ($A = 3$) than in the low-value version ($A = 1.5$), holding the length of the game (T) constant?*

As we have seen, the free-rider problem is absent in the high-cost game ($K = NE$) where provision occurs with probability one when the horizon is long ($T = 5$). It is therefore natural to compare provision in the high-cost and baseline games.

Question 3 (cost) *Is provision rate higher in the high-cost game ($K = 3$) than in the baseline game ($K = 2$), holding the value of the good (A) and the length of the game (T) constant?*

Finally, in the high-endowment game, the free-rider problem is exacerbated (Proposition 6), thus suggesting that provision may be lower than in the baseline game, although the multiplicity of equilibria prevents us from making a precise prediction.

Question 4 (endowment) *Is provision rate lower in the high-endowment game ($E = 2$) than in the baseline game ($E = 1$), holding the value of the good (A) and the length of the game (T) constant?*

In dynamic games with a large number of sequential equilibria, it is natural to simplify the analysis by restricting attention to a subset of equilibria. For example, one might restrict attention to equilibria in pure strategies, to equilibria in Markov strategies, or to equilibria with symmetric strategies.

We consider each of these refinements in turn and ask whether the data suggest that the refinement is satisfied. We begin by looking for evidence of pure and mixed strategies.

Question 5 (mixed strategies) *Are behaviors consistent with mixed-strategy (pure-strategy) equilibrium?*

A strategy is called *Markov* if it depends only on payoff-relevant variables. By limiting the variables on which behavior is conditioned, the Markov property reduces the set of sequential equilibria, sometimes substantially. When each player has an endowment of one token ($E = 1$), the payoff-relevant states of the game are denoted by (n, τ), where n is the total number of contributions and τ is the number of periods remaining after the current period. When each player has an endowment of two tokens ($E = 2$), the payoff relevant states for subject i are denoted by (n, τ, n_i), where n and τ have the usual meaning and n_i is the number of contributions to date by player i. To test the Markov hypothesis, we look at different histories of play that lead to the same payoff-relevant state, (n, τ) or (n, τ, n_i), and test whether the behavior is the same for all histories corresponding to a given state.

Question 6 (Markov) *Are subjects' decision rules consistent with the Markov hypothesis?*

If we assume that strategies are symmetric and Markovian, we are led to consider the class of symmetric Markov perfect equilibria (SMPE), which take a relatively simple form. A general characterization of the SMPE is provided in Section 8 (Proposition 7).

Question 7 (SMPE) *Do the SMPE approximate the observed behavior in some treatments?*

4 Experimental procedures

The experiment was run at the Experimental Economics Laboratory of the Center for Experimental Social Sciences (C.E.S.S.) at New York University. The subjects in this experiment were recruited from undergraduate classes at NYU. Throughout the experiment we ensured anonymity and effective isolation of subjects in order to minimize any interpersonal influences that could stimulate cooperation. After subjects read the instructions, the instructions were read aloud by an experimental administrator. Each experimental session comprised 18 subjects (except for two sessions in which 15 subjects were used) and 15 independent decision-rounds (except for the one-shot game, which comprised 30 rounds). A single treatment (A, E, K, N, T) was used for each session.

Each round began with the computer randomly forming three-person groups. The groups formed in each round were independent of the groups formed in any of the other rounds. At the beginning of the round, each subject had an endowment of E tokens. In the first period, each subject was asked to allocate his tokens to either an x-account or a y-account. Investing a token in the y-account was irreversible. After all subjects had made a decision, each subject observed the decisions of all the subjects in his group. In the second period, each subject was asked to allocate the token(s) remaining in his x-account between the two accounts. At the end of this period, each subject again observed the decisions of all the subjects in his group. This procedure was repeated until T decisions had been made.

When the first round ended, the computer informs subjects of their payoffs. Earnings in each round were determined as follows: if K or more tokens had been contributed to the y-accounts by the subjects, each subject received A tokens plus the number of tokens remaining in his x-account. Otherwise, each subject only received the number of tokens in his x-account. This process was repeated until all decision rounds were completed. Subjects received their payment privately as they left the experiment.

5 Experimental results

In this section we present our experimental results and use them to address the questions listed above. The analysis is mainly focused on quantitative changes in the provision rates, defined as the percentage of rounds in which the public good was provided, and the qualitative shifts in subjects' behavior resulting from changes in the model parameters. We next describe the sensitivity of provision to changes in the parameters A, E, K and T that define the various treatments.

5.1 Provision

One of the central objectives of this paper is to see whether backward induction in dynamic games allows for greater cooperation in the provision of the public good. Therefore, we begin with an analysis of the sensitivity of provision to changes in T and then consider the sensitivity of provision to changes in the other parameters. Table 1 presents the provision rates for each treatment.

[Table 1 here]

5.1.1 The time effect (T)

The static or one-shot game differs from the baseline game only in respect to the number of periods (T). Referring to Table 1, when the value of the good is high ($A = 3$), the provision rates are 0.76 when $T = 1$, 0.74 when $T = 2$, and 0.81 when $T = 5$. A joint chi-square nonparametric test indicates that the difference in the provision rates is not statistically significant (p-value = 0.530).[1] By contrast, when the value of the good is low ($A = 1.5$), the provision rate decreases sharply to 0.10 when $T = 1$, while remaining at 0.47 when $T = 2$ and 0.678 when $T = 5$. These provision rates are significantly different at all conventional significance levels (p-value = 0.000). Hence, the effect of time length on public-good provision is significant when the value of the public good is low, but not when the value of the public good is high. The collapse in the provision rate in the one-shot game when the value is low suggests that backward induction is essential to high provision. By contrast, when the value is high, provision remains positive in the one-shot case and all we can say is that this is consistent with equilibria in which the good is provided with positive probability (Proposition 1).

A crucial feature of the theoretical model used in this paper is the indivisibility of the public good. As Bagnoli and Lipman (1992) showed, the indivisibility of the public good makes each contributing player "pivotal" in the sense that *at the margin* his contribution is both necessary and sufficient for provision, so there is no free rider problem. The Bagnoli-Lipman pivotal player argument applies to games of any length and ensures the existence of efficient equilibria. It is also consistent with the existence of inefficient or non-provision equilibria. The backward induction argument, by contrast, only applies to games with a sufficiently long time horizon and guarantees positive provision in all sequential equilibria if certain conditions are satisfied. A central theme of this paper is the difference between static and dynamic games, as illustrated by the low provision observed in the (low-value) static game and the high provision observed in all the dynamic games.

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[1] Unless otherwise noted, throughout this section, we use chi-square nonparametric tests (see Siegel and Castellan (1988)). We make the usual assumption that observations are independent across groups and rounds.

