

Minimum Wage Effects on Labor Market Outcomes under Search, Matching, and Endogenous Contact Rates¹

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Abstract

Building upon a continuous-time model of search with Nash bargaining in a stationary environment, we analyze the effect of changes in minimum wages on labor market outcomes and welfare. While minimum wage increases may or may not lead to increases in unemployment in our model, they can be welfare-improving to labor market participants on both the supply and demand sides of the labor market. Key determinants of the welfare impact of a minimum wage increase is the Nash bargaining power parameter and the matching function. We discuss identification of this model using Current Population Survey data on accepted wages and unemployment durations, and demonstrate that the bargaining power parameter is not identified when the distribution of match values belongs to a location-scale family. By incorporating a limited amount of information from the demand side of the market, we are able to obtain credible and precise estimates of all primitive parameters, including bargaining power and estimable parameters characterizing the matching function. We show that an efficient level of the minimum wage in September 1996 would have been approximately twice its then current level of \$4.25. When we allow for endogenous contact rates, the efficient minimum wage would have been \$3.36. However, we find no empirical evidence to support the hypothesis that contact rates were impacted by minimum wage changes observed over this period, casting some doubt on the advisability of reductions in the minimum wage.

1 Introduction

Determining the equilibrium effects of minimum wage changes on labor market outcomes is a challenging modeling and estimation problem; arriving at policy recommendations is a task even more daunting. Faced with the inherent difficulties of modeling equilibrium labor market events given the limited amount of data to which researchers have access, much recent research has been performed outside of an explicit behavioral framework, with researchers pursuing the more limited objective of carefully describing the observed effects of recent minimum wage changes using quasi-experimental methods [see Card and Krueger (1995) for a summary of these studies and a comprehensive, critical survey of most of the previous research done in this area]. In our view, these recent studies have been particularly useful in indicating that the “textbook” competitive model of the labor market, which has been used as an interpretive framework for the bulk of empirical work performed using aggregated time series data, may have serious deficiencies in accounting for minimum wage effects on labor market outcomes when confronted with disaggregated information.

While the quasi-experimental results have raised a number of interesting challenges to orthodox theory, few cogent models have been advanced that are consistent with the results that have been found. Some of the explanations for these empirical findings (e.g., lack of significant employment losses, impacts on the wage distribution above the minimum) do not seem to be testable given our current data resources. In fact, it appears difficult to operationalize many of the explanations proposed even for the more modest purpose of empirical implementation.

It has long been recognized that for the imposition of minimum wages to have beneficial effects on the welfare distribution of workers and searchers, firms must have some degree of monopsony power [see for example Manning (1995)]. Search frictions and the existence of match-specific capital are capable of providing this. The existence of both seems in accord with common sense and empirical findings [on the existence of match-specific capital see Miller (1984) and Flinn (1986), for example]. Below we formulate a model which includes match-specific capital, job search, and worker-firm bargaining over match-specific rents, that is capable of accounting for many of the empirical observations cited by CK in their survey of minimum wage research based on individual level data (1995, Ch. 7).¹

In the standard monopsony model of minimum wage effects, minimum wages lead to higher wages and employment levels but obviously lead to lower levels of firm welfare. By adding some general equilibrium elements to the model it is possible for binding minimum wages to lead to increases in the welfare of firms, as well. We utilize a matching function formulation of the worker-firm contact rate to accomplish this (for a thorough summary of this approach see Pissarides (2000)). From Hosios (1990) we know the efficiency of equilibrium in such a framework requires that the elasticity of the matching function with respect to the size of the set of searchers be equal to the workers’ share of the match surplus. In a world in which workers’ receive a low share of the surplus in comparison with the matching function elasticity, increases in the minimum wage can be viewed as a way to increase their “effective” bargaining power, leading to increased search activity on their part and an equilibrium outcome closer to what is obtained when the Hosios condition is satisfied. A goal of the empirical research is to obtain estimates of bargaining power and the matching function elasticity so that optimal - in the sense of efficient - minimum wages can be determined in a general equilibrium setting.

¹As Kennan (1995) points out in his informative review of CK, there exist a number of studies of minimum wage impacts within local labor markets that utilize disaggregated data, some of which were conducted at the beginning of the last century. While the local nature of the labor market makes generalization somewhat difficult, the fact that these minimum wage changes were substantial in many cases alleviates the problem of conducting policy analysis outside of the narrow range of historical federal minimum wage rates.

Estimation of the parameters characterizing our model is necessary to carry out the policy experiments we conduct. We will discuss some of the features of wage distributions (and unemployment phenomena) that our model must fit using the histograms in Figures 1 and 2. Figure 1 contains plots of hourly wage distributions for individuals 16-24 years of age in the Outgoing Rotation Groups (ORG) of the Current Population Survey (CPS) in September 1996 and February 1997 (by the sample design of the CPS, these are different individuals). The federal minimum wage was officially changed from \$4.25 to \$4.75 on September 30, 1996. Thus the data for September are from the month preceding the change and those from February 1997 were collected approximately five months after the change. The hourly wage distributions that appear in panels 1.a and 1.b exhibit a number of common features. In both cases, there are hourly wage rates below the mandated minimum wage, about 5 percent in each month. In addition, both distributions contain a large amount of probability mass concentrated on relatively few points. In September 1996 the minimum wage value \$4.25 has the fourth largest probability mass, while in August 1997 the probability mass at the prevailing minimum wage \$4.75 is the third largest. In terms of an overall impression of these distributions, if one were to smooth the probability mass away from the spikes (except at the minimum wage rates) and ignore the small number of wage observations below the minimum wage rates, the data seem consistent with draws from a truncated lognormal distribution.² In the sequel we focus on the lognormal distribution because of its predominance in empirical studies of wage determination.

The model developed below will generate at most one mass point in the wage distribution, at the prevailing minimum wage, so it may be worthwhile to comment on the appearance of so many mass points in the empirical distribution. In the descriptive statistics presented in Table 1, we see that the proportion of employed individuals paid \$4.25 in September 1996 was 0.053 while in February 1997 the proportion paid \$4.75 was 0.063. The largest probability mass in both months is associated with the value \$5.00, which is the wage reported for about 12 percent of the employed youth in each month. Other especially large mass points are associated with the values of \$6.00, \$7.00, and \$8.00. We believe that it is reasonable to suppose that these values are simply “focal points” for respondents, who in many cases are not even the individual whose wage is being reported.³ The fact that the minimum wage was not set at values that could be characterized as natural focal points may lend some strength to our claim that the mass point at the minimum wage is the only “real” one in these empirical wage distributions.

Figures 1.c and 1.d contain plots of the simulated wage distributions which are generated from the estimates of primitive parameters obtained below. In the formulation of the model we will not allow for wages below the active minimum wage; this explains the lack of mass below \$4.25 and \$4.75 in the respective figures. From an underlying continuous productivity distribution, the model produces an equilibrium wage distribution which is mixed continuous-discrete, with the one mass point occurring at the minimum wage. We see that the model predicts slightly more mass at the minimum wage than is observed.⁴ For wages above the minimum, the behavioral model smoothes out the masses at the focal points, as would a simple kernel estimator, and produces a more regular looking distribution. In terms of general shape the distributions seem similar.

²Of course, the observed data are consistent with truncated forms of a large number of other parametric distributions as well. We make no claim that the truncated lognormal provides a better statistical fit to the data than other competitors.

³Labor market information for all eligible household members is reported by one household member only. Since many of the sample members live with parents, in these cases typically their mother or father will be the “proxy” respondent.

⁴In the estimation phase we first round all minimum wages below the minimum wage up to the minimum wage. After this transformation, the proportion of the sample at the minimum wage is over 10 percent in both months, so the overprediction at the mass point is not substantial.

Figure 2 contains histograms of the durations of on-going search spells at the time of the interview (the CPS only collects information on the search durations of respondents who report that they are currently looking for a job).⁵ The model we employ assumes a constant economic environment (except for changes in the minimum wage rate) and Poisson arrival rates for individual-specific events. Then in the steady state, the distribution of on-going unemployment spells from point-samples should be negative exponential, implying a constant hazard rate out of the unemployed state. In terms of the histograms from the CPS samples, we see a fairly regular pattern of decreasing sample proportions with duration, though there are a number of exceptions, which tend to occur at natural focal points.⁶ From Table 1 we note that the average unemployment duration actually is lower after the minimum wage rate was increased, while the unemployment rate of labor market participants is substantially greater in February 1997 than in September 1996.

While the shapes of the unemployment spells in the data and from the simulations are not greatly at odds, we go a bit further and conduct some simple statistical tests for negative exponentiality in the sample distributions. The first test conducted is a completely parametric one, in which maximum likelihood estimates of a Weibull distribution are obtained. The negative exponential is nested within the Weibull when the Weibull shape parameter is equal to 1. Estimates of the Weibull shape parameter are 0.861 (0.044) and 1.015 (0.048) for September and February, where estimates of the asymptotic standard errors are in parentheses. Thus there is a reasonably strong indication that the negative exponential restriction is not appropriate for September, while it would seem to be for February. We also conducted nonparametric tests of the negative exponential assumption and were led to reasonably similar conclusions.⁷ While the negative exponential duration distribution implication of the model is not fully consistent with the data, we feel that the assumption is not so at odds with the data as to invalidate the usefulness of the econometric analysis.

The model we develop is a logical descendent of the econometric model of search of Flinn and Heckman (1982) - hereafter referred to as FH - and the econometric model of minimum wage effects on the distribution of wages and the probability of employment estimated by Meyer and Wise (1983a,b). FH formulated an equilibrium continuous-time search model in which searchers encountered potential employers according to a Poisson process; upon meeting, the potential value of the contact was determined by a draw from a fixed distribution G . In their analysis the match value was arbitrarily assumed to be divided evenly between workers and firms. The setup of the basic model proposed here is similar, with the important difference that rents are divided using an explicit Nash-bargaining criteria. The bargaining power parameter which appears in this formulation is of key importance in determining the welfare consequences of imposing, or increasing, a minimum wage.

It is worth elaborating on the role of the bargaining power parameter in this research. As we have noted, this parameter did not appear in the formulation of FH - for one reason because it is a more difficult parameter to interpret than are the others in their model. We think of it as comprising a sort of summary statistic of the labor market “position” of a particular group. For example, the match value distribution for low-skilled workers may be stochastically dominated by the match value distribution for high-skilled workers, but in addition low-skilled workers may be

⁵For both the sample and simulated duration distributions, we have rounded all durations above 12 months to 12 months for ease of graphical presentation.

⁶The respondent reports search duration in weeks, which we have converted to months here since the econometric model utilizes that time period. Heaping in the weekly data is especially pronounced at 4, 8, 12, 36, and 52 weeks.

⁷In particular, we computed values of various Kolmogorov statistics, the Cramer-von Mises W^2 statistic, and the Anderson-Darling A^2 statistic under the null of a negative exponential distribution which allow for sampling error (i.e., uncertainty regarding the negative exponential parameter). Using the critical values in Stephens (1974), most statistics indicated strong rejection of the null for the September data, and marginal rejection of the null for the February data.

at a disadvantage due to their having little bargaining power. Their low bargaining power may derive from there being many substitutes for them in the production process, or to the relative number of low-skilled workers to the number of positions for this type of worker. In this sense, the parameter cannot be really thought of as “primitive” since significant policy changes - such as a doubling of the minimum wage - may result in participation effects or substitution responses by firms which change the labor market “position” of the group, and hence change the bargaining power parameter. Thus comparative statics exercises and policy experiments performed with the estimates obtained from this model will only be valid locally, that is, for small changes in policy variables. This limitation is not too disturbing, since it probably applies to all empirical research which has been conducted in this area.

