Prudential Regulation of Banks

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The structure developed in Section 7 of "Banking and Markets" (BM) can be adapted to study the prudential regulation of banks. With early-consumer, late-consumer preferences, we can restrict our attention to deposit contracts offering depositors a choice of d_1 units of consumption at date 1 or d_2 units at date 2. We can assume $d_2 = \infty$, without essential loss of generality, to ensure that late consumers receive whatever is left at date 2. In what follows, we write d in place of d_1 and refer to d as the deposit contract.

As before, since ex ante types are symmetric we can restrict attention to symmetric equilibria, in which every bank of type i chooses the same deposit contract $d_i = d$ and portfolio $y_i = y$. The price of future consumption at date 1 is non-stochastic.

Bank regulation can be analyzed using techniques similar to those in BM. The main difference is that intermediaries do not face the risk of default, whereas banks can default. The deposit contract promises early consumers d units of the good at date 1 and incentive compatibility requires the late consumers get at least d units at date 2. So default can be avoided if and only if the bank can give both early and late consumers at least d units. The bankruptcy point is given by a critical proportion of early consumers σ^* such that the bank can just afford to give both types d units of consumption. Formally, σ^* is implicitly defined by the condition that

$$\sigma^* d + p(1 - \sigma^*)d = y + pr(1 - y).$$

The bank can afford to give early and late consumers at least d units of consumption if and only if $\sigma_k \leq \sigma^*$. Note that $d \leq y + pr(1 - y)$ implies that $\sigma^* \geq 1$. Clearly, the bank can always afford to offer early and late consumers $d \leq y + pr(1 - y)$ units of consumption, for any value of σ_k. Thus, default only occurs if $d > y + pr(1 - y)$.

The bank's optimization problem is to choose (d, y) to maximize the objective function

$$\sum_k \lambda_k \left\{ \sigma_k U(x_1(\sigma_k)) + (1-\sigma_k) U(x_2(\sigma_k)) \right\}$$

subject to the "budget constraints"

$$x_1(\sigma_k) = \begin{cases} d & \forall \sigma_k \leq \sigma^*, \\ y + pr(1-y) & \forall \sigma_k > \sigma^* \end{cases}$$

$$x_2(\sigma_k) = \begin{cases} \frac{r(y+pr(1-y)-\sigma_k d)}{(1-\sigma_k)} & \forall \sigma_k \leq \sigma^*, \\ y + pr(1-y) & \forall \sigma_k > \sigma^* \end{cases} .$$

If the bank does not default, the early consumers receive $x_1(\sigma_k) = d$ at date 1 and the late consumers receive the remainder

$$\begin{aligned} (1-\sigma_k) x_2(\sigma_k) &= r(y + pr(1-y) - \sigma_k x_1(\sigma_k)) \\ &= r(y + pr(1-y) - \sigma_k d) \end{aligned}$$

at date 2. If the bank defaults, every depositor receives $y + pr(1-y)$ at date 1.

A *symmetric banking equilibrium* consists of an array (p, d, y) such that (d, y) is optimal for the bank at the equilibrium price p and the market-clearing conditions

$$\begin{aligned} &\textstyle\sum_{k=1}^K \lambda_k \sigma_k x_1(\sigma_k) = ny, \\ &\textstyle\sum_{k=1}^K \lambda_k (1-\sigma_k) x_2(\sigma_k) = nr(1-y). \end{aligned}$$

are satisfied. (By a previous argument, the equilibrium price $p = p^* = 1/r$. Note that $p = p^*$ implies that none of the short asset is held between date 1 and date 2. Thus, consumption at date 1 is equal to the returns on the short asset and consumption at date 2 is equal to the returns on the long asset.)

Proposition 1 *Let d be an optimal choice for the bank, for a given value of (p, y). Then $d < y + pr(1-y)$ if the degree of relative risk aversion is less than one and $d > y + pr(1-y)$ if the degree of relative risk aversion is greater than one.*

Proof. See Section 1. ■

Thus, default is possible only if the degree of relative risk aversion is greater than one.