The data summarized in Table 1 also show differences in the provision rates corresponding to $T = 2$ and $T = 5$ for both the high-cost and high-endowment games. When the value of the good is high ($A = 3$), provision rates in the high-cost treatments amount 0.88 and 0.99 when $T = 2$ and $T = 5$. Although the provision rates are both high, they are significantly different at the 1 percent significance level (p-value = 0.003). When the value of the good is low ($A = 1.5$), provision rates decrease to 0.30 and 0.64 for $T = 2$ and $T = 5$, respectively. The difference between these provision rates is also significant at the 1 percent level (p-value = 0.000).

Finally, holding the value of the good (A) constant, the differences in the provision rates in the high-endowment treatments are significant at the 1 and 10 percent levels for $T = 2$ and $T = 5$, respectively (the respective p-values are 0.008 and 0.051). In summary, the data supports the following result.

Result 1 (time) (i) *Comparing the one-shot game with the baseline game, when the value of the good is low (*$A = 1.5$*), the provision rate is highest when* $T = 5$ *and lowest when* $T = 1$*. When the value of the good is high (*$A = 3$*), there are no significant differences between the provision rates in the static and baseline treatments. (ii) Within the high-cost and high-endowment treatments, the provision rate is significantly higher when* $T = 5$ *than when* $T = 2$*.*

5.1.2 The value effect (A)

Since all games come in low-value ($A = 1.5$) and high-value ($A = 3$) versions, we can study the effect of the value of the good within each game. In the baseline game there exists a mixed strategy equilibrium in which the good is provided with probability zero when the value of the good is low (Proposition 3). This might suggest that a lower value of A will be associated with lower provision rates. The data summarized in Table 1 above confirm this conjecture. In the low-value treatments, the provision rates are 0.47 and 0.68 for $T = 2$ and $T = 5$, respectively. These provision rates are significantly lower than those in the high-value treatments at the 1 and 5 percent significance levels for $T = 2$ and $T = 5$, respectively (the p-values are 0.000 and 0.040).

Similarly, the provision rates in the high-endowment treatments amount to 0.49 and 0.63 for $T = 2$ and $T = 5$, respectively, when the value is low ($A = 1.5$). These rates are significantly lower than the corresponding rates when the value is high ($A = 3$). The differences are significant at the 1 percent level (the respective p-values are 0.006 and 0.001). Most interestingly, when the value is low, the high-endowment game has a pure strategy equilibrium in which the good is provided with probability zero (Proposition 6). Finally, the provision rates in the low-value treatments are also significantly lower than those in the corresponding high-value treatments in both the one-shot and high-cost games (p-values = 0.000). Our next result summarizes these findings.

Result 2 (value) *Within each game, the provision rate is significantly higher when the value of the good is high (*$A = 3$*) than when it is low (*$A = 1.5$*), holding the game length* T *constant.*

5.1.3 The cost effect (K)

To explore the effects of the cost of the public good, we compare the provision rate in the baseline game ($K = 2$) with that in the high-cost game ($K = 3$), holding the game length (T) and the value of the good (A) constant. When the value is high ($A = 3$), the good is produced with probability one in *any* sequential equilibria of the high-cost game when the horizon is long ($T = 5$) and in some pure-strategy sequential equilibria when the horizon is short ($T = 2$) (Proposition 5). The provision rates in the high-value treatments are 0.88 and 0.99, when $T = 2$ and $T = 5$, respectively, confirming that the absence of the free rider problem ($K = NE$) leads to high provision. These provision rates are significantly higher than those in the corresponding baseline treatments at the 1 and 5 percent significance levels (the respective p-values are 0.000 and 0.022). When the value of the good is low ($A = 1.5$), there is no significant difference between the provision rates in the high-cost and baseline treatments when $T = 5$, and the provision rate is even significantly lower in the high-cost treatment when $T = 2$ (p-value = 0.021). In summary, the data support the following result.

Result 3 (cost) *Comparing the high-cost game (*$K = 3$*) with the baseline game (*$K = 2$*) while holding the length* T *constant, the provision rates are higher in the high-cost treatment when the value of the good is high (*$A = 3$*). When the value of the good is low (*$A = 1.5$*), the provision rates are not significantly different when* $T = 5$ *and lower in the high-cost treatment when* $T = 2$*.*

5.1.4 The endowment effect (E)

Finally, we compare the provision rate in the baseline game ($E = 1$) with that in the high-endowment game ($E = 2$), holding the game length (T) and the value of the good (A) constant. In the high-endowment game, the potential asymmetry of contributions to the provision of the public good may intensify the free-rider problem (Proposition 6). Referring to Table 1, when the value of the good is high ($A = 3$), provision rates in the high-endowment treatments are 0.69 and 0.86 when $T = 2$ and $T = 5$, respectively. When the value of the good is low ($A = 1.5$), the corresponding provision rates decrease to 0.49 and 0.63, respectively. However, none of the differences in the provision rates between the baseline and high-endowment treatments is statistically significant at any conventional significance level. We thus report the following result.

Result 4 (endowment) *The provision rates in the high-endowment game (*$E = 2$*) are not significantly different from those in the baseline game (*$E = 1$*), holding the value* A *and length* T *constant.*

5.2 Properties of equilibrium

In this section we discuss the implications of the data for equilibrium selection. We first organize the data by calculating the relative frequencies of contributions for each of the payoff-relevant states in each dynamic treatment. The data are presented in Table 2. The payoff-relevant states of the game are represented by (n, τ) when $E = 1$ and by (n, τ, n_i) for each player i when $E = 2$, where n is the total number of contributions, τ is the number of periods remaining after the current period, and n_i is the total number of contributions to date by player i. The number in parentheses in each cell represents the number of subjects who have an endowment left for contribution.

[Table 2 here]

5.2.1 Mixed versus pure strategies

We first address the evidence for mixed strategies in the experimental data. Referring back to Table 1 above, in all dynamic games, apart from the high-cost game ($K = 3$), the provision rate is positive but less than one. In this respect, the behavior of subjects is not consistent with pure-strategy equilibrium. By contrast, in the high-cost game, when the horizon is long

($T = 5$) and the value of the good is high ($A = 3$), the probability the good is provided is virtually one and this is well approximated by a pure-strategy equilibrium. Note that even in the case where provision occurs with probability one, one cannot conclude that players are necessarily using a pure strategy. There exist mixed-strategy equilibria, including some in which mixing occurs on the equilibrium path, in which the good is provided with probability one. The most that one can say is that this outcome is consistent with the use of pure strategies.