Meyer and Wise estimated a model of minimum wage effects using individual-level data which allowed them to infer what the wage distribution and employment level would have been in the absence of a minimum wage. Their model was both original and suggestive. While it has been criticized by a number of researchers [e.g., Card and Krueger (pp.232-236) and Dickens et al. (1998)], primarily for relying on functional form assumptions for identification and for choosing a parameterization that rules out the possibility of employment increases in response to a minimum wage increase, it remains one of the better econometric attempts to identify minimum wage effects using individual-level data in the literature [another example is Heckman and Sedlacek (1981)]. From our perspective, the main weakness of their model is the arbitrary specification of the manner in which a minimum wage “distorts” the preexisting wage distribution. In our model, optimizing behavior by searchers and firms determines the nature of this “distortion,” and it is roughly consistent both with the Meyer and Wise specification and with the empirical evidence cited in Card and Kueger.

In this research we attempt to integrate theory and measurement in analyzing minimum wage effects on the labor market. There are a number of papers in the literature which consider the possibility of welfare-enhancing minimum wage rates [e.g., Drazen (1986), Lang (1987), Rebitzer and Taylor (1996), Swinnerton (1996)], though the frameworks in which the models are set tend to be relatively abstract and the models themselves are typically unestimable. A recent addition to this literature is van den Berg (2003), in which minimum wages are used as equilibrium selection devices when there exist multiple equilibria in the economy (and a minimum wage can be used to exclude the possibility of a Pareto-dominated equilibrium).⁸ In this paper the same model that is used to define conditions under which minimum wages can be welfare-improving is estimated using individual-level data. Using the parameter estimates we are able to check whether actual minimum wage changes have been welfare-improving for searchers and employees, as well as to make some informed speculation as to the optimal level of the minimum wage under certain explicit welfare criteria.

There do exist a number of empirical studies that examine the role of minimum wages within different equilibrium labor market models than the one used here. Eckstein and Wolpin (1990) generalize and estimate the equilibrium search model of Albrecht and Axell (1984). In these models firms are heterogeneous in terms of profitability, so that a minimum wage effectively serves as an equilibrium selection device in that it selects less profitable (and hence lower wage) firms out of the market. By so doing the wage distribution is shifted to the right, but the cost is a lower rate of job finding due to the exit of firms. Using their estimates, Eckstein and Wolpin find an optimal minimum wage of zero, though they qualify this result because they feel that the model does not adequately fit the data.

Van den Berg and Ridder (1998) modify the Burdett and Mortensen (1998) equilibrium search

⁸This model generally implies that the equilibrium wage distribution would not have a mass point at the minimum wage. A mass point is clearly indicated in the data used in our application.

model and estimate it using data from The Netherlands. The equilibrium in these papers relies critically on the existence of on-the-job search, which is lacking in the equilibrium model estimated here (primarily due to data limitations) and in the Albrecht and Axell (1984) and Eckstein and Wolpin (1990) models. Van den Berg and Ridder (1998) Their extensions involve adding firm heterogeneity in innate profitability and the addition of a legislated minimum wage. The first generalization is required to produce an econometric model that does not imply a monotonically increasing steady state wage density. In their empirical implementation, the authors allow for permanent productivity differences across individuals. The imposition of an economy wide minimum wage can result in certain sets of individuals becoming permanently unemployable, and also selects certain low productivity firms out of the market, as in Eckstein and Wolpin (1990). Based on their empirical analysis the authors argue that the relatively high minimum wage rate in The Netherlands has resulted in a potentially large amount of structural (i.e., permanent) unemployment. While they present no quantitative measures of the welfare effect of minimum wage changes, they suggest that it the minimum wage is probably too high.

The implications of these models are similar to that of the one developed here in the sense that unemployment is generally increased by the imposition of a minimum wage. In Eckstein and Wolpin (1990) and van den Berg and Ridder (1998), unemployment cannot decrease as a result of an increase in the minimum wage. In the model considered here, with endogenous contact rates and labor market participation decisions, it is not necessarily the case that unemployment increases when the minimum wage does. Moreover, even when it does, it is possible for the steady state employment rate to increase due to inflows into the labor market (which occur whenever the value of unemployed search increases as a result of the minimum wage change). Thus the impact of the minimum wage on the steady state distribution of the population by labor market status is a bit more general in the setup considered here.

The biggest difference between those models and the current one, however, is in the form of the steady state wage distribution. The Eckstein and Wolpin (1990) model produces a mass point at the minimum wage, as does the one developed here, though their framework also implies a discrete wage distribution, something only problematically matched to individual level wage observations. While we do find evidence that wages are concentrated on only a few values (see Table 1), for reasons given below we believe that this phenomenon is more likely to be the result of response error than discreteness in the underlying equilibrium wage distribution.

As is the case in Burdett and Mortensen (1998), the van den Berg and Ridder (1998) model implies an absolutely continuous equilibrium wage distribution. In particular, there exists no mass point at the minimum wage. The authors present an extensive justification for the reasonableness of this implication, at least within the context of the labor market data they use to estimate the model. We argue below that in the labor market we examine the mass point at the minimum wage is real and significant; that the mass shifts in a way consistent with the structure of our model when the minimum wage is increased, and that as a result it is necessary to estimate a model consistent with this observation if we are to perform informative policy exercises.

The model we develop should be viewed as complementary to those we have just discussed, though our emphasis is somewhat different. We essentially ignore firm- and individual-specific heterogeneity, which to varying degrees is an important component of the other models, and emphasize match-specific heterogeneity. By treating the match productivity distribution as a primitive and the minimum wage as a side constraint on the bilateral Nash bargaining problem, we feel that we are able to generate an equilibrium wage distribution that captures the features of the data of most concern to us in the analysis of minimum wage effects on labor market equilibrium and welfare. The results of our policy exercise generally agree with those of Eckstein and Wolpin (1990) and van den Berg and Ridder (1998) when the contact rate between searching individuals and firms

is endogenously determined. That is, we find the aggregate welfare could have been improved by decreasing the minimum wage of \$4.25 in September 1996. However, we find little empirical support for the endogeneity of the contact rate. When this rate is treated as exogenous, we find that aggregate welfare could have been significantly increased by doubling the minimum wage. As a result, we do not come to a firm conclusion regarding the “optimal” minimum wage in the labor market we study.

The plan of the paper is as follows. In Section 2 we develop a bargaining model in a continuous-time search environment, both in the absence and in the presence of a binding minimum wage, in which contact rates between searching individuals and firms are endogenously determined. Section 3 contains a discussion of the impact of minimum wage changes on labor market equilibrium outcomes and welfare. In Section 4 we present the econometric framework within which the model is estimated, and devote a great deal of attention to identification issues. We present the estimation results in Section 5 obtained using various specifications of the econometric model. Section 6 employs model estimates to perform welfare experiments associated with changing the minimum wage and discusses the implications of the results for finding an optimal minimum wage. A conclusion is offered in Section 7.

2 Labor Market Search with Bargaining

In this section we describe the behavioral model of labor market search with matching and bargaining. The model is formulated in continuous time and assumes stationarity of the labor market environment. In the first subsection we derive the decision rules for terminating search and for dividing the match value between worker and firm in the absence of minimum wages. In the following subsection we describe the manner in which minimum wages affect search behavior and the division of the match value. In the final subsection, we explicitly bring in labor market participation decisions of individuals and the vacancy creation decisions of firms. Taken together, the addition of these decisions allows us to endogenously determine the rate of contact between both sides of the market. Thus our strategy in the first two subsections will be to simply condition on a fixed value of the contact rate. In the third subsection we expand the bargaining with binding minimum wage case, the one which is empirically relevant, to include the determination of equilibrium contact rates between agents on the two sides of the market

Throughout we assume that there exists an invariant, technologically-determined distribution of worker-firm productivity levels which is given by $G(\theta)$. When a potential employee and a firm meet the productive value of the match (θ) is immediately observed by both the applicant and the firm. At this point a division of the match value is proposed using a Nash bargaining framework. The searcher’s instantaneous discount rate is given by $\rho > 0$. The rate of (exogenous) termination of employment contracts is $\eta \geq 0$. While unemployed individuals search, their instantaneous utility is given by b , which can assume positive or negative values. For simplicity, we assume that employed individuals do not receive alternative offers of employment, i.e., there is no on-the-job search. It is straightforward to adapt the current framework to that case, however.⁹

2.1 Labor Market Decisions without Minimum Wages

We assume that the only factor of production is labor, and that total output of the firm is simply the sum of the productivity levels of all of its employees. Then if the firm “passes” on the applicant

⁹Since the CPS data used in the empirical analysis are essentially cross-sectional, it is virtually impossible to construct the kind of event history data required to estimate a model with sophisticated turnover phenomena.

- that is, does not make an employment offer - its “disagreement” outcome is 0 [it earns no revenue but makes no wage payment].¹⁰ As mentioned above, in this subsection we assume that the arrival rate of contacts (from the point of view of searchers and firms) is equal to λ , a predetermined “primitive” parameter. The applicant’s disagreement value is the value of continued search, which we denote by V_n . For any given value of V_n there exists a corresponding critical “match” value $\theta^* = \rho V_n$ (ρ is the instantaneous discount rate), which has the property that all matches with values at least as great as θ^* will result in employment while all those matches of lower value will not. For any $\theta \geq \theta^*$, the Nash-bargained wage is given by

$$w(\theta, V_n) = \arg \max_w [V_e(w) - V_n]^\alpha \left[\frac{\theta - w}{\rho + \eta} \right]^{1-\alpha}, \quad (1)$$

where $V_e(w)$ is the value of being employed at wage w and α is the bargaining power parameter, which takes values in $(0, 1)$. Without loss of generality it has been assumed that the firm shares the employee’s effective rate of discount, $\rho + \eta$.

