

Microeconomic Theory I

Final examination

Saturday, December 18, 2004

Answer the following questions completely, clearly and concisely. You will be graded on how clearly you explain your answer as well on the content. Write the answers to Questions #1 and #2 in one blue book and the answers to Questions #3 and #4 in another. Write the number of the questions along with your name in BLOCK LETTERS and your ID number on the front of each blue book.

- (Ten points) Give formal definitions of the following terms or expressions:
 - an **attainable allocation** for an (Arrow-Debreu) economy with production;
 - a **(strongly) Pareto-efficient allocation** for an exchange economy;
 - the **transversality condition** for a production path $\{(x_t, y_t)\}$ and a price path $\{p_t\}$.
- (Fifteen points) Indicate whether each of the following statements is true or false. If true, provide a short proof; if false, provide a counter-example.
 - Any competitive equilibrium of an Arrow-Debreu exchange economy is Pareto-efficient.
 - A profit-maximizing production plan is efficient.
 - An efficient production plan is profit-maximizing for some vector of prices.
- (Twenty-five points) Consider a small open economy in which two goods $h = 1, 2$ are produced subject to a production technology

$$(y_1)^2 + (y_2)^2 \leq 1,$$

where $y_1 \geq 0$, $y_2 \geq 0$ are the outputs of good 1 and good 2 respectively. There is a large number of identical consumers whose preferences are represented by the utility function $U(x_1, x_2)$ where

$$U(x_1, x_2) = x_1$$

for any consumption bundle $(x_1, x_2) \geq 0$. Consumers have no endowments but receive the profits from production

$$\pi = p_1 y_1 + p_2 y_2.$$

The international goods prices are $p^* = (1, 1)$.

(a) Define a competitive equilibrium for this economy under the assumption that there are no barriers to international trade. Solve for the equilibrium price vector and allocation.

(b) Suppose the government decides to impose a tariff t on each unit good 1 imported, in order to raise a revenue of $g > 0$ units of good 1. Solve for the equilibrium prices and outputs. What is the value of the tariff t required to raise revenue g in equilibrium?

(c) Show that consumer welfare would be higher if the government raised g units of good 1 from a consumption tax τ on good 1 rather than from a tariff. Describe the corresponding competitive equilibrium and find the value of the consumption tax τ .

4. (Twenty-five points) Consider an economy with two dates $t = 0, 1$, two equally probable states of nature $s = 1, 2$, two goods $h = 1, 2$, and two consumers $i = 1, 2$. The consumers have endowments of the goods at the second date only (there is no consumption at the first date). Consumers i 's endowment is denoted by $e_i(s)$ in state s and defined by

$$e_1(s) = \begin{cases} (1, 2) & \text{if } s = 1 \\ (3, 4) & \text{if } s = 2 \end{cases} \quad \text{and} \quad e_2(s) = \begin{cases} (4, 3) & \text{if } s = 1 \\ (2, 1) & \text{if } s = 2 \end{cases} .$$

Any non-negative bundle of goods is feasible for the consumers. Consumer i 's preferences are described by a von Neumann Morgenstern utility function $U_i(c_1, c_2)$, where c_h is consumption of good $h = 1, 2$. We assume that

$$U_1(c_1, c_2) = \ln c_1 + 2 \ln c_2$$

and

$$U_2(c_1, c_2) = \ln c_1 + \ln c_2.$$

(a) Suppose the economy described above has a complete set of markets for contingent commodities at date 0, that is, it is an Arrow-Debreu economy. Give a formal definition of the competitive equilibrium for the Arrow-Debreu economy and solve for an equilibrium price vector.

(b) Now suppose that there are no markets for contingent commodities at date 0. Instead, consumers can trade two real securities $k = 1, 2$ at date 0 and can trade goods on spot markets in each state s at date 1. Security k is a promise to deliver a vector of goods $a_k(s)$ in each state s at date 1. The securities are defined by their payoffs as follows

$$A = \begin{bmatrix} a_1(1) & a_2(1) \\ a_1(2) & a_2(2) \end{bmatrix} = \begin{bmatrix} (1, 1) & (1, 2) \\ (1, 1) & (2, 1) \end{bmatrix} .$$

The securities are assumed to be in zero net supply. Give a formal definition of a Radner equilibrium for this economy. Use the data from part (a) to define security prices and goods prices corresponding to a Radner equilibrium of this economy. Be sure to check that, at these prices, the securities allow the consumers to achieve the optimal expenditure in each state of nature.