In analyzing prudential regulation, there are two cases to consider. When the degree of relative risk aversion is less than 1, the equilibrium deposit contract satisfies $d < 1$ and there is no possibility of default. In this case, we can use a familiar argument to show that raising p decreases the efficiency of risk sharing and reduces welfare. If the degree of relative risk aversion is greater than 1, the equilibrium deposit contract satisfies $d > 1$ and there is the possibility of default (bank runs). The analysis of the effect of an increase in p is more complicated in this case. In states where the bank is solvent, the analysis of a change in p is essentially the same as in the case of no default. The increase in p will reduce the welfare of the late consumers but leave unchanged the welfare of the early consumers, so the net effect on welfare is negative. However, we must also take into account the effect on welfare in states where default occurs. Here the welfare effect of an increase in p is clearly positive, since $y + pr(1 - y)$ is increasing in p and independent of y when $p = p^*$. Thus we have two offsetting effects which, on their face, are ambiguous. However, we know that the positive and negative effects are the result of a transfer from individuals with high consumption to those with low consumption. It can be shown that this results in an increase in overall welfare.

Define a *regulated banking equilibrium* to be an array (p, d, y) such that d is the optimal choice of deposit contract for the given value of (p, y) and the market-clearing equations are satisfied. As before, if (p, d, y) is a regulated banking equilibrium, then (d, y) will be the optimal choice for the banks subject to a lower bound on y if $p > p^*$ and an upper bound if $p < p^*$. The next result characterizes the impact of prudential regulation in the absence of default.

Proposition 2 *Assume that the degree of relative risk aversion is less than one. For any value of p sufficiently close to p^* there exists a unique regulated banking equilibrium (p, d, y) and for $p < p^*$ sufficiently close to p^* the expected utility of the typical depositor is greater than in the banking equilibrium.*

Proof. See Section 1. ■

With the possibility of default, the analysis of prudential regulation becomes more complicated. We restrict attention to *regular equilibria*, in which

$\sigma^* \neq \sigma_k$ for any k. Let

$$w(p, y) \equiv y + pr(1 - y)$$

and let $D(p, y)$ denote the optimal deposit contract given the value (p, y). Then $D(p, y)$ maximizes

$$\sum_{\sigma_k < \sigma^*} \lambda_k \left\{ \sigma_k U(d) + (1 - \sigma_k) U\left(\frac{w(p, y) - \sigma_k d}{p(1 - \sigma_k)} \right) \right\}.$$

The strict concavity of the utility functions guarantees that $D(p, y)$ is well defined and satisfies the first-order condition:

$$\sum_{\sigma_k < \sigma^*} \lambda_k \sigma_k \left\{ U'(D(p, y)) - U'\left(\frac{w(p, y) - \sigma_k D(p, y)}{p(1 - \sigma_k)} \right) \frac{1}{p} \right\} = 0 \qquad (1)$$

for all (p, y) sufficiently close to (p^*, y^*). Since $D(p^*, y^*) > 1$ an increase in σ_k reduces consumption at the second date and increases marginal utility. Hence, the term in braces declines with σ_k. This implies that

$$\sum_{\sigma_k < \sigma^*} \lambda_k \left\{ U'(D(p^*, y^*)) - U'\left(\frac{w(p, y) - \sigma_k D(p^*, y^*)}{p^*(1 - \sigma_k)} \right) \frac{1}{p^*} \right\} \geq 0. \qquad (2)$$

This inequality will be used in the sequel.

Restricting attention to states in which the bank is solvent, $\sigma_k < \sigma^*$, the analysis of a change in p is essentially the same as in the case of no default. The increase in p will reduce the welfare of the late consumers but leave unchanged the welfare of the early consumers, so the net effect on welfare is negative.

However, we must also take into account the effect on the welfare in states where default occurs. Here the welfare effect of an increase in p is clearly positive, since $w(p, y)$ is increasing in p and independent of y when $p = p^*$. Thus we have two offsetting effects which, on their face, are ambiguous. However, we know that the positive and negative effects are the result of a transfer from individuals with high consumption to those with low consumption. This results in an increase in overall welfare.

Proposition 3 *Assume that the degree of relative risk aversion is greater than one. Let (p^*, d^*, y^*) be a regular banking equilibrium in which default occurs with positive probability. For any p sufficiently close to p^* there exists a unique regulated banking equilibrium (p, d, y) and welfare is higher in (p, d, y) than in (p^*, d^*, y^*) if and only if $p > p^*$.*

Proof. See Section 1. ■

Remark: Banks and intermediaries liquidate assets by selling them on the market. Ex post, liquidation does not impose deadweight costs and equilibrium is efficient. Prudential regulation increases welfare by improving ex ante risk sharing, not by reducing the probability of a crisis. Ex ante risk sharing is inefficient because of the incompleteness of markets. One of the important lessons of this welfare analysis is that efficiency does not require the prevention of crises per se. It simply requires optimal allocation of risk bearing.