The dynamic patterns of contributions also appear to confirm the use of mixed and pure strategies. The overall pattern, in which most contributions have been made in early periods in the high-cost game, is consistent with pure-strategy equilibria. Referring to Table 2, in the high-cost treatment with $T = 2$ and $A = 3$, the relative frequencies of contributions in the second period are, respectively, close to zero when no one has contributed and close to one when exactly one player has contributed. These cases approximate, on the one hand, a pure-strategy equilibrium with no provision and, on the other, a pure-strategy equilibrium with certain provision, at the corresponding states. Given these continuation equilibria, mixing at the first period is clearly not sustainable in equilibrium. On the other hand, in other games, the relative frequencies of contributions at the last two periods are similar to theoretical (symmetric) mixed-strategy equilibria, as will be shown in Figure 1 below. The use of mixed strategies in continuation games is significant both because it discourages contributions in earlier periods and because it supports outcomes with a (positive) probability of provision strictly less than one. The next result summarizes the findings.

Result 5 (mixed-strategies) *We find behavior consistent with non-provision and full-provision pure-strategy equilibria and with mixed-strategy equilibria in the data.*

5.2.2 Markov behavior

Our next question is whether subjects' behavior is consistent with Markov strategies, which depend only on payoff-relevant states and not on the complete history of play. We focus on the baseline and high-endowment games where the horizon is long ($T = 5$), since in the high-cost game ($K = 3$) the group outcomes are so highly clustered that it does not seem possible to have a meaningful test. The procedure for testing the Markov property is as follows. We first select each of the three states, $(n, \tau) = (1, 2), (1, 1), (1, 0)$, where one token has already been contributed in the past. In the high-endowment game ($E = 2$), we only consider the case where the individual contribution variable $n_i = 0$ in each of the three states.

At each payoff-relevant state (n, τ) there are $(4 - \tau)$ distinct histories reaching the state (n, τ). These histories are denoted by $h(t)$, where $h(t)$ represents the history where one token was contributed at time period t. For example, when $(n, \tau) = (1, 2)$, there are two different histories reaching the current state: one where one token was contributed at $t = 1$, $h(1)$, and the other where the contribution of one token was made at $t = 2$, $h(2)$. Then the (joint) null hypothesis at each of the three states is that the relative frequencies of contributions from different histories reaching the current state are equivalent.

Table 3 below summarizes the relative frequencies of contributions from the different histories reaching each of the three states $(n, \tau) = (1, 2), (1, 1), (1, 0)$ in the baseline and high-endowment treatments with $T = 5$. The number in parentheses in each cell represents the number of observations. At the last column of Table 3, p-values are reported under the null hypothesis. The numbers in parentheses in each cell of the last column represent the value of the chi-square test statistic and its degrees of freedom. In both high-endowment treatments, we cannot reject the Markov restriction in all three states at the 10 percent significance level. In each of the baseline treatments, we reject the null hypothesis only in one out of the three states at the 10 percent significance level. Thus, the data supports the following result.

Result 6 (Markov) *The overall outcomes in the test of the Markov property confirm that subjects' behavior supports the Markov specification.*

[Table 3 here]

5.2.3 Symmetric Markov perfect equilibrium (SMPE)

If we add symmetry to the Markov property, we are led to consider the ability of the SMPE to account for the broad features of the experimental data. A general characterization of the SMPE is provided in Section 8 (Proposition 7). The main predictions from SMPE can be summarized by two facts. First, within each game, the class of SMPE predicts no provision of the public good in early periods when the value is high ($A = 3$) and no provision at all when the value is low ($A = 1.5$). The use of mixed strategies in the continuation games at later periods discourages an initial contribution and, especially when the value of the public good is low enough, supports a unique SMPE with no provision. Secondly, SMPE allow us to make comparative-static comparisons of contributions to various changes in the model parameters. Most importantly, the theory predicts that, within each game, the contribution probability at each state when the value is high is no lower than that when the value is low.

In what follows, we again ignore the high-cost game where pure-strategy equilibria seem like a reasonable explanation for the data. For the other dynamic games, we compare the empirical patterns of contributions with the theoretical predictions of SMPE. Since the SMPE are not unique, we have to select an equilibrium for some treatments. When the value of the good is high ($A = 3$), there are two mixed-strategy SMPE. When the value is low ($A = 1.5$) there is a unique SMPE. Table 4 reports the strategies corresponding to the SMPE in each of the treatments with $T = 5$. Due to the isomorphism between games, the theoretical predictions in treatments with $T = 2$ and $T = 1$ correspond to outcomes in continuation games with two and one periods remaining when $T = 5$.

[Table 4 here]

Comparing the empirical contribution rates in Table 2 above with the predictions from the SMPE in Table 4, we find the qualitative features of the theory and the experimental data surprisingly similar, although there are systematic deviations. Referring to Table 2, the relative frequencies of contributions in the first three periods are around 10 percent in both the baseline and high-endowment games. The strategic delay in early periods reflects the theory's prediction of zero contributions and is clearly robust to changes in the model parameters. Any deviation from the SMPE must increase contributions in the early periods and this probably increases the provision of the public good overall. The data also confirm that provision of the public good is higher than the SMPE predict. This is most clearly seen in the treatments where the value is low ($A = 1.5$).

Referring to Table 4, the SMPE predict that the contribution probabilities in a low-value treatment will be lower in the last two periods than those in the corresponding high-value treatment. It is interesting to note that the comparative-static prediction of the SMPE holds true: in the baseline treatments, the contribution probability at each state in the high-value treatment ($A = 3$) is no lower than that in the low-value treatment ($A = 1.5$). In summary, the dynamic patterns of contribution behavior are sensitive to changes in the parameters, especially the value of the public good. Such sensitivity of contribution behavior is often closely related to the predictions of SMPE.

Figure 1 below compares the empirical contribution frequencies with the SMPE contribution probabilities in the baseline and high-endowment games split by low-value (black) and high-value (white) treatments. The clustering around the 45 degree line in Figure 1 shows that, for those states, the SMPE probabilities are close to the empirical frequencies. Nonetheless, the empirical contribution probabilities are often higher than those from the SMPE in the treatments where the value is low ($A = 1.5$). The deviation in the subjects' behavior from SMPE in the last two periods is toward a lower probability of contribution. Hence, the analysis detects both successes and failures of the SMPE in accounting for different features of the data, as summarized in the next result.