The value of employment at a wage of w is easily determined. Consider an infinitesimally small period of time ε . Over this “period,” either the individual will continue to be employed at wage w or will lose their job, which occurs at rate η . Then

$$V_e(w) = (1 + \rho\varepsilon)^{-1} \{w\varepsilon + \eta\varepsilon V_n + (1 - \eta\varepsilon) V_e(w) + o(\varepsilon)\}, \quad (2)$$

where the term $(1 + \rho\varepsilon)^{-1}$ is an “infinitesimal” discount factor associated with the small interval ε , $\eta\varepsilon$ is the approximate probability of being terminated from one’s current employment by the end of ε , and $o(\varepsilon)$ is a term which has the property that $\lim_{\varepsilon \rightarrow 0} (o(\varepsilon)/\varepsilon) = 0$. Note that the first term on the right hand side of [2] is the value of the wage payment over the interval, which is the total payment $w\varepsilon$ multiplied by the “instantaneous” discount factor [think of the payment as being received at the end of the interval ε]. After collecting terms and taking the limit of [2] as $\varepsilon \rightarrow 0$, we have

$$V_e(w) = \frac{w + \eta V_n}{\rho + \eta}. \quad (3)$$

We now substitute [3] into [1] so as to simplify the problem as follows:

$$\begin{aligned} V_e(w) - V_n &= \frac{w + \eta V_n}{\rho + \eta} - V_n \\ &= \frac{w - \rho V_n}{\rho + \eta}, \end{aligned}$$

so that we get the well-known expression

$$\begin{aligned} w(\theta, V_n) &= \arg \max_w [w - \rho V_n]^\alpha [\theta - w]^{1-\alpha} \\ &= \alpha\theta + (1 - \alpha)\rho V_n. \end{aligned}$$

We can now move onto computing the value of nonemployment. Using the same setup as above for defining the value of employment, we begin with the ε -period formulation

$$\begin{aligned} V_n &= (1 + \rho\varepsilon)^{-1} \{b\varepsilon + \lambda\varepsilon \int \max[V_n, V_e(w(\theta, V_n))] dG(\theta) \\ &\quad + (1 - \lambda\varepsilon) V_n + o(\varepsilon)\}, \end{aligned}$$

¹⁰The zero outside option assumption made in this section is justified under the analysis presented in the third subsection below. A free entry assumption assures that the ex ante value of an unfilled vacancy is zero.

where $b\varepsilon$ is the utility associated with unemployment over the interval ε and $\lambda\varepsilon$ is the approximate probability of encountering one potential employer within the period of duration ε . Rearranging and taking limits, we have

$$\rho V_n = b + \lambda \int_{\rho V_n} [V_e(w(\theta, V_n)) - V_n] dG(\theta).$$

Since

$$\begin{aligned} V_e(w(\theta, V_n)) &= \frac{\alpha\theta + (1 - \alpha)\rho V_n + \eta V_n}{\rho + \eta} \\ &= \frac{\alpha\theta - \alpha\rho V_n}{\rho + \eta} + V_n, \end{aligned}$$

we have

$$V_e(w(\theta, V_n)) - V_n = \frac{\alpha\theta - \alpha\rho V_n}{\rho + \eta}.$$

Then the final (implicit) expression for the value of search is

$$\rho V_n = b + \frac{\alpha\lambda}{\rho + \eta} \int_{\rho V_n} [\theta - \rho V_n] dG(\theta). \quad (4)$$

Note that this expression is identical to the expression for the reservation value in a model with no bargaining when θ is the payment to the individual except for the presence of the factor α . This is not unexpected, since when $\alpha = 1$, the entire match value is transferred to the worker, and thus search over θ is the same as search over w .

Now we can summarize the important properties of the model. The critical “match” value θ^* is equal to ρV_n , which is defined by [4]. Since at this match value the wage payment is equal to $w^* \equiv w(\theta^*, V_n) = \alpha\theta^* + (1 - \alpha)\theta^* = \theta^*$, the reservation wage is identical to the reservation match value. The probability that a random encounter generates an acceptable match is given by $\tilde{G}(\theta^*)$, where \tilde{G} denotes the survivor function, $1 - G$. The rate of leaving unemployment is $\lambda\tilde{G}(\theta^*)$. As we can see from [4], since θ^* is an increasing function of α , rates of unemployment are higher when searchers have more bargaining power. Since supply-side welfare, no matter how measured, is a non-decreasing function of α , it is clear that in this type of model high unemployment rates are not necessarily associated with low welfare values for employees and unemployed searchers.

The observed wage density is a simple mapping from the matching density. Since

$$\begin{aligned} w(\theta, V_n) &= \alpha\theta + (1 - \alpha)\theta^* \\ \Rightarrow \tilde{\theta}(w, V_n) &= \frac{w - (1 - \alpha)\theta^*}{\alpha}, \end{aligned}$$

then the density function of observed wages is given by

$$h(w) = \begin{cases} \frac{\alpha^{-1}g(\tilde{\theta}(w, V_n))}{\tilde{G}(\theta^*)} & w \geq \theta^* \\ 0 & w < \theta^* \end{cases}.$$

It will be useful to carry an example through the first part of the paper, particularly in order to emphasize the key role the bargaining power parameter α plays in determining the substantive implications of the model. In our example labor market we set the rate of arrival of offers (λ) to the value .5 (so that job contacts occur every 2 “periods” on average), the rate of job dissolutions (η) is set to .02 (so that the average length of a job is 50 periods), ρ is set to .01, and the instantaneous

return from search (b) is set to -1. The firm-searcher matching distribution is assumed to be uniform with support $[0, 10]$. We will compute the equilibrium wage distribution for $\alpha = .3$ and $\alpha = .6$.

Figure 3.a plots the uniform p.d.f. which represents $g(\theta)$ in this case. Figure 3.b plots the mapping from draws of θ into wage offers under the alternative values of α , that is $w_\alpha(\theta, V_n(\alpha)) = \alpha\theta + (1-\alpha)\rho V_n(\alpha)$. Note that α affects the equilibrium mapping both directly through the slope and indirectly through the disagreement point $\rho V_n(\alpha)$. Figures 3.c-d plot the equilibrium wage p.d.f.s for the two α values. Increasing α in the uniform case simply results in increases in the lower and upper bound of the support of the equilibrium wage distribution, which is itself uniform.

2.2 Labor Market Decisions in the Presence of Minimum Wages

Now consider the case in which the interactions between applicants and firms are constrained by the presence of a minimum wage. The minimum wage, m , is set by the government and is assumed to apply to all potential matches. We assume that the only compensation provided by the firm is the wage. Thus there are no other forms of compensation the firm can adjust so as to “undo” the minimum wage payment requirement.

We impose the minimum wage in the framework established in the previous section; in particular, we continue to assume that the contact rate is fixed at λ . As should be clear, any $m \leq \theta^*$ has no effect on the behavior of applicants or firms and thus would be a meaningless constraint. For this reason we consider only the effects of the imposition of an $m > \theta^*$.

Recall that the ex post value of the match from the point of view of the firm is proportional to $(\theta - w)$. Firms cannot earn positive profits on matches which have a value less than m . Since $m > \theta^*$, an immediate implication of the imposition of the minimum wage is that fewer contacts will result in jobs - the standard employment effect.¹¹

In terms of wage payments, the minimum wage acts solely as a side constraint on the Nash bargaining problem. Formally, the revised problem is given by

$$w(\theta, V_n) = \arg \max_{w \geq m} [V_e(w) - V_n]^\alpha \left[\frac{\theta - w}{\rho + \eta} \right]^{1-\alpha},$$

where the only difference from [1] is the restriction $w \geq m$. The effect on the solution is relatively intuitive. Under the “constrained” Nash bargaining problem, there will exist a value of search which we denote $V_n(m)$ [this value is *not* equal to the V_n defined in the previous subsection - it will be defined below]. If we ignore the minimum wage constraint in determining the wage payment given a match value of θ and the search value $V_n(m)$, we get

$$w(\theta, V_n(m)) = \alpha\theta + (1-\alpha)\rho V_n(m). \tag{5}$$

Under this division of the match value, the worker would receive a wage of m when $\theta = \hat{\theta}$, where

$$\hat{\theta}(m, V_n(m)) = \frac{m - (1-\alpha)\rho V_n(m)}{\alpha}.$$

Then if $\hat{\theta} \leq m$, all “feasible” matches would generate wage offers at least as large as m . When $\hat{\theta} > m$, this is not the case. When θ belongs to the set $[m, \hat{\theta})$, the offer according to [5] is less than m . However, when confronted with the choice of giving some of its surplus to the worker versus a return of 0, the firm pays the wage of m for all $\theta \in [m, \hat{\theta})$. Wages for acceptable θ outside of this set are determined according to [5].

¹¹This implication critically depends on the assumption of a fixed contact rate. We will return to this issue in the following subsection.

We can now consider the individual's search problem given this wage offer function. Using the ε look ahead formulation,

$$\begin{aligned}
V_n(m) &= (1 + \rho\varepsilon)^{-1} \{ b\varepsilon + \lambda\varepsilon [\int_m^{\hat{\theta}(m, V_n(m))} \left[\frac{m + \eta V_n(m)}{\rho + \eta} - V_n(m) \right] dG(\theta) \\
&\quad + \int_{\hat{\theta}(m, V_n(m))} \left[\frac{\alpha\theta + (1 - \alpha)\rho V_n(m) + \eta V_n(m)}{\rho + \eta} - V_n(m) \right] dG(\theta) + V_n(m)] \\
&\quad + (1 - \lambda\varepsilon)V_n(m) + o(\varepsilon) \} \\
&= (1 + \rho\varepsilon)^{-1} \{ b\varepsilon + \lambda\varepsilon [\int_m^{\hat{\theta}(m, V_n(m))} \left[\frac{m - \rho V_n(m)}{\rho + \eta} \right] dG(\theta) \\
&\quad + \int_{\hat{\theta}(m, V_n(m))} \left[\frac{\alpha(\theta + \rho V_n(m))}{\rho + \eta} \right] dG(\theta) + V_n(m)] + (1 - \lambda\varepsilon)V_n(m) + o(\varepsilon) \}.
\end{aligned}$$

Taking limits after collecting terms, we have

$$\begin{aligned}
\rho V_n(m) &= b + \frac{\lambda}{\rho + \eta} \left\{ \int_m^{\hat{\theta}(m, V_n(m))} [m - \rho V_n(m)] dG(\theta) \right. \\
&\quad \left. + \alpha \int_{\hat{\theta}(m, V_n(m))} [\theta - \rho V_n(m)] dG(\theta) \right\}.
\end{aligned} \tag{6}$$

We shall often refer to the value $\rho V_n(m)$ as the “implicit” reservation wage. Unlike the situation in which a binding minimum wage is not present, this value is *not* the minimal acceptable wage and match value. The acceptable wage/match value is rather the imposed minimum value m . Nonetheless, the value $\rho V_n(m)$ is of critical importance in determining equilibrium wages and the welfare impacts of minimum wage changes.

Conditional on the value of a binding minimum wage m , the equilibrium wage distribution is described by

$$p(w; m) = \begin{cases} \frac{\alpha^{-1} g(\hat{\theta}(w, V_n(m)))}{\tilde{G}(m)} & w > m \\ \frac{\tilde{G}(m) - \tilde{G}(\hat{\theta}(m, V_n(m)))}{\tilde{G}(m)} & w = m \\ 0 & w < m \end{cases} \tag{7}$$

The minimum wage side constraint produces an equilibrium wage distribution which has a mass point at m and has wages being continuously distributed on the interval (m, ∞) .¹²

Let us reconsider our uniform example after a minimum wage of 7 has been imposed; since the distribution now has a mass point, it is necessary to plot the c.d.f. as opposed to the p.d.f. Figure 4.a plots the c.d.f. of the matching distribution. Figure 4.b contains the equilibrium wage offer mapping from θ to w when $m = 7$. Note that for the case of $\alpha = .3$ the equilibrium wage function maps all values of $\theta \geq 7$ into a wage offer of $w = 7$. At least when the distribution G has bounded support, this demonstrates that the imposition of a minimum wage can result in a degenerate wage offer distribution at the minimum, as we see in Figure 4.c. In the case of $\alpha = .6$ (Figure 4.d), the equilibrium wage distribution has a substantial mass point at 7, with a relatively narrow range of wages above it.