1 Proofs

1.1 Proof of Proposition 1

Substituting the definition of $x_t(\sigma_k)$ from the budget constraints into the objective function, we obtain

$$\sum_k \lambda_k \left\{ \sigma_k U(d) + (1-\sigma_k) U\left(\frac{y + pr(1-y) - \sigma_k d}{p(1-\sigma_k)} \right) \right\}.$$

Consider the relaxed problem of choosing d to maximize this expression, taking (p, y) as given and ignoring the incentive constraint. Because the objective function is concave in d, the first-order condition

$$\sum_k \lambda_k \sigma_k \left\{ U'(d) - U'\left(\frac{y + pr(1-y) - \sigma_k d}{p(1-\sigma_k)} \right) \frac{1}{p} \right\} = 0$$

is necessary and sufficient for a feasible value of d to solve the relaxed problem. If $d \leq y + pr(1-y)$ solves relaxed problem, then the incentive constraint is satisfied and d is also a solution to the bank's problem. Conversely, if d solves the bank's problem and $d < y + pr(1-y)$ then d satisfies the first-order condition and solves the relaxed problem.

If the degree of relative risk aversion is less than one, set $d = y + pr(1-y)$ and check that

$$\begin{aligned} & \sum_k \lambda_k \sigma_k \left\{ U'(d) - U'\left(\frac{y + pr(1-y) - \sigma_k d}{p(1-\sigma_k)} \right) \frac{1}{p} \right\} \\ = & \sum_k \lambda_k \sigma_k \left\{ U'(d) - U'\left(\frac{d}{p} \right) \frac{1}{p} \right\} \\ < & \ 0. \end{aligned}$$

Since the objective function is strictly concave in d, a solution of the first-order condition must satisfy $d < y + pr(1-y)$. Hence, the unique value of d that solves the bank's problem for the given (p, y) must satisfy $d < y + pr(1-y)$.

If the degree of relative risk aversion is greater than one, note that $d \leq y + pr(1-y)$ implies

$$\frac{y + pr(1-y) - \sigma_k d}{p(1-\sigma_k)} \geq \frac{d}{p}$$

and that

$$\begin{aligned} U'(d) - U'\left(\frac{y + pr(1-y) - \sigma_k d}{p(1-\sigma_k)}\right)\frac{1}{p} &\geq U'(d) - U'\left(\frac{d}{p}\right)\frac{1}{p} \\ &> 0, \end{aligned}$$

for every k. Hence, a solution to the bank's problem must satisfy $d > y + pr(1-y)$.

To sum up, the solution to the bank's problem entails $d \leq y + pr(1-y)$ if the degree of relative risk aversion is less than one and $d > y + pr(1-y)$ if the degree of relative risk aversion is greater than one.

1.2 Proof of Proposition 2

The existence and uniqueness of a regulated banking equilibrium for p sufficiently close to p^* follows by a familiar argument (see the proof of Proposition 6 in BM).

Consider a fixed value of $p < p^*$ close to p^* and let (p, d, y) denote a regulated equilibrium. Since there is no default in equilibrium, the bank chooses d to maximize

$$\sum_k \lambda_k \left\{ \sigma_k U(d) + (1-\sigma_k) U\left(\frac{y + pr(1-y) - \sigma_k d}{p(1-\sigma_k)}\right)\right\}.$$

Let $D(p, y)$ denote the optimal choice of d for a given value of (p, y). Then $D(p, y)$ satisfies the necessary and sufficient first-order condition

$$\sum_k \lambda_k \left\{ \sigma_k U'(D(p,y)) - \sigma_k U'\left(\frac{y + pr(1-y) - \sigma_k D(p,y)}{p(1-\sigma_k)}\right)\frac{1}{p}\right\} = 0$$

and by the implicit function theorem $D(p, y)$ is well defined and continuously differentiable for all p in some small neighborhood of p^*. The market-clearing condition can be written

$$\sum_k \lambda_k \sigma_k D(p, y) = ny,$$

since for all p close to p^* no one will hold the short asset between date 1 and date 2. To show that there exists a regulated bank equilibrium (p, d, y) for every p sufficiently close to p^*, it is sufficient to note that

$$\sum_k \lambda_k \sigma_k \frac{\partial D(p^*, y^*)}{\partial y} - n = -n$$

and use the implicit function theorem.