Result 7 (SMPE) *The SMPE explains the qualitative patterns of contributions in the games. Nonetheless, there are systematic departures from the predictions of the SMPE and the deviations from the equilibrium probabilities at earlier and later periods go in opposite directions.*

[Figure 1 here]

6 Quantal Response Equilibrium (QRE)

We have noted many points at which the theory appears to be confirmed by the data, or where the theory provides a natural interpretation for the observed behavior. A more systematic comparison of the theory with the data requires a model that can account for the data, both in the sense of reproducing important moments of the observed data and in the sense of explaining the data in terms of the theory. Observed behavior inevitably contains random mistakes, which can be interpreted, following Harsanyi and Selten, as the effect of a trembling hand, so we cannot expect the theory to fit perfectly; but a version of the theory that allows for random errors may fit the data quite well.

The Quantal Response Equilibrium (QRE) of McKelvey and Palfrey (1995, 1998) allows for the possibility of errors and yet retains a role for best response behavior. At each state, a player's contribution probability is assumed to be a function of the difference between the payoffs from contributing and not contributing. The predictions of the QRE model are different from those of the SMPE for two reasons: first, because the QRE allows for the possibility of mistakes and, secondly, because it assumes that players take into account the possibility that others are also making mistakes.

To explain the subject's propensity to choose an action different from the one predicted by the theory, we assume that each agent's payoff is perturbed by an idiosyncratic preference shock that has a logistic distribution. When each player has an endowment of one token ($E = 1$), the logit equilibrium can be summarized by a choice probability function following a binomial logit distribution:

$$\lambda_{(n,\tau)} = \frac{1}{1 + \exp\left(-\beta_{(n,\tau)}\Delta_{(n,\tau)}\right)},$$

where $\lambda_{(n,\tau)}$ is the equilibrium probability of making a contribution at state (n, τ), $\Delta_{(n,\tau)}$ is the difference between the expected payoffs from contributing and not contributing at state (n, τ), and $\beta_{(n,\tau)}$ is a coefficient. The choice of action becomes purely random as $\beta_{(n,\tau)}$ goes to zero, whereas the optimal action is chosen for sure as $\beta_{(n,\tau)}$ goes to the infinity. For positive values of $\beta_{(n,\tau)}$, the choice probability is increasing in $\Delta_{(n,\tau)}$. To calculate the payoff difference $\Delta_{(n,\tau)}$, we need to take account of the stochastic behavior of other players in the current and future periods. Specifically, it is assumed that players predict the choice probabilities in the continuation games correctly and use them to calculate $\Delta_{(n,\tau)}$.

We use standard maximum likelihood (ML) method for the estimation of the QRE model (the program and details are available from the authors upon request). We focus on the baseline game, since in the high-cost game ($K = 3$) the group outcomes are highly clustered and in the high-endowment game ($E = 2$) the state (n, τ, n_i) is more complex so it does not seem possible to have a meaningful test. Table 5 presents the estimation results with a single parameter value, $\beta_{(n,\tau)} = \beta$ for all (n, τ), and the resulting contribution probabilities at each state $\lambda(n, \tau)$.[2] The parameter estimates are positive and highly significant, showing that the theory does help predict the subjects' behavior.

[Table 5 here]

Comparing the estimated predictions of the QRE with the empirical patterns of contributions in Table 2 and the theoretical predictions of SMPE in Table 4 shows that the QRE captures the subjects' tendency to make early contributions, something which the SMPE cannot reproduce. The estimated QRE also generates a pattern of contributions that is qualitatively similar to the SMPE and is very close in the last two periods, when most of the action occurs.

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[2] $\beta_{(n,\tau)} = \beta$ for all (n, τ) is the most parsimonious account of the data. We also estimated less restrictive specifications where $\beta_{(n,\tau)} = \beta_{(n,\tau')}$ for each n and all t and t', and $\beta_{(n,\tau)} = \beta_{(n',\tau)}$ for each t and all n and n'.

Finally, Figure 2 compares the empirical contribution probabilities with the predicted logit contribution probabilities when the horizon is long ($T = 5$). Each series is a function of the payoff differences estimated by the QRE. The predicted logit choice probabilities across treatments are graphed using the corresponding β estimates. The horizontal axis measures the difference between the expected payoffs from contributing and not contributing, $\Delta_{(n,\tau)}$, and the vertical axis measures the probability of making a contribution, $\lambda_{(n,\tau)}$. These graphical comparisons give a rough indication for goodness of fit and suggest that the logistic specification of the QRE model is confirmed by the data.

[Figure 2 here]

7 Conclusion

We have undertaken an experimental investigation of a class of monotone games – voluntary contribution games - and focused on using several equilibrium refinements to interpret the data generated by the experiments. Our approach it to let theory drive design and our results suggest that the theory adequately accounts for large-scale features of the data. Most importantly, although the multiplicity of equilibria means the theory lacks predictive power, the qualitative features of equilibrium match the data surprisingly well. Additionally, in sharp contrast to the experimental results in one-shot settings, there is a very high level of provision of the public good in the dynamic games. This emphasizes the role of backward induction in supporting cooperative outcomes. Furthermore, both pure- and mixed-strategy equilibria and several key features of the symmetric Markov perfect equilibrium are replicated in the data. Finally, to interpret the data more systematically, we adopt and estimate the model of QRE. The estimation results suggest that the QRE model does help predict the subjects' behavior. The QRE replicates the tendency of early contributions in games, which could not be captured by the SMPE, and at the same time predicts the choice behavior at later periods quite precisely.

Our paper contributes to a small body of work on monotone games. Gale (2001) developed the theory applied in this paper in general environments. Admati and Perry (1991) studied a voluntary contribution game in which two players make alternate contributions to the provision of a public good, and showed that under certain conditions there is a unique (possibly inefficient) equilibrium. Marx and Matthews (2000) extended the model of Admati and Perry (1991) in a number of ways, including allowing for simultaneous moves.

Duffy, Ochs and Vesterlund (2006), henceforth DOV, investigate the model of Marx and Matthews (2000) experimentally and replicate higher provision in a dynamic laboratory setting. While our paper shares a number of features with DOV, there are differences. Most importantly, in DOV contributions are not sunk and the project is partly divisible (there exists a positive marginal benefit of contributing prior to completion of the public good and a discrete positive benefit jump upon reaching the completion threshold). The two papers also address different questions. DOV mainly focus on the sensitivity of provision in static and dynamic games, whereas we focus explicitly on the correspondence between theory and empirical behavior.

Finally, we also note that DOV also have treatments in which providing a public good is not an equilibrium in both static and dynamic games, but they found that increasing the number of periods appears to increase contributions. These findings might suggest that pure trembling plays a role in increasing the provision rates in our dynamic games. However, our finding that the pattern of contributions differs significantly across treatments, *while holding the time length constant*, implies that something other than pure trembling is needed to explain the high provision rates.