¹²This statement is predicated on θ being a continuously distributed random variable with unbounded support on R_+ .

2.3 Endogenous Contact Rates

As in Pissarides (2000), we adopt a highly stylized model of the decision to participate in the labor market. While we continue to assume that agents are identical when entering the market, in the ex ante sense, we assume that individuals in the population are heterogeneous in the value of remaining outside of the market. In this case, outside options include schooling, leisure, etc., and since a large proportion of the 16-24 year old population that is the focus of our analysis does not participate in the market, this is an important consideration. Let the (normalized) value of not participating in the labor market for an individual be given by ρV_o , and assume that in the population this random variable has a parametric distribution $Q(\rho V_o; \zeta)$, where ζ is a finite-dimensional parameter vector. If an agent enters the labor market they enter as an unemployed searcher, the normalized value of which is $\rho V_n(m)$ given minimum wage m . Thus all agents in the population with a value of V_o less than or equal to $V_n(m)$ will enter the market under m . Then, in the steady state, the participation rate is given by $Q(\rho V_n(m); \zeta)$.¹³

We adopt the standard set up (in the macroeconomics literature) for modeling firms' decisions to create vacancies. Let there exist a constant returns to scale matching technology,

$$M(\tilde{u}, v) = vq(\kappa),$$

where $\kappa \equiv \tilde{u}/v$, \tilde{u} is the size (measure) of the set of unemployed searchers, and v is the size of the set of vacancies. The contact rate per vacancy is given by $M(\tilde{u}, v)/v = q(\kappa)$, and the contact rate per unemployed agent is $M(\tilde{u}, v)/\tilde{u} = q(\kappa)/\kappa$. We will make a parametric assumption regarding q , and write $q(\kappa; \omega)$, where ω is a finite-dimensional parameter vector.

Assuming that there exists a population of potential (firm) entrants with an outside option value of 0, firms create vacancies until the point that expected profits are zero. Let the cost of creating a vacancy be given by $\psi > 0$. Then the expected value of creating a vacancy is given by

$$\rho V_v = -\psi + q(\kappa; \omega)\tilde{G}(r)(J - V_v),$$

where r denotes the acceptance match value (equal to the maximum of $(\rho V_n(m), m)$), $q(\kappa; \omega)\tilde{G}(r)$ is the rate at which a firm fills a vacancy, J is the expected value of a filled vacancy (where the expectation is taken with respect to the distribution of acceptable matches $\theta \geq r$), and V_v is the value of a vacancy. Under the free entry assumption (FEC), $V_v = 0$ and we have

$$0 = -\psi + q(\kappa; \omega)\tilde{G}(r)J. \quad (8)$$

The [8] can be used to solve for equilibrium number of vacancies given the expected value of a filled vacancy J and the size of the set of unemployed searchers. In the steady state, the probability that a labor market participant is unemployed is given by

$$u = \frac{\eta}{\eta + \tilde{G}(r)q(\kappa; \omega)/\kappa}.$$

If we denote the size of the set of labor market participants by l , then the size of the set of unemployed searchers (relative to the entire population) is

$$\begin{aligned} \tilde{u} &= lu \\ &= \frac{\eta l}{\eta + \tilde{G}(r)q(\tilde{u}/v; \omega)/(\tilde{u}/v)}. \end{aligned} \quad (9)$$

¹³This is only strictly true in the steady state. For example, if the minimum wage is changed and as a result the value of unemployed search declines, those individuals currently working at a match at which the value of the employment contract exceeds the value of the outside option will remain in the market even if the value of search is now less than the value of their outside option. Once they lose their job however, which occurs with probability one in finite time, they will no longer continue to search and will instead leave the labor market.

With endogenous contact rates, a labor market equilibrium in the presence of a minimum wage (that may or may not be binding) is characterized by the quadruplet $(l, u, v, \rho V_n(m))$, which is a solely a function of the primitive parameters $(\rho, b, \eta, \alpha, G, \zeta, q, \psi)$. After replacing λ with $q(\kappa)/\kappa$ in (6), the three equations (6), (8), and (9), together with $l = Q(\rho V_n(m); \zeta)$, serve to uniquely determine $(l, u, v, \rho V_n(m))$.

Example 1 Assume that ρV_o is distributed as $N(5, 4)$ in the population, and that the distribution of $\ln \theta$ is $N(2, .25)$. The discount rate, in “monthly” units, is $.05/12$, the exogenous dismissal rate η is $.038$, the utility flow in unemployment b is -20 , and the flow cost of a vacancy ψ is 120 . The matching production function M is given by

$$M(\tilde{u}, v) = \tilde{u}^\omega v^{1-\omega}, \tag{10}$$

and the Cobb-Douglas parameter ω is set at $.4$. The equilibrium of this model, with and without a binding minimum wage of $m = 6$, is as follows:

Outcome	m = 0	m = 6
u	0.098	0.121
l	0.583	0.620
v	0.017	0.018
$\rho V_n(m)$	5.416	5.610
\tilde{u}	0.057	0.075
e	0.529	0.545

The outcome e in the last row of the table is the proportion of the population employed in equilibrium. We note that the imposition of a binding minimum wage of 6 in this labor market environment has some notable impacts. First, the unemployment rate among labor force participants increases by about one-third. The percentage of population members in the labor market increases from 0.583 to 0.620; this increase is large enough to counterbalance the increased unemployment rate among labor market participants so that the proportion of the total population that is employed actually increases from 0.529 to 0.545. Another striking feature of the new equilibrium is the increase in the value of unemployed search. Thus an imposition of this particular binding minimum wage in the particular labor market resulted in increased levels of unemployment among labor market participants, increased levels of employment in the population, and an increased value of unemployed search. In the next section we turn to the consideration of constructing welfare measures to quantify the value of minimum wage changes to agents on the supply and demand side of the labor market.

3 The Impacts of Minimum Wage Changes on Labor Market Outcomes and Welfare

We begin by describing a fairly general framework within which to consider welfare impacts. Since the model is stationary in nature, we will focus our attention on the long run welfare impact; this is consistent with the approach taken by Hosios (1990). At any point in time, all agents who are (potentially, at least) on the supply side of the market will be nonparticipants, others will be unemployed, and the remainder will be employed. On the demand side of the market, some set

of agents will not have created vacancies, others will currently be carrying an unfilled vacancy, and others will have a filled vacancy. We will assume that the minimum wage is the only policy instrument available to a social planner, and that the planner's objective can be expressed as a Benthamite social welfare function, defined as follows:

$$S(m, a) = a_1(T_o(m))\bar{V}_o(m) + a_2(T_n(m))V_n(m) + a_3(T_e(m))\bar{V}_e(m) + a_4(T_f(m))\bar{J}(m), \quad (11)$$

where $T_o(m)$ is the size of the set of individuals who are out of the labor force under minimum wage m and $\bar{V}_o(m)$ is their average welfare level, $T_n(m)$ is the proportion of the population unemployed (all unemployed individuals have the same welfare level $V_n(m)$), and $T_e(m)$ is the proportion of the population who are employed in the steady state with $\bar{V}_e(m)$ denoting their average welfare level. On the demand side of the market, we have ignored the welfare contribution of agents who have chosen not to create a vacancy and those currently holding one, since the free entry condition implies that the welfare of firms (or potential firms) in either of these classes is zero. Thus the only term from the demand side that enters is associated with firms with a filled vacancy. The size of this set is denoted by $T_f(m)$ and the average value of $J(m)$ in this set is $\bar{J}(m)$.

While it may seem natural to specify total welfare in terms of group weighted averages, we have allowed for slightly more generality by specifying weight functions a that depend on group size. This additional flexibility facilitates the discussion of one welfare measure developed below. In general, however, we will be considering welfare weights of the form

$$a_i(T_{(i)}(m)) = a_i^0 T_{(i)}(m),$$

where $T_{(i)}(m)$ is the size of the set of individuals to whom the scalar weight a_i^0 is applied.

First consider the size of the set of individuals who do not participate in the market. Given an equilibrium outcome of $\rho V_n(m)$, we have

$$T_1(m) = 1 - Q(\rho V_n(m)) \equiv \tilde{Q}(\rho V_n(m)).$$

Since we assume that ρV_o is continuously distributed in the population, T_1 is a monotonically decreasing function of $V_n(m)$. Thus any minimum wage change that results in an increasing value of search leads to greater labor force participation.

The size of the set of searching participants is given by the participation rate multiplied time the steady state probability of unemployment (given participation). Thus

$$T_2(m) = Q(\rho V_n(m)) \times \frac{\eta}{\eta + \lambda(m)\tilde{G}(r)},$$

where $r = \max\{\rho V_n(m), m\}$ and where $\lambda(m)$ signifies that the contact rate (from the searching individual's perspective) depends on the equilibrium participation and vacancy creation decisions which are a function of m (when m is binding, or $r = m$). Defining the minimal acceptable θ in this way, that is, by r , allows for the possibility that the minimum wage m is not binding.

The size of the set of employed individuals is defined in a similar way,

$$T_3(m) = Q(\rho V_n(m)) \times \frac{\lambda(m)\tilde{G}(r)}{\eta + \lambda(m)\tilde{G}(r)}.$$

In the literature it is common to assume that each firm creates at most one vacancy, which will be filled or unfilled at a given moment in time. In this case, the set of firm owners that have a filled vacancy must be equal to the size of the set of employed searchers, or

$$T_4(m) = T_3(m).$$

3.1 Specific Welfare Criteria

We now discuss particular welfare measures of interest, given the general social welfare function (11).¹⁴ Before doing so, we want to point out that two “welfare measures” commonly employed in the empirical literature on minimum wages are not derivable from (11). These are the unemployment rate of labor market participants and the population employment rate. Given the definitions of specific welfare weights developed above, the steady state unemployment rate is simply $T_2(m)/(1 - T_1(m))$, while the steady state employment rate among population members is $T_3(m)$. These numbers are simple to compute but have a problematic welfare interpretation. The basic problem emanates from the fact that the size of these particular sets is not perfectly informative regarding the distribution of welfare among members of the set, much less about the distribution of welfare of other agents in the population. The welfare measures we briefly discuss below all are explicitly take into account the distribution of welfare in at least some of the population classes.

3.1.1 Rawlsian Criterion

Under this criterion the welfare impact of a minimum wage is measured by its impact on the worse-off members of the population. For this to be an interesting exercise we must ignore the firm side of the market, since under the FEC the least well-off agent has a value of 0 no matter what the value of m . Thus we restrict attention to the supply side in this case.

On the supply side of the market, the lowest welfare value at a moment in time belongs to those agents who are in the labor market and unemployed. Thus our Rawlsian criterion is represented by a choice of welfare weights

$$\begin{aligned}a_1 &= 0 \\a_2 &= 1 \\a_3 &= 0 \\a_4 &= 0.\end{aligned}$$

As a result, the (supply side) Rawlsian welfare measure is equal to $V_n(m)$.