Let $y(p)$ denote the equilibrium value of y corresponding to the regulated bank equilibrium. Substituting $y(p)$ into the objective function, the welfare of the typical depositor can be written as a function of p:

$$\sum_k \lambda_k \left\{ \sigma_k U(D(p, y(p))) + (1 - \sigma_k) U\left(x_2(p, y(p))\right) \right\},$$

where

$$x_2(p, y(p)) \equiv \frac{y(p) + pr(1 - y(p)) - \sigma_k D(p, y(p))}{p(1 - \sigma_k)}.$$

By direct calculation,

$$\begin{aligned}
& \frac{d}{dp} \sum_k \lambda_k \left\{ \sigma_k U(D(p, y(p))) + (1 - \sigma_k) U\left(x_2(p, y(p))\right\} \right. \\
= & \sum_k \lambda_k \left(\sigma_k U'(D(p, y(p))) - \sigma_k U'\left(x_2(p, y(p))\right) \frac{1}{p} \right) \left(\frac{\partial D}{\partial p}(p, y(p)) \right. \\
& \left. + \frac{\partial D}{\partial y}(p, y(p)) y'(p) \right) + \sum_k \lambda_k U'\left(x_2(p, y(p))\right) \left(\frac{(1 - pr) y'(p)}{p} - \frac{y(p)}{p^2} \right) \\
= & \sum_k \lambda_k U'\left(x_2(p, y(p))\right) \left(-\frac{y(p)}{p^2} \right) < 0
\end{aligned}$$

when $p = p^*$. A change in (p, y) induces a change in $D(p, y)$, but this has no effect on welfare because of the envelope theorem. The change in y has no direct effect on welfare because at the price p^* the two assets have the same returns. So we are left with the direct effect of a change in p, which reduces welfare by reducing the consumption of the late consumers.

1.3 Proof of Proposition 3

The existence and uniqueness of the regulated equilibrium follow by a familiar argument (see the proof of Proposition 6 in BM).

The welfare effect of a change in p can be derived by differentiating the objective function and applying the envelope theorem. Substituting the optimal value of the deposit $D(p, y)$ into the objective function, we obtain an expression for expected utility as a function of (p, y):

$$\begin{aligned} W(p,y) \equiv & \sum_{\sigma_k < \sigma^*} \lambda_k \left\{ \sigma_k U(D(p,y)) + (1-\sigma_k) U\left(\frac{w(p,y) - \sigma_k D(p,y)}{p(1-\sigma_k)} \right) \right\} \\ & + \sum_{\sigma_k > \sigma^*} \lambda_k \left\{ U\left(w(p,y)\right) \right\}. \end{aligned}$$

Since y is chosen optimally,

$$\frac{\partial W}{\partial y}(p^*, y^*) = 0,$$

and since $D(p, y)$ is chosen optimally,

$$\begin{aligned} \frac{\partial W}{\partial p}(p^*, y^*) = & -\sum_{\sigma_k < \sigma^*} \lambda_k \left\{ U' \left(\frac{w(p^*,y^*) - \sigma_k D(p^*,y^*)}{p^*(1-\sigma_k)} \right) \frac{y^* - \sigma_k D(p^*,y^*)}{(p^*)^2} \right\} \\ & + \sum_{\sigma_k > \sigma^*} \lambda_k \left\{ U'\left(w(p^*,y^*)\right) r(1-y^*) \right\} \\ > & \; rU'\left(w(p^*,y^*)\right) \left\{ -\sum_{\sigma_k < \sigma^*} \lambda_k \left(y^* - \sigma_k D(p^*,y^*)\right) + \sum_{\sigma_k > \sigma^*} \lambda_k (1-y^*) \right\}, \end{aligned}$$

where the last inequality follows from (2) and $U'\left(w(p^*, y^*)\right) > U'(D(p^*, y^*)$. From the market-clearing condition for consumption at date 1 we have

$$\sum_{\sigma_k < \sigma^*} \lambda_k \sigma_k D(p^*, y^*) + \sum_{\sigma_k > \sigma^*} \lambda_k = y^*$$

which can be re-written as

$$-\sum_{\sigma_k < \sigma^*} \lambda_k \left(y^* - \sigma_k D(p^*, y^*)\right) + \sum_{\sigma_k > \sigma^*} \lambda_k (1 - y^*) = 0.$$

Substituting this into the right hand side of the inequality for $\partial W(p^*, y^*)/\partial p$ gives us

$$\frac{\partial W}{\partial p}(p^*, y^*) > 0$$

as required.