More broadly, this paper also contributes to several large experimental game theory literatures, such as those on coordination games, on mixed-strategy equilibria, and on sequential rationality, which explicitly test the predictions of game theory. Camerer (2003) provides a comprehensive discussion of these literatures. Although we are sympathetic to the findings in the experimental literature showing that the use of backward induction is limited in some dynamic games, our findings indicate that additional careful studies are needed before drawing any conclusions about this important issue.

8 Proofs

Let x_{it} denote the number of tokens contributed by player i up to the end of period t and let $x_t = (x_{1t}, ..., x_{Nt})$ denote the profile of cumulative contributions at the end of period t. We refer to x_t as the state of the game in period t. Since a player's choice is irreversible, the state is monotonically non-decreasing over time, $x_t \geq x_{t-1}$, $\forall t$. By convention, $x_0 \equiv 0$. A player's payoff is a function of the terminal state x_T. The payoff function for player

i is denoted by $U_i(x_T)$ and defined by

$$U_i(x_T) = \begin{cases} A + E - x_{iT} & \text{if } \sum_{i=1}^{n} x_{iT} \geq K; \\ E - x_{iT} & \text{if } \sum_{i=1}^{n} x_{iT} < K. \end{cases}$$

Proof of Proposition 1. If player i expects his opponents to contribute nothing, the best response is to contribute nothing. If he contributes $x_i > 0$, then either $x_i < K$ and his payoff is $E - x_i < E$ or $E \geq x_i \geq K > A$ and his payoff is $A + E - x_i \leq A + E - K < E$. In either case, he is better off contributing nothing. On the other hand, we can find at least one outcome x^* such that $\sum_{i=1}^{N} x_i^* = K$ and $A + E - x_i^* \geq E$ for every i. Note that a player will be worse off if he increases his contribution and a player who makes a positive contribution in equilibrium will be no better off if he reduces it because in that case the good is not provided. Thus, x^* is a Nash equilibrium. ■

Let k denote the number of tokens that need to be contributed in order for the public good to be provided and let τ denote the number of time periods remaining in the game, otherwise specified.

Proof of Proposition 2. If $k = 1$ and $\tau \geq 1$ the good will be provided in equilibrium as long as $A > 1$. If not, a single uncommitted player would provide the good and increase his payoff. Since only pure strategies are allowed, the good is provided with probability one in any sequential equilibrium of the game with $k = 1$ and $\tau \geq 1$.

Now consider an arbitrary $k < K$ and $\tau \geq k$ and assume that, for any sequential equilibrium of the game with k tokens needed and τ periods remaining, the good is provided for certain. Then consider the game with $k + 1$ tokens needed and $\tau + 1$ periods remaining . If a player contributes one unit in the game, the good is provided for certain and his payoff will be at least A. Suppose, contrary to what we want to prove, that there exists a sequential equilibrium of this game in which the good is provided with probability zero. In such an equilibrium the player's payoff cannot be greater than E. Clearly, the player would be better off contributing the token and ensuring the good is provided. The desired result follows by induction. ■

Proof of Proposition 3. Suppose that $T = 2$. We construct the necessary mixed-strategy equilibrium. If $k = 0$, the best response for any uncommitted player is to contribute nothing. If $k = 1$, there is only one period left (one token has already been contributed in the first period) and there are two uncommitted players. We assume they play the unique symmetric mixed-strategy equilibrium of the one-shot game. If $k = 2$, we assume that no one contributes. To show that this strategy profile is a sequential equilibrium, it is sufficient to show that no player wants to deviate in the first period. But we have already shown that if $A < A^*$ and the player anticipates a mixed strategy equilibrium in the continuation game, it is not worth contributing in the first period. Thus, we have the required equilibrium.

The proof for $T > 2$ uses a variation of the argument in the proof for $T = 2$. Suppose, contrary to what we want to prove, there exists a sequential equilibrium in which provision is zero. Then the last two periods of the game are isomorphic to the game with $T = 2$. Suppose one player deviates by contributing a unit in period $T - 1$. There is a unique continuation equilibrium at T in which both players randomize with the probability that makes the other indifferent. As we showed earlier, if $A > A^*$ the probability that the good is provided is sufficiently high that the first player is strictly better off by deviating. This contradicts the equilibrium conditions and proves the corollary. ■

Proof of Proposition 4. If $k = 1$ and $\tau \geq 1$ the probability of provision in equilibrium must be positive as long as $A > 1$. If not, a single uncommitted player would provide the good and increase his payoff. Thus, the probability of provision must be positive in any sequential equilibrium of the game with $k = 1$ and $\tau \geq 1$.

Now consider an arbitrary $k < K$ and $\tau \geq k$ and assume that $A_k > 1$ and $\lambda_k > 0$ are such that, for any sequential equilibrium of the game with k tokens needed and τ periods remaining, the good is provided with probability at least λ_k if $A \geq A_k$. Then consider the game with $k+1$ tokens needed and $\tau + 1$ periods remaining . If an uncommitted player contributes one unit in the game, the good is provided with probability at least λ_k and his payoff will be at least A.

Suppose, contrary to what we want to prove, that there exists a sequential equilibrium of this game in which the good is provided with probability zero. In such an equilibrium the player's payoff cannot be greater than E. Then choose $A_{k+1} = \frac{2E}{\lambda_k}$. Then $\lambda_k A \geq 2E > E$ for any $A \geq A_{k+1}$. By forcing the provision of the good, the player can guarantee an equilibrium payoff greater than E, contradicting our assumption that the good is provided with probability zero. In fact, the probability λ that the public good is provided must satisfy

$$\lambda \geq \lambda_{k+1} \equiv \frac{\lambda_k A - E}{A} \geq \lambda_k - \frac{\lambda_k}{2} = \frac{\lambda_k}{2} > 0.$$

This proves by induction that there are numbers A_K and $\lambda_K > 0$ such that the probability of provision is at least λ_K in any game (A, E, K, N, T) satisfying $A \geq A_K \equiv A^*(E, K, N, T)$, as required. ■

Proof of Proposition 5. If $k = 1$ and $\tau \geq 1$ not contributing is strictly dominated by contributing a token since $A > E \geq 1$. Thus, the good must be provided with probability one in any sequential equilibrium of the game with $k = 1$ and $\tau \geq 1$. Note that we do allow the use of mixed strategies along the equilibrium path.