3.1.2 Total Welfare (TW)

The focal point of our analysis is social welfare, broadly construed, in which each agent’s welfare is equally weighted. In this case we have

$$\begin{aligned}a_1^0 &= 1 \\a_2^0 &= 1 \\a_3^0 &= 1 \\a_4^0 &= 1.\end{aligned}$$

For convenience we will define $S(m, a)$ in this case by $TW(m)$.

3.1.3 Participants’ Welfare (PW)

In his study of efficient labor market equilibrium, Hosios (1990) considered explicitly only the welfare of labor market participants. This would be consistent with $TW(m)$ if we were to make

¹⁴A number of specific welfare criteria are developed in greater detail in Flinn (2002).

an analogous free entry condition for the supply side of the market by normalizing the value of the nonparticipation state to equal 0 for all population members. Since we don't want to impose such a condition, we will define a separate welfare measure that applies only to market participants (on both the supply and demand side). The weighting scheme in this case is

$$\begin{aligned} a_1^0 &= 0 \\ a_2^0 &= 1 \\ a_3^0 &= 1 \\ a_4^0 &= 1. \end{aligned}$$

This measure will be the focus of our analysis in what follows, both because we can relate optimization of this social welfare function to the Hosios (1990) analysis and since our specification of welfare of nonparticipants on the supply side of the market is essentially arbitrary (i.e., is determined outside of the model).

3.1.4 Discussion

Within a relatively general bargaining framework (consistent with ours in the absence of a binding minimum wage) and with a constant returns to scale matching technology, Hosios (1990) showed that the maximization of aggregate welfare PW required that the effects of search externalities be appropriately recognized in the allocation of surplus shares to the unemployed searchers and vacancy creators. Thus the policy variable of his analysis was α .

Given that we will only be able to estimate extremely simple matching functions given the data available to us, we consider the implications of this result when the matching technology is Cobb-Douglas as in [10]. Then if the elasticity of the matching function with respect to \tilde{u} is ω , the Hosios condition for efficiency is

$$\alpha = \omega.^{15}$$

When this condition holds, there is no scope for a minimum wage to increase aggregate welfare as defined by $PW(m)$.

The discussion in Hosios (1990) treats α as a policy variable that can be set by the social planner when designing an optimal compensation scheme. Our perspective is decidedly different, in that we consider α to be set by market forces, preference characteristics, and the generic bargaining environment, all of which are considered to be beyond the purview of the social planner.¹⁶ In the econometric analysis below, a substantial amount of effort is devoted to the identification and estimation of α and ω based on Current Population Survey data and information taken from the aggregate economy or a private-sector firm. Our estimates suggest strongly that $\alpha \neq \omega$.¹⁷ In this case a minimum wage may improve welfare, as measured by PW or TW , by increasing the "effective" bargaining power of searchers on the supply side and inducing them to increase their

¹⁵Hosios considers a slightly more general decision problem since he allows both firms and searchers to choose a search intensity. Given the severe econometric identification problems we face, the estimation of a model allowing for the choice of intensity is not possible, even though it would clearly be desirable.

¹⁶Our axiomatic Nash bargaining framework provides no explicit mechanism through which α is determined. One possibility for rectifying this situation is to employ a strategic approach to the bargaining problem as in Rubinstein (1982). In the case of an alternating offers game to determine the share of a pie allotted to the two parties, α was found to be a function of the discount rates of the two parties. If this discount rate is considered to be a part of the preference map of the individual parties, then it is not reasonable to consider it a policy variable.

¹⁷While we are able to estimate a standard error associated with our estimate of α , we do not do so for our estimate of ω . Thus this claim is based on the fact that the standard error of α is small and the distance between the point estimates of the two parameters is large given the fact that both are constrained to lie in the unit interval.

participation rate to approach what it would be under $\alpha = \omega$. This increased participation rate can have positive impacts on all market participants by increasing the size of the employed population (which is equal to the size of the set of filled vacancies).

From this perspective the minimum wage can be viewed as an instrument to increase total welfare of labor market participants or the entire society. It is crude in the sense that its performance will not be as good as would altering the value of α to equal ω . In its favor is the relative ease of implementation. We should note that the minimum wage can properly be viewed as a part of a larger policy that entails truncating the wage distribution, whether from below or above. Thus if it is found that vacancy creators get too low a share of the surplus relative to $(1 - \omega)$, an efficiency-enhancing wage truncation policy could result in wage caps.

While we do not derive explicit propositions concerning the conditions under which minimum (or maximum) wages will be welfare-enhancing under our three welfare measures, we do analyze optimal policy choices given our estimates of primitive parameters. Generally speaking, we find that the welfare impacts of the minimum wage depend critically on the endogeneity of the contact rate, λ .

3.2 Employment and Unemployment Effects

We will consider the impact of minimum wage changes on the steady state employment and unemployment levels. While our policy analysis will focus on the impacts of minimum wages on the three welfare measures described above, most empirical analyses of minimum wage impacts on labor market outcomes has focused on their effects on unemployment and employment (see, e.g., Card and Krueger (1995)). It may be useful to understand the extent to which our model can deliver implications consistent with empirical observation under a variety of assumptions regarding the endogeneity of the contact rate and labor market participation decision.

We begin by considering the simplest case, that in which λ is a primitive parameter and the market participation rate is fixed. We then allow for a participation decision with a fixed contact rate, after which we reverse the situation and allow for endogenous contact rates and a closed labor market. We conclude by considering the case of endogenous contact rates and participation decisions.

3.2.1 Fixed Contact and Participation Rates

In this setting the impact of a minimum wage increase on steady state employment and unemployment rates is immediate. Since the steady state unemployment rate (as a proportion of the labor market) is given by

$$u(m) = \frac{\eta}{D(m)},$$

where $D(m) \equiv \eta + \lambda\tilde{G}(m)$, then

$$\begin{aligned} u'(m) &= \frac{u(m)}{D(m)}\lambda g(m) \\ &> 0. \end{aligned}$$

Since $e(m) = 1 - u(m)$, the steady state employment rate decreases in m .

3.2.2 Fixed Contact and Endogenous Participation Rates

The impacts in this case are dependent upon whether the rates are computed relative to the labor market population or the entire population. Letting $l(m)$ denote the proportion of the population

in the labor market, the unemployment rate defined with respect to the total population is

$$\tilde{u}(m) = l(m)u(m),$$

so that the impact of the minimum wage on the unemployment rate in the population is given by $\tilde{u}(m) = l'(m)u(m) + l(m)u'(m)$. The sign of the second term on the right hand side is unambiguously positive, as we saw above, while the first term can be positive or negative. Given our assumption regarding the nature of the participation decision, we have

$$\begin{aligned} l(m) &= Q(\rho V_n(m)), \\ \Rightarrow l'(m) &= q(\rho V_n(m))\rho V_n'(m). \end{aligned}$$

A necessary condition for an increase in the minimum wage to lead to a lower proportion of unemployed in the population is that the value of search decrease.

Now since the population employment rate is $\tilde{e}(m) = l(m)(1 - u(m))$, it follows that an increase in \tilde{e} requires that the value of search be increasing in m . Combining these two necessary conditions, we note that the “one factor” structure of the steady state population unemployment and employment rates implies that a minimum wage increase cannot simultaneously yield an increase in the employment rate and a decrease in the unemployment rate. Minimum wage hikes always involve a trade-off between unemployment and employment when λ is fixed.

3.2.3 Fixed Participation and Endogenous Contact Rates

We now return to the case of a fixed participation rate, so that l is independent of m , but allow for an endogenous contact rate, $\lambda(m)$. We again consider the unemployment rate within the set of labor market participants, so that

$$u'(m) = \frac{u(m)}{D(m)}(\lambda(m)g(m) - \lambda'(m)\tilde{G}(m)).$$

We have seen that when λ is a primitive parameter, so that $\lambda'(m) = 0$ for all m , $u'(m)$ is unambiguously positive. With an endogenous contact rate, for the minimum wage to decrease the unemployment rate requires that

$$\frac{\lambda'(m)}{\lambda(m)} > \frac{g(m)}{\tilde{G}(m)}, \tag{12}$$

where the right hand side of (12) is the hazard rate of the matching distribution evaluated at m .

We can say more about this condition if the matching function is Cobb-Douglas. In this case we have

$$\begin{aligned} \lambda(m) &= \frac{(u(m)l)^\delta v(m)^{1-\delta}}{u(m)l} \\ &= \left(\frac{v(m)}{u(m)l}\right)^{1-\delta}. \end{aligned}$$

Now the FEC under the minimum wage implies

$$\psi = \left(\frac{v(m)}{u(m)l}\right)^{-\delta} \tilde{G}(m)J(m),$$

or

$$v(m) = A(m)^{1/\delta} u(m)l,$$

where

$$A(m) = \frac{\tilde{G}(m)J(m)}{\psi}.$$

Then

$$\lambda(m) = A(m)^{\frac{1-\delta}{\delta}}.$$

After some simplification we find that for a minimum wage increase to decrease the unemployment rate requires that

$$\frac{J'(m)}{J(m)} > \frac{1}{1-\delta} \frac{g(m)}{\tilde{G}(m)}.$$

A necessary condition for this to occur is $J'(m) > 0$. By the definition of $J(m)$, a necessary condition for this to occur is that $V'_n(m) < 0$, so that the unemployment rate can only decrease if the value of search decreases. This necessity condition is the same as what we found for the case of fixed λ and variable participation. Because the size of the set of labor market participants is fixed in this case, the employment rate increases only when the unemployment rate decreases, so a necessary condition for employment gains is a decrease in the value of search.

3.2.4 Endogenous Participation and Contact Rates

As in the preceding section, we will confine our attention to the case of a Cobb-Douglas matching function. If we measure the unemployment rate as a proportion of the population in the unemployment state, then

$$\tilde{u}'(m) = l'(m)u(m) + l(m)u'(m).$$

From the preceding section, a necessary condition for a $u'(m) < 0$ is $V'_n(m) < 0$. Under this condition, $l'(m) < 0$. If $V'_n(m) > 0$, then $u'(m) > 0$ and $l'(m) > 0$. Thus $V'_n(m) < 0$ is necessary for a decrease in the unemployment rate \tilde{u} .

In contrast, the proportion of the total population employed, $\tilde{e}(m)$, can increase even with $V'_n(m) > 0$ through the participation effect. While the proportion of individuals in the labor market in the employed state will decrease in this instance, the positive participation effect will counterbalance it. On the other hand, when $V'_n(m) < 0$, the participation rate decreases though the employment rate in the labor market increases. Thus a decrease in the value of search can lead to a decrease in the population unemployment rate and an increase in the population employment rate. An increase in the value of search leads to an unambiguous increase in the population unemployment rate and has an ambiguous effect on the population employment rate.

3.3 Effects on the Wage Distribution

Within this model the effects of imposing a minimum wage on the accepted wage distribution are somewhat complex, though the Nash bargaining structure of the problem does impose a number of restrictions on the relationship between the pre- and post-change distributions. In essence a change in a binding minimum wage has three impacts on the accepted wage distribution. For purposes of discussion we consider the impact of a minimum wage change from m to m' , $m' > m$.

1. It changes the support of the wage distribution. In particular, if G has unbounded support equal to the positive real line, the lower support point of the wage distribution changes from m to m' .