Now consider an arbitrary $k < K$ and $\tau \geq k$ and assume that, for any sequential equilibrium of the game with k tokens needed and τ periods remaining, the good is provided for certain. Then consider the game with $k+1$ tokens needed and $\tau+1$ periods remaining. If an uncommitted player contributes one unit in the game, the good is provided for certain and his payoffs will be A. Suppose, contrary to what we want to prove, that there exists a sequential equilibrium of this game in which the good is provided with strictly less than probability one. In such an equilibrium, the player's payoff is less than the payoff A. Clearly, the player would be strictly better off contributing the token and ensuring the good is provided. The desired result follows by induction. ■

Proof of Proposition 6. We construct a pure-strategy equilibrium by considering a number of possible situations. If no one has contributed in the past, so that $k = K$, then no player contributes in the current or future periods. Now suppose that $0 < k < K$ and that exactly one player has contributed a positive amount M, say, where $M < [A+1]$, the largest integer less than $A+1$. Then that player is assumed to contribute $[A+1] - M$ in the current period.

If $0 < k < K$ and $M \geq [A+1]$ we can construct a pure strategy equilibrium along the lines of Proposition 2 to ensure that the good is provided with probability 1. The efficiency of provision implies that we can do this without requiring that any player to contribute more than $[A+1]$ tokens and we assume this property is satisfied in what follows. Finally, if $k = 0$ no player contributes in the current or future periods.

To show that this is a pure strategy equilibrium, we need to prove two facts. First, we need to show that it is optimal for the distinguished player to contribute $[A+1] - M$ when called on to do so. By construction, this contribution is less than A and ensures that the good is provided in the pure-strategy continuation equilibrium, so the player is clearly better off making the contribution. Secondly, we need to show that no player wants to deviate by contributing when $k = K$. By construction, any player who deviates when $k = K$ will end up contributing $[A+1] \geq A$, so the deviation cannot make him better off. This completes the proof. ■

Proposition 7 (SMPE) *A symmetric Markov perfect equilibrium (SMPE) of the voluntary contribution game is a choice probability* $\lambda(m,\tau)$*, satisfying the conditions:*

$$\sum_{k=0}^{N-1-n} B\left(k; N-n-1, \lambda\left(n,\tau\right)\right)\left[V\left(1, n+k+1, \tau-1\right) - V\left(0, n+k, \tau-1\right)\right] = 0,$$

if $1 > \lambda(m,\tau) > 0$*;*

$$\sum_{k=0}^{N-1-n} B\left(k; N-n-1, \lambda\left(n,\tau\right)\right)\left[V\left(1, n+k+1, \tau-1\right) - V\left(0, n+k, \tau-1\right)\right] \geq 0,$$

if $\lambda(m,\tau) = 1$*; and*

$$\sum_{k=0}^{N-1-n} B\left(k; N-n-1, \lambda\left(n,\tau\right)\right)\left[V\left(1, n+k+1, \tau-1\right) - V\left(0, n+k, \tau-1\right)\right] \leq 0,$$

if $\lambda(m,\tau) = 0$*; where*

$$V\left(a, n, 0\right) = U_i\left(a, n\right), \text{ for } a \in \{0, 1\}.$$

Proof. The SMPE strategy λ must satisfy the following equilibrium condition for every state (n,τ):

$$\begin{aligned}
&\sum_{k=0}^{N-1-m} B\left(k; N-n-1, \lambda\left(n,\tau\right)\right)\left\{\lambda\left(n,\tau\right) V\left(1, n+k+1, \tau-1\right)\right. \\
&\qquad\qquad \left. +\ \left(1-\lambda\left(n,\tau\right)\right) V\left(0, n+k, \tau-1\right)\right\} \\
&\qquad \geq \sum_{k=0}^{N-1-n} B\left(k; N-n-1, \lambda\left(n,\tau\right)\right)\left\{\lambda V\left(1, n+k+1, \tau-1\right)\right. \\
&\qquad\qquad\qquad\qquad \left. +\ \left(1-\lambda\right) V\left(0, n+k, \tau-1\right)\right\},
\end{aligned}$$

for any $\lambda \in [0,1]$ and where $B\left(k; N-n-1, \lambda\right)$ represents the probability of k successes in $N-n-1$ independent Bernoulli trials with probability of success λ in each trial and $V(a,n,\tau)$ denotes the expected utility of an agent who has chosen $a \in \{0,1\}$ when the current state is (n,τ).

In order to derive the equilibrium condition, we need to apply for the backward induction. First, it is easy to see that the value function at the terminal date is the same as the payoff function. Then, at the final date T and at the state x_T where $\sum_{i=1}^{N} x_{iT} = n < K$, player i who has not contributed yet should be indifferent between contributing and not contributing, given the fact that others who have not contributed use an equilibrium choice probability $\lambda(n,0)$:

$$\sum_{k=0}^{n-1-n} B\left(k; N-n-1, \lambda(n,0)\right)\left[U_i\left(1,n+k\right)-U_i\left(0,n+k\right)\right]=0.$$

Since $U_i(1,n)-U_i(0,n)=A-1$ when $n=K-1$ and -1 where $n\neq K-1$, we have

$$B\left(K-1-n; n-1-n, \lambda(n,0)\right)=\frac{1}{A}.$$

And, analogously, at any date t and at the state (n,τ) where $\sum_{i=1}^{N} x_{it}=n<K$, in order to get an equilibrium choice probability $\lambda(n,\tau)$, we can apply for the condition of the indifference between contributing and not contributing, which results in the equilibrium condition. ■

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Table 1. Provision rate by treatment

Game	*T*	*A*	Provision	Std. Err.
One-shot	1	1.5	0.100	0.025
	1	3	0.761	0.032
Baseline	2	1.5	0.467	0.053
	2	3	0.744	0.046
	5	1.5	0.678	0.050
	5	3	0.811	0.041
High cost	2	1.5	0.300	0.049
	2	3	0.878	0.035
	5	1.5	0.644	0.051
	5	3	0.989	0.011
High endowment	2	1.5	0.489	0.053
	2	3	0.689	0.049
	5	1.5	0.633	0.051
	5	3	0.856	0.037

Table 2. The relative frequencies of contribution at payoff-relevant states across treatments

Baseline

$T=5$, $A=3$

τ / n	0	1	2
4	0.09 (270)		
3	0.08 (207)	0.11 (38)	0 (2)
2	0.11 (165)	0.07 (54)	0.25 (8)
1	0.37 (117)	0.07 (76)	0.10 (10)
0	0.36 (36)	0.60 (94)	0.08 (24)

$T=5$, $A=1.5$

τ / n	0	1	2
4	0.09 (270)		
3	0.05 (207)	0.03 (36)	0 (3)
2	0.06 (177)	0.06 (54)	0.25 (4)
1	0.26 (144)	0.19 (70)	0.17 (6)
0	0.20 (57)	0.48 (88)	0.09 (23)

$T=2$, $A=3$

τ / n	0	1	2
1	0.18 (270)		
0	0.62 (159)	0.54 (54)	0 (9)

$T=2$, $A=1.5$

τ / n	0	1	2
1	0.18 (270)		
0	0.35 (150)	0.33 (64)	0 (7)

High-cost

$T=5$, $A=3$

τ / n	0	1	2
4	0.76 (270)		
3	0.67 (6)	0.42 (24)	0.89 (36)
2	N/A	0.25 (8)	0.58 (12)
1	N/A	0.33 (6)	0.20 (5)
0	N/A	1 (2)	0.83 (6)

$T=5$, $A=1.5$

τ / n	0	1	2
4	0.38 (270)		
3	0.15 (66)	0.37 (78)	0.79 (24)
2	0.10 (39)	0.23 (40)	0.68 (31)
1	0.74 (27)	0.94 (32)	0.29 (17)
0	0.00 (21)	0.10 (30)	0.33 (15)

$T=2$, $A=3$

τ / n	0	1	2
1	0.68 (270)		
0	0.22 (9)	0.82 (38)	0.95 (39)

$T=2$, $A=1.5$

τ / n	0	1	2
1	0.37 (270)		
0	0.93 (75)	0.35 (68)	0.75 (28)

High-endowment

T=5, A=3

τ /(n, n_i)	(0,0)	(1,0)	(1,1)
4	0.14 (270)		
3	0.03 (165)	0.02 (52)	0.12 (26)
2	0.07 (153)	0.04 (50)	0.08 (25)
1	0.3 (126)	0.08 (60)	0 (30)
0	0.53 (45)	0.46 (84)	0.26 (42)

T=5, A=1.5

τ /(n, n_i)	(0,0)	(1,0)	(1,1)
4	0.06 (270)		
3	0.05 (228)	0.09 (22)	0.00 (11)
2	0.13 (195)	0.05 (40)	0.15 (20)
1	0.21 (126)	0.07 (70)	0.00 (35)
0	0.04 (63)	0.39 (92)	0.07 (46)

T=2, A=3

τ /(n, n_i)	(0,0)	(1,0)	(1,1)
1	0.34 (270)		
0	0.44 (75)	0.34 (70)	0.11 (35)

T=2, A=1.5

τ /(n, n_i)	(0,0)	(1,0)	(1,1)
1	0.26 (270)		
0	0.13 (111)	0.38 (70)	0.00 (35)

Table 3. The relative frequencies of contributions from the different histories

Game	*A*	(*n*,*τ*)	*h*(1)	*h*(2)	*h*(3)	*h*(4)	*p*-value
Baseline	1.5	(1,2)	0.03 (34)	0.10 (20)			0.63 (0.23, 1)
		(1,1)	0.06 (32)	0.25 (16)	0.32 (22)		0.05 (6.20, 2)
		(1,0)	0.54 (28)	0.25 (8)	0.30 (10)	0.52 (42)	0.30 (3.67, 3)
	3	(1,2)	0.00 (30)	0.17 (24)			0.07 (3.24, 1)
		(1,1)	0.00 (30)	0.06 (18)	0.14 (28)		0.21 (3.15, 2)
		(1,0)	0.47 (30)	0.75 (18)	0.60 (20)	0.64 (28)	0.27 (3.91, 3)
High endowment	1.5	(1,2)	0.56 (18)	0.45 (22)			0.25 (1.33, 1)
		(1,1)	0.00 (10)	0.05 (20)	0.10 (40)		0.50 (1.40, 2)
		(1,0)	0.50 (10)	0.33 (18)	0.47 (32)	0.31 (32)	0.12 (5.91, 3)
	3	(1,2)	0.05 (44)	0.00 (6)			0.10 (2.70, 1)
		(1,1)	0.11 (38)	0.00 (6)	0.06 (16)		0.60 (1.02, 2)
		(1,0)	0.43 (30)	0.67 (6)	0.57 (14)	0.41 (34)	0.53 (2.23, 3)

Table 4. The probability of contribution in SMPE across games

Baseline

A=3

τ / n	0	1
4	0	
3	0	0
2	0	0
1	0.55, 0	0
0	0.21, 0.79	0.67

High-endowment

A=3

τ /(n, n_i)	(0,0)	(1,0)	(1,1)
4	0		
3	0	0	0
2	0	0	0
1	0.48, 0	0	0
0	0.21, 0.79	0.42	0.42

A=1.5

τ / n	0	1
4	0	
3	0	0
2	0	0
1	0	0
0	0	0.33

A=1.5

τ /(n, n_i)	(0,0)	(1,0)	(1,1)
4	0		
3	0	0	0
2	0	0	0
1	0	0	0
0	0	0.21	0.21

Table 5. QRE estimation results and the probability of contribution

T=5, A=3
Log_lik =-435.13, β=10.18 (0.05)

τ / n	0	1
4	0.11	
3	0.13	0.07
2	0.17	0.10
1	0.19	0.17
0	0.75	0.65

T=5, A=1.5
Log_lik =-444.96, β=12.36 (0.05)

τ / n	0	1
4	0.08	
3	0.09	0.05
2	0.12	0.08
1	0.19	0.13
0	0.00	0.36

T=2, A=3
Log_lik = -278.54, β = 10.5 (1.26)

τ / n	0	1
1	0.19	
0	0.76	0.65

T=2, A=1.5
Log_lik = -294.35, β = 2.2 (0.20)

τ / n	0	1
1	0.39	
0	0.31	0.42

Figure 1. The theoretical (SMPE) and empirical contribution probabilities in the baseline and high-endowment games