2. It changes the probability mass at the minimum wage. Recall that the mass point at the minimum wage is given by

$$P(w = m) = 1 - \frac{\tilde{G}\left(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha}\right)}{\tilde{G}(m)}.$$

While it is straightforward to show that $\hat{\theta}(m) = (m - (1 - \alpha)\rho V_n(m))/\alpha$ is monotone in m , the impact on the probability mass is ambiguous and depends on the properties of the matching distribution G .

3. Changes in the density of the distribution above m . Consider a wage $w > m$. Then the partial derivative of the wage density at w with respect to m is given by

$$\begin{aligned} \frac{\partial}{\partial m} \left(\frac{g\left(\frac{w-(1-\alpha)\rho V_n(m)}{\alpha}\right)}{\tilde{G}(m)} \right) = \\ -\frac{1-\alpha}{\alpha} \rho V_n'(m) \frac{g'\left(\frac{w-(1-\alpha)\rho V_n(m)}{\alpha}\right)}{\tilde{G}(m)} + \frac{g\left(\frac{w-(1-\alpha)\rho V_n(m)}{\alpha}\right)g(m)}{\tilde{G}(m)^2}. \end{aligned}$$

The second term is always positive and represents the “truncation” effect of the increasing minimum wage which results from concentrating the probability mass on a small support. The sign of the first term is ambiguous in general. In special cases, it may be possible to sign this expression for all $w > m$. One example is the case in which g is monotone decreasing on R_+ and $V_n'(m) > 0$.

Flinn (2002) contains a detailed analysis of the effect of finite minimum wage changes on changes in the wage offer distribution, and constructs stochastic dominance tests to test whether the value of search increased as the result of the change from m to m' . The interested reader is referred to that paper for details.

4 Econometric Issues

The model as formulated relies heavily on stationarity assumptions, which enable us to use what are essentially cross-sectional data sets to estimate dynamic models. The CPS point-in-time sample only contains information on the length of time individuals currently unemployed have been actively searching for a job and the hourly wage rate and/or gross weekly earnings and usual hours worked per week for those reporting that they are currently employed. No information is available on the length of time an employed individual has worked for their current employer. A substantial amount of demographic information is available for each individual as well, information which is sparingly used in most of the empirical analysis that we conduct.

Given the data to which we have access, which is a point-in-time sample, the data from the CPS will not be useful for learning the parameters characterizing the vacancy creation process. That is, there will be an equilibrium value of the contact rate, from the perspective of searchers, that is given by $\lambda = q(\kappa)/\kappa$, where $\kappa = \tilde{u}/v$ and $\tilde{u} = ul$. While we will be able to provide conditions under which \tilde{u} is estimable from the CPS data, data on vacancies are not available. We will show that given an estimator for λ based on the CPS data, and a consistent estimator for \tilde{u} , we will be able to estimate the steady state vacancy rate v if q contains no unknown parameters. In this case estimating v given q or simply estimating λ are equivalent. For this reason, the bulk of

our identification analysis is for the case of a given λ . At the end of this section we consider the estimation of parameters relevant to the vacancy creation process.

We will begin our analysis by laying out the information content of the CPS data for purposes of estimating the labor market model developed above. We shall see that a major stumbling block to the estimation of such a model is the identification of the bargaining power parameter α . The difficulties of estimating the bargaining power parameter have been appreciated for quite some time. In their discussion of an estimable equilibrium search model, FH impose the assumption that the match value (θ here) is split evenly between the firm and worker, but provide little justification for doing so (pp. 142-143). Kooremen and Kapteyn (1990) provide an enlightening discussion and example of the identification of a “sharing” parameter, much like our α , in the context of a cooperative household labor supply model. Eckstein and Wolpin (1995) provide a deeper discussion of the problem of identifying α when using only supply side data than do FH, and in the end impose the restriction $\alpha = .5$ when estimating their model. The same authors revisit this issue at a later date (Eckstein and Wolpin, 1999), and in that research use a clever restriction on the model to obtain an upper bound estimate of the black-white difference in the logarithm of the share parameter (akin to our α). As do FH and Eckstein and Wolpin (1995), they assume that wages are a constant proportion of the match value θ , which is not the case in the model estimated here.

A number of studies using aggregated time series data have obtained estimates of α , as well as the matching function in some cases. The most comparable to ours is probably that of Yashiv (2003), in which he estimates the bargaining power parameter and a Cobb-Douglas matching function parameter using aggregate time series data from Israel. Macro time series studies can potentially avail themselves of information on vacancies, unemployment rates, wage distributions, and firm profitability. The major limitation of these studies is the necessity of relying on a representative agents formulation of the economy, which neglects individual and firm heterogeneity for the most part. This feature is a particular liability when looking at policies with strong distributional impacts, such as the minimum wage.

The recent availability of matched employer-employee data has led to the availability of information on firm performance measures as well as employee characteristics. Cahuc et al. (2003) use French data of this type to estimate a bargaining model, and utilize methods similar to that employed here to identify the bargaining power parameter for different classes of employees. They use a different matching structure in which the match value is given by $\theta = A \times B$, where A is the fixed productivity characteristic value of the worker and B is the productivity characteristic of the firm, and they allow for on-the-job search and contract negotiation as in Postel-Vinay and Robin (2002) and Dey and Flinn (2005). With this matching structure they are able to recover estimates of A for the firms in their large sample. This specification of match heterogeneity is particularly useful with these types of data since it is able to generate systematic firm size differences, something which our i.i.d. match heterogeneity distribution cannot. Presumably a minimum wage could be introduced into their model as well, and it would have similar impacts to those produced by the models estimated by Eckstein and Wolpin (1990) and van den Berg and Ridder (1998) in that low quality firms would drop out of the market, a result beneficial in terms of the equilibrium wage distribution coming at the cost of a reduction in contact rates.

We will explore a number of ways in which the model can be estimated using the CPS data available to us (and in one case auxiliary information from other sources), although some of these will be mentioned only in passing. Possible approaches to the under-identification problem include: (1) normalization, (2) the use of demand-side information, (3) the use of observable heterogeneity in conjunction with exclusion restrictions, and (4) natural experiments. Each approach has its drawbacks, though we shall argue that method (2) provides the most “robust” method for model

identification if one is willing to make some distributional assumptions. We shall present model estimates based on approaches (1) and (2).

In most empirical implementations of behavioral search models an allowance is made for measurement error in the wage data (but not in the duration measures). Allowing for measurement error may be indicated when the sample contains very low wage observations, which without measurement error would imply a perhaps incredibly small reservation wage. When allowing for on-the-job search, which we do not do here, measurement error is required for the model of expected wealth maximization to be consistent with direct job-to-job transitions in which the destination job offers a lower wage than the job which was left.

In the presence of a binding minimum wage, it is not possible to add “classical” measurement error to the model. If the reported wage distribution is assumed to be given by the convolution of the distribution of actual wages and some continuously distributed i.i.d. random variable, then even though the true wage distribution is of the mixed continuous-discrete type, the observed wage distribution will be absolutely continuous. Since this is inconsistent with even the most casual inspection of the data (see Figures 1.a and 1.c, for example), classical measurement error cannot be introduced. While one could consider allowing for “contamination” in the data, i.e., that some unknown proportion τ of wages are measured with classical error while the remainder are not, the estimation of τ would seem not to be possible without further arbitrary assumptions. As a result, we assume throughout that all observations are measured exactly. While we have already noted that a large number of observations are “heaped” at focal points, our hope is that the regular pattern of the heaping process will not bias our estimates to any great degree.

The likelihood function based on the CPS data is constructed as follows. Each individual observation can be characterized by the pair of observations (t_i, w_i) , $i = 1, \dots, N$. The variable t_i is the length of the on-going spell of unemployed search, which is positive if the sample member is unemployed at the time of the survey and is otherwise equal to 0. If the individual is employed, the hourly wage w_i is recorded. If the individual is paid an hourly wage, w_i corresponds to the hourly wage rate they report. If the individual is not paid on an hourly basis, the hourly rate is imputed by dividing the gross weekly wage by the usual hours of work per week. This imputation procedure is standard, but is particularly problematic in this application because of the likely undercount of individuals paid exactly the minimum wage that will result. On the positive side, the vast majority of our sample members report an hourly wage rate so that the imputation procedure has to be utilized for less than 20 percent of the sample in any given year.

There are essentially three components of the likelihood function, one for the unemployed, one for the employed paid the minimum wage, and one for those paid more than the minimum. Let us first consider the contribution of a sample member who is unemployed with an on-going unemployment spell of length t . Given being in the unemployed state, the likelihood of finding someone in an on-going spell of length t in the steady state is given by $r_u(t) = \tilde{F}_u(t)/E(t)$, where \tilde{F}_u is the population survivor function of unemployment durations and $E(t)$ is the population mean duration of unemployment spells. When the population unemployment spell duration distribution is of the negative exponential form, it is well-known that $r_u(t)$ is equal to the population density. Thus, given unemployment, the density associated with an on-going spell length of t is

$$f_u(t|u) = \lambda \tilde{G}(m) \exp(-\lambda \tilde{G}(m)t),$$

where the acceptance match value of m presumes the existence of a binding minimum wage (in the absence of one m is replaced by ρV_n in the above expression). Using standard ergodic results, the steady state probability of unemployment is given by

$$p(u) = \frac{\eta}{\eta + \lambda \tilde{G}(m)},$$

so that the joint likelihood of (t, u) is given by

$$f(t, u) = \frac{\eta \lambda \tilde{G}(m) \exp(-\lambda \tilde{G}(m)t)}{\eta + \lambda \tilde{G}(m)}.$$

Next we consider the likelihood contribution of an individual who is paid the minimum wage m . We will assume that whenever two or more population members are paid the minimum wage m the minimum wage is binding for their population type. Assuming that it is, conditional on employment the likelihood of being paid m is given by

$$p(w = m|e) = \frac{\tilde{G}(m) - \tilde{G}(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha})}{\tilde{G}(m)}.$$

Therefore the likelihood of observing $(w = m, e)$ is

$$p(w = m, e) = \frac{\lambda[\tilde{G}(m) - \tilde{G}(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha})]}{\eta + \lambda \tilde{G}(m)}.$$

The contribution of individuals paid more than the minimum wage is determined as follows. To be paid a wage $w > m$, it must be the case that the match value exceeds $(m - (1 - \alpha)\rho V_n(m))/\alpha$. Thus the likelihood of being a paid a wage w given employment and $w > m$ is

$$f(w|w > m, e) = \frac{\frac{1}{\alpha}g(\frac{w-(1-\alpha)\rho V_n(m)}{\alpha})}{\tilde{G}(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha})}.$$

Furthermore, the probability that a sample member is paid a wage greater than m given that she is employed is

$$p(w > m|e) = \frac{\tilde{G}(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha})}{\tilde{G}(m)},$$

so that the likelihood contribution for such an individual is given by

$$\begin{aligned} f(w, w > m, e) &= \frac{\frac{1}{\alpha}g(\frac{w-(1-\alpha)\rho V_n(m)}{\alpha})}{\tilde{G}(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha})} \frac{\tilde{G}(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha})}{\tilde{G}(m)} \frac{\lambda \tilde{G}(m)}{\eta + \lambda \tilde{G}(m)} \\ &= \frac{\frac{\lambda}{\alpha}g(\frac{w-(1-\alpha)\rho V_n(m)}{\alpha})}{\eta + \lambda \tilde{G}(m)}. \end{aligned}$$

The log likelihood for the parameters describing the labor market environment of a given population can be written as

$$\begin{aligned} \ln L &= N[\ln(\lambda) - \ln(\eta + \lambda \tilde{G}(m))] + N_U[\ln(\eta) + \tilde{G}(m)] \\ &\quad - \lambda \tilde{G}(m) \sum_{i \in U} t_i + N_M \ln(\tilde{G}(m) - \tilde{G}(\frac{m - (1 - \alpha)\theta^*}{\alpha})) \\ &\quad - N_H \ln(\alpha) + \sum_{i \in H} \ln(g(\frac{w_i - (1 - \alpha)\theta^*}{\alpha})), \end{aligned} \tag{13}$$

where N is the number of individuals in the sample, N_U is the number of unemployed, N_M is the number paid the minimum wage, N_H is the number paid more than the minimum wage, U is the

set of indices of sample members who are unemployed, H is the set of indices of sample members who are paid more than the minimum wage, and $\theta^* \equiv \rho V_n(m)$ is the “implicit” reservation wage in this population. Recall that G will be assumed to belong to a parametric family, so that it can be characterized in terms of a finite-dimensional parameter vector τ . Then the parameters that appear “directly” in the log likelihood are λ , η , τ , α , and θ^* . The value θ^* is not a primitive parameter per se, but rather is a scalar that is determined endogenously and is a function of all the parameters of the model, including ρ and b . For purposes of estimation, however, we shall treat it as a parameter of the model.¹⁸

4.1 Model Identification

We now present a discussion of the identification of the model using only CPS data. We begin by considering the case in which there is no binding minimum wage. After describing the identification problems faced in this case we will be in a better position to understand the situation in which a binding minimum wage is present.

Before we begin, let us review the identification results of FH, which are particularly relevant here. They demonstrated that the basic search model with only accepted wage and duration information is fundamentally underidentified, even in the absence of bargaining (i.e., $\alpha = 1$). In particular, they showed that a parametric assumption on G was required, and that only certain distributions with support on a subset of R_+ were identified even under parametric assumptions. In the case we consider here, that of parametric G with support given by R_+ , all of this class of distributions satisfy what they term the “recoverability condition.” Thus we need not consider this particular problem further.

They also demonstrated that the parameters (ρ, b) were not individually identified, essentially since they both enter the likelihood function only through the critical value θ^* . Because of this we can treat θ^* as a parameter to be estimated without loss of generality. When all parameter estimates have been obtained, the functional equation that determines θ^* may be used with point estimates of all identified parameters to “back out” the locus of estimates of (ρ, b) . This is the procedure followed here. Thus we do not discuss identification of ρ and b explicitly below, but instead consider identification of θ^* along with the other true primitive parameters of the model.

4.1.1 No Minimum Wage

We begin by writing the log likelihood function for this problem in a transparent manner:

$$\ln L = N \ln h_U - N \ln D + N_U \ln \eta - h_U \sum_{i \in U} t_i + \sum_{i \in E} \ln \frac{\frac{1}{\alpha} g\left(\frac{w_i - (1-\alpha)\theta^*}{\alpha}\right)}{\tilde{G}(\theta^*)}, \quad (14)$$

where $h_U \equiv \lambda \tilde{G}(\theta^*)$, $D \equiv \eta + h_U$, and E is the set of sample members employed at the survey date. Just as η is the hazard rate from the employment state, h_U is the hazard rate from the unemployment state; it is defined as the product of the rate of arrival of offers and the probability

¹⁸Treating θ^* as a parameter in the estimation does not imply that all of the behavioral restrictions are not being imposed. The model is inherently underidentified in the sense that θ^* is determined by ρ and b in addition to the other parameters that appear “directly” in [13]. Once we estimate θ^* , we can assume a value for ρ or b and then (uniquely) solve for the other. Conversely, we can directly estimate ρ or b after fixing the value of the other, and our estimate of the parameter along with the others in the likelihood function will uniquely determine θ^* . This last procedure is less attractive from a computational standpoint due to the necessity of imposing a complicated, implicit restriction on the parameter space so as to ensure that the (implied) value of θ^* be strictly less than m .

that an offer will be acceptable. Without auxiliary information to enable the identification of G , it will be impossible to disentangle the two components of h_U .

In discussing identification rigorously we will need to work with a class of matching distributions. For reasons that will become immediately clear, we will work with distributions with support R_+ that belong to a location-scale family.¹⁹ To be precise, the distribution $G(x; c, d)$ belongs to a location-scale family if and only if

$$G(x; c, d) = G_0\left(\frac{x - c}{d}\right),$$

where $c \in R$ and $d \in R_+$ are two constants and G_0 is a known function. Assuming that G is continuously differentiable on its support R_+ , we have

$$g(x; c, d) = \frac{1}{d}g_0\left(\frac{x - c}{d}\right)$$

as the probability density function of x . Many commonly used parametric distributions in economics belong to this class.

Given that G belongs to a location-scale family, we can rewrite the last term in [14] as follows. Note that

$$\frac{\frac{1}{\alpha}g\left(\frac{w_i - (1-\alpha)\theta^*}{\alpha}; c, d\right)}{\tilde{G}(\theta^*; c, d)} = \frac{\frac{1}{\alpha d}g_0\left(\frac{w_i - (1-\alpha)\theta^* - \alpha c}{\alpha d}\right)}{\tilde{G}_0\left(\frac{\theta^* - c}{d}\right)}.$$

Since $w^* = \theta^*$ when there is no binding minimum wage, we can write

$$w^* = \alpha\theta^* + (1 - \alpha)\theta^*,$$

or

$$\theta^* = \frac{w^* - (1 - \alpha)\theta^*}{\alpha}.$$

Substituting this expression into the denominator, we arrive at

$$\frac{\frac{1}{\alpha d}g_0\left(\frac{w_i - (1-\alpha)\theta^* - \alpha c}{\alpha d}\right)}{\tilde{G}_0\left(\frac{w^* - (1-\alpha)\theta^* - c}{\alpha d}\right)} = \frac{\frac{1}{\alpha d}g_0\left(\frac{w_i - (1-\alpha)\theta^* - \alpha c}{\alpha d}\right)}{\tilde{G}_0\left(\frac{w^* - (1-\alpha)\theta^* - \alpha c}{\alpha d}\right)}.$$

This is the p.d.f. of a random variable generated by a truncated location-scale distribution, with lower truncation point w^* . The "new" location parameter is

$$c' = (1 - \alpha)\theta^* + \alpha c$$

and the new scale parameter is

$$d' = \alpha d.$$

Since a super-consistent estimator for θ^* ($= w^*$) is available, the two parameters c' and d' will be a function of three unknown parameters: α , c , and d .

Let us begin with the easiest problem, that of estimating the parameters η and h_U . Taking the partial derivatives of $\ln L$ with respect to these two parameters, we obtain the following first order conditions

$$\begin{aligned} 0 &= -\frac{N}{D} + \frac{N_U}{\eta} \\ 0 &= \frac{N}{h_U} - \frac{N}{D} - \sum_{i \in U} t_i. \end{aligned}$$

¹⁹We could consider slightly more general classes of distributions, such as those that also possess a shape parameter, though the negative results of this section regarding the identifiability of α would only be reinforced.

From the first equation we get

$$\frac{\hat{\eta}}{D} = \frac{N_U}{N}, \quad (15)$$

while manipulation of the second equation yields

$$\begin{aligned} \frac{N(D - \hat{h}_U)}{\hat{h}_U D} &= \sum_{i \in U} t_i \\ \Rightarrow \frac{N\eta}{\hat{h}_U D} &= \sum_{i \in U} t_i. \end{aligned}$$

After substituting [15] for $\frac{\eta}{D}$ in the above expression, we get

$$\hat{h}_U = N_U / \sum_{i \in U} t_i.$$

Since $D = \eta + h_U$, we can now recover η directly as

$$\hat{\eta} = \frac{N_U^2 / \sum_{i \in U} t_i}{N_E}.$$

Note that the estimators of η and h_U are independent of the specification of G . Of course, the specification of G will be critical in “decomposing” h_U into the products of the terms λ and $\tilde{G}(\theta^*)$.

We now consider the more difficult problem, that of identifying the parameters (c, d, α, θ^*) . A (super) consistent estimator for θ^* is trivial to obtain, as originally noted by Flinn and Heckman (1982), and is given by

$$\hat{\theta}^* = \hat{w}^* = \min_{i \in E} \{w_i\}.$$

This estimator converges at rate N and therefore can be treated as a constant when it appears as an argument in any other estimator whose convergence rate is less than N . Since all other estimators in our model converge at rate \sqrt{N} , we can simply treat $\hat{\theta}^*$ as equal to θ^* when deriving the \sqrt{N} asymptotic sampling distributions for these estimators.

Now we are left with the determination of the three unknown parameters that appear only in the wage distribution associated with the employed sample. For any given specification of G_0 , and in fact even when G_0 is treated as unknown, there exist a number of consistent estimators of the location and scale parameters c' and d' . Thus we assume with no loss of generality that consistent estimators of these parameters exist, and denote them by \hat{c}' and \hat{d}' . With no prior information concerning c , d , or α , no further progress toward identification of the individual parameters can be made. This is the fundamental underidentification problem associated with the asymmetric Nash bargaining model.

Proposition 2 *Let G belong to a location-scale family with parameters (c, d) . Then a necessary condition for identification of (c, d, α) is that one of the parameters be expressible as a known, invertible function of the other two.*

Proof: If there exist no restrictions relating (c, d, α) , then consistent estimators of c' and d' are insufficient to determine all three parameters. Suppose that there exists one exact restriction on the parameter vector (c, d, α) of the form $0 = r(c, d, \alpha)$, and without loss of generality let us use this

restriction to determine a value of α given (c, d) , or $\alpha = \alpha(c, d)$. Then with consistent estimators for c' and d' , we can solve for

$$\begin{aligned}\widehat{c}' &= (1 - \alpha(c, d))\widehat{\theta}^* + \alpha(c, d)c \\ \widehat{d}' &= \alpha(c, d)d,\end{aligned}$$

a system of two equations in two unknowns. Depending on the form of the function $\alpha(c, d)$ and the values of \widehat{c}' and \widehat{d}' there may exist no solution, a unique solution, or multiple solutions. We will say that the model is just identified if

$$\begin{aligned}c' &= (1 - \alpha(c, d))\theta^* + \alpha(c, d)c \\ d' &= \alpha(c, d)d\end{aligned}$$

has exactly one solution for all values of (c, d) in the parameter space. Note that in this case we have replaced all estimated quantities with their limiting values, so that this should be interpreted as a large sample identification condition.

The most common restriction will be a direct one on one of the parameters c or d . For example, if $c = 0$ we have

$$\begin{aligned}\widehat{c}' &= (1 - \alpha)\widehat{\theta}^* \\ \widehat{d}' &= \alpha d.\end{aligned}$$

This is a two equation system in two unknowns, and yields estimators for α and d of

$$\begin{aligned}\widehat{\alpha} &= 1 - \frac{\widehat{c}'}{\widehat{\theta}^*} \\ \widehat{d} &= \widehat{d}'/\widehat{\alpha}.\end{aligned}$$

While in small samples it is possible for these estimators to yield values outside of the parameter space, the likelihood of such an event will be a decreasing function of sample size.