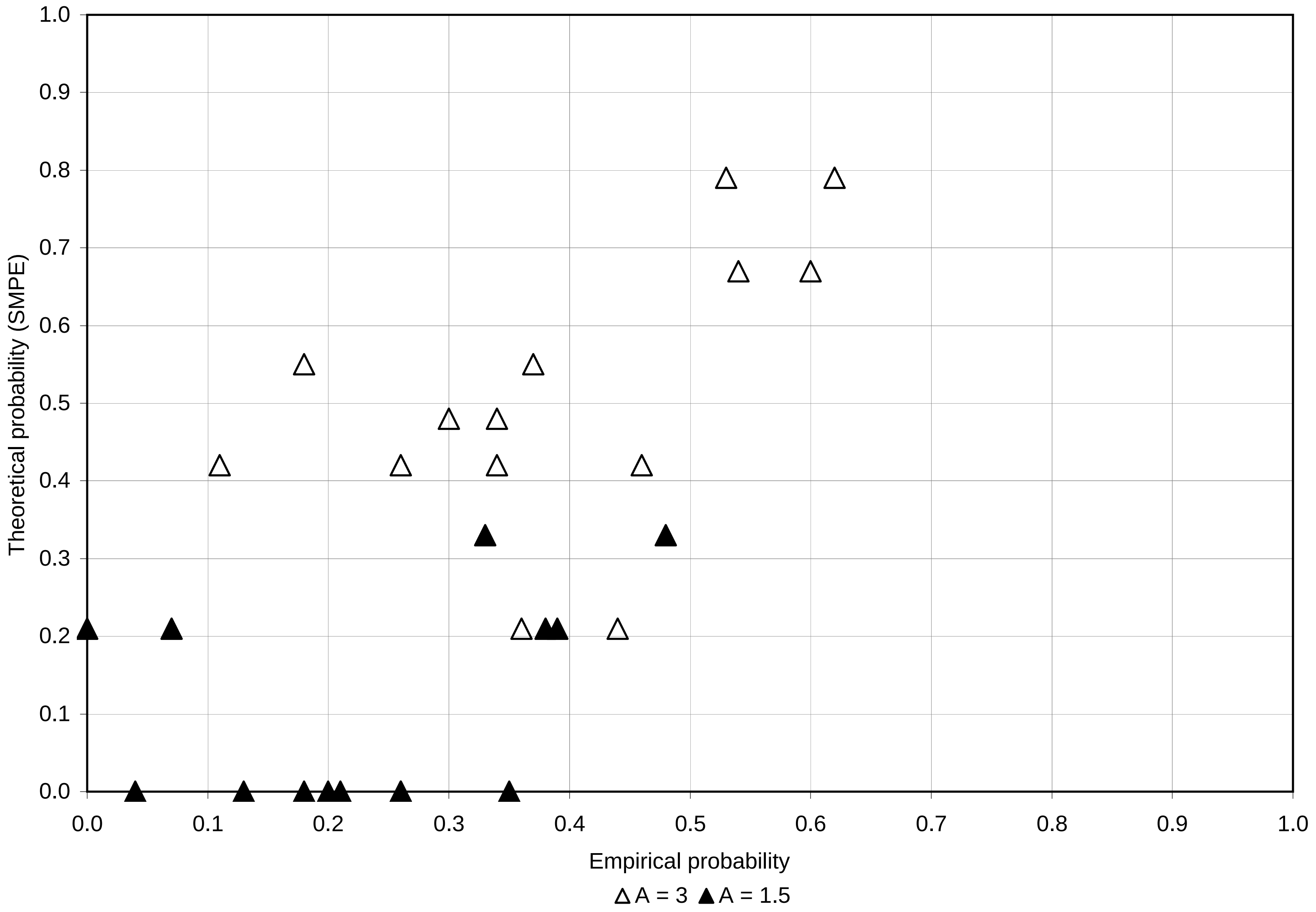

Figure 2A. The predicted (QRE) and empirical contribution probabilities
(Baseline treatment with T =5 and A =3)

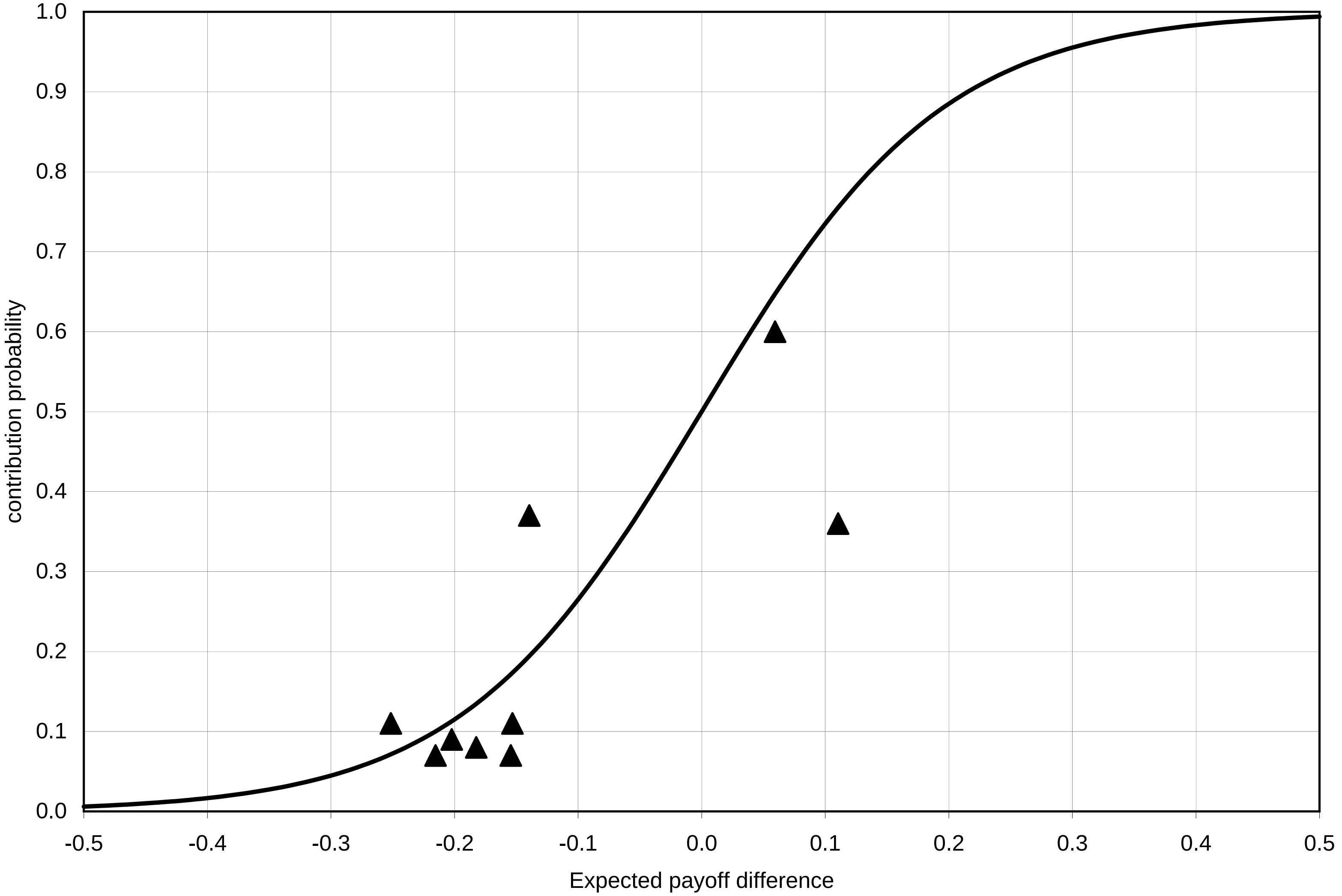

Figure 2B. The predicted (QRE) and empirical contribution probabilities

(Baseline treatment with *T* =5 and *A* =1.5)