We now establish why a restriction relating (c, d, α) is not sufficient to ensure identification. Assume that G is negative exponential with support R_+ . In this case the location parameter is 0 and the scale parameter $d > 0$ completely characterizes the distribution. The likelihood of wage observation w_i is given by

$$\begin{aligned}\frac{\frac{1}{\alpha d} g_0\left(\frac{w_i - (1-\alpha)\theta^*}{\alpha d}\right)}{\widetilde{G}_0\left(\frac{w^* - (1-\alpha)\theta^*}{\alpha d}\right)} &= \frac{\frac{1}{\alpha d} \exp\left(-\frac{w_i - (1-\alpha)\theta^*}{\alpha d}\right)}{\exp\left(-\frac{w^* - (1-\alpha)\theta^*}{\alpha d}\right)} \\ &= \frac{1}{\alpha d} \exp\left(-\frac{w_i - w^*}{\alpha d}\right).\end{aligned}$$

Thus the wage distribution can only identify αd , not the two components individually. This is the only continuous distribution with support R_+ that we have found that is not identified under a restriction on c or d so that it may be a pathological case. ■

The fact that a restriction on c or d may be sufficient to enable identification of the other parameter characterizing G and the bargaining power parameter α must not be viewed with too much optimism. While the knowledge that $c = 0$, probably the most common type of restriction that will be imposed in this case, does lead to “theoretical” identification in large samples (as long as G is not negative exponential), the small sample properties of the estimator tend to be poor. This last conclusion comes from conducting Monte Carlo experiments with a variety of scale

distributions. If one fixes the scale parameter (d), instead, the performance of an estimator for α tends to be better in reasonably-sized samples. Unfortunately, it is rarely the case that a researcher will feel comfortable fixing the value of a scale parameter *a priori*. Thus this case is not particularly germane for empirical researchers, and we are left with the impression that under most “reasonable” *a priori* restrictions on the parameters (c, d), identification of the parameter α will remain tentative at best.

The fundamental reason for the identification problem is not hard to understand intuitively. From payoff data from one side of the market we are attempting to simultaneously determine the total size of the “pie” and the share received by the worker. While incredibly restrictive assumptions on the form of the underlying matching distribution may make α identifiable in theory, these restrictions are probably not believable and even imposing them is likely to produce an estimator with a large (though not infinite) standard error. We next consider whether having CPS information from a labor market with a binding minimum wage alleviates the identification problem to any extent.

4.1.2 Binding Minimum Wage

In this case we can write the log likelihood function most transparently as

$$\begin{aligned} \ln L = & N \ln h_U - N \ln D + N_U \ln \eta - h_U \sum_{i \in U} t_i \\ & + N_M \ln \left(1 - \frac{\tilde{G}(\frac{m-(1-\alpha)\theta^*}{\alpha})}{\tilde{G}(m)} \right) + \sum_{i \in H} \ln \frac{\frac{1}{\alpha} g(\frac{w_i-(1-\alpha)\theta^*}{\alpha})}{\tilde{G}(m)}, \end{aligned} \quad (16)$$

where $h_U = \lambda \tilde{G}(m)$. The critical acceptance match value is now known to be equal to m , however we have to estimate the “implicit” critical match value θ^* along with the other primitive parameters of the model. We shall consider the identification of the parameters ($\eta, h_U, c, d, \alpha, \theta^*$).

With respect to the rate parameters η and h_U no new issues arise. The maximum likelihood estimators for these two parameters are exactly as were given above for the case of no binding m .

Does the mass point at m provide enough additional information to enable identification of α ? The answer to this question is no in general, and this is primarily due to the necessity of estimating θ^* simultaneously with the other parameters, something we did not have to do when there was no binding minimum wage. Let us begin by rewriting the terms involving G in their location-scale form. For those paid the minimum wage, the term involving G is written as

$$1 - \frac{\tilde{G}(\frac{m-(1-\alpha)\theta^*}{\alpha})}{\tilde{G}(m)} = 1 - \frac{\tilde{G}_0(\frac{m-c'}{d'})}{\tilde{G}_0(\frac{m-c}{d})},$$

where $c' = (1-\alpha)\theta^* + \alpha c$ and $d' = \alpha d$. For an individual i who belongs to set H , their log likelihood contribution involving G is written as

$$\ln \frac{\frac{1}{d'} g_0(\frac{w_i-c'}{d'})}{\tilde{G}_0(\frac{m-c}{d})}.$$

The fact that (c, d) and (c', d') appear independently in these expressions may bring false optimism. If it were possible to consistently estimate these four terms, then we would be able to consistently estimate α and θ^* since

$$\begin{aligned} \alpha &= d'/d \\ \theta^* &= \frac{c' - \frac{d'}{d}c}{1 - \frac{d'}{d}}. \end{aligned} \quad (17)$$

Thus all terms are clearly identified given consistent estimates of those four parameters. Unfortunately, it is not possible to consistently estimate c and d from these expressions. These terms only appear in the expression $\tilde{G}_0(\frac{m-c}{d})$, which is the probability of encountering an acceptable match. Even given knowledge of this probability and the minimum wage m , there are in general a continuum of values of (c, d) which can produce the same value of the argument $(m - c)/d$. Thus (c, d) is not uniquely identified. Given this lack of uniqueness, we cannot uniquely determine α and θ^* from [17]. As was the case in the last subsection, identification may be obtainable if we are willing to impose *a priori* restrictions on the parameters (c, d) . For good reason, most analysts will be hesitant to resort to such devices.

In the empirical results reported below we have for the most part assumed that G is lognormal. This is in keeping with much of the literature in empirical search or even earnings function estimation. The lognormal is not a location-scale distribution and thus the “exact” identification analysis just conducted for that family of distributions does not strictly apply.²⁰ In practical estimation situations the departures from the truncated location-scale characterization of the empirical distribution of accepted wages are not sufficiently great to enable identification of α .²¹ Thus sample information from one side of the market can not be expected to enable identification of both the total surplus distribution and the sharing rule. This is the larger point to take away from the identification analysis.

4.2 Some Additional Identification Devices

In empirical work utilizing a Nash-bargaining structure, it is common to assume symmetric bargaining, i.e., $\alpha = .5$ (e.g., Flinn and Heckman (1982), Eckstein and Wolpin (1993), and Del Boca and Flinn (1994)). Such a “normalization” is imposed both because the difficulty of identifying the parameter α has been apparent for some time, and also due to modeling considerations. In terms of the second point, recall that in our formulation of the behavioral model we assumed that both workers and firms shared a common effective discount rate. If we were to ignore the fact that the searcher has an “outside option” when bargaining with the firm, i.e., the value of continued search, bargaining over the match surplus could be placed in the framework of a Rubinstein (1982) alternating-offers game. It is well known that in the case of equal discount rates, the solution is equivalent to the symmetric Nash bargaining solution.

As the first step in the empirical analysis we simply restrict $\alpha = .5$ for every subpopulation considered. Besides conforming with common practice, this case serves as something of a benchmark. We were also interested in determining whether positive “optimal” minimum wages were ever indicated given this normalization. Roughly speaking, we know that the likelihood that binding minimum wages will improve the welfare of labor market participants is decreasing in α . The empirical question we address is whether the restriction that $\alpha = .5$ effectively rules out a welfare-improvement role for minimum wages. Using the samples below, this indeed appears to be the case.

It is also possible to obtain identification of some parameters by redefining the parameter space with respect to observable characteristics of individuals. From the previous subsection, we know that if we sort sample members into a small number of observationally-distinguishable classes,

²⁰Given that the location parameter is set to 0, the lognormal distribution is characterized in terms of its scale parameter μ and shape parameter σ . Thus there are two unknown parameters of the distribution, as in the location-scale case. The difference is that sample moments above the second order convey information on these parameters, which is not the case for location-scale distributions. The content of this sample information is not sufficiently great to enable identification in practical situations, for the reasons described in the subsequent footnote.

²¹Whether in Monte Carlo experiments or using CPS data, the log likelihood function invariably is monotone in α , resulting in maximum likelihood estimates of this parameter that converge to 0 or 1.

identification of the labor market parameters for each class will not be possible. However, if we are willing to restrict the values of certain subsets of the primitive parameters to be constant across at least some of the classes, it will in general be possible to obtain identification of the reformulated model. The parameters that will be identified are those that are taken to be constant across at least some groups and those that are considered to be group-specific. Since our original model did not entail any across-group restrictions, this would have to be considered a nonequivalent reparameterization of the original model. It seems difficult to imagine that a researcher could specify a subset of primitive parameters that was common to a number of subpopulations in anything other than an arbitrary manner. We have chosen not to estimate the model under these types of “exclusion restrictions” because we could not find any reasonable ones to impose.

We have also attempted estimation of a subset of parameters by imposing an intertemporal restriction. A different nominal minimum wage was in place in March of 1996, 1997, and 1998. We allow for “approximately” neutral price changes across the years, and assume that changes in nominal minimum wages are unanticipated. Thus we view the wage distributions in each of these three years as being determined by a time invariant “real” match value distribution G , a time invariant bargaining power parameter α , and a year-specific implicit reservation wage, θ_t^* . At the imposition of a new minimum wage, all existing contracts are rebargained instantaneously, so that the current wage distribution reflects the influence of the contemporaneous θ_t^* and the existing “real” minimum wage m_t . We only use wage distribution information when imposing these restrictions, since even if agents continue to use stationary rules the contemporaneous probability of employment and durations of unemployed search will reflect past environmental conditions.

4.3 Use of Demand Side Information to Estimate α

Our analysis of minimum wages takes place in the context of a simple equilibrium model of the labor market. In the end it is not terribly surprising that identification of an equilibrium model using data from outcomes for only one side of the market is a daunting task. In this section we explore identification strategies that utilize both the CPS data and some information readily available from the financial statements of publicly held corporations.

The datum we utilize from the firm side of the market is labor’s share of revenue at some “representative” firm j , which we denote by $\pi_W^j(m)$. Now from the model

$$\pi_W^j(m) = \frac{\sum_{i \in E^j} w(\theta_i; m, \rho V_n(m))}{\sum_{i \in E^j} \theta_i},$$

where E^j is the set of employees of firm j at some arbitrary point in time (where it is assumed that $E^j \neq \emptyset$). Dividing both by the numerator and denominator by the number of employees at the firm we get the ratio of average wages to average productivity,

$$\pi_W^j(m) = \frac{\bar{w}^j}{\bar{\theta}^j}.$$

If the number of employees at firm j is large enough for the law of large numbers to apply, then

$$\pi_W^j(m) \doteq \frac{mp(w = m) + \int_{m - \frac{(1-\alpha)\rho V_n(m)}{\alpha}}^m (\alpha\theta + (1-\alpha)\rho V_n(m)) dG(\theta)/\tilde{G}(m)}{\int_m \theta dG(\theta)/\tilde{G}(m)}. \quad (18)$$

If there exists no particular firm j with enough employees for the law of large numbers to be applicable, it must be the case that the average wage share of revenues across all firms in the market,

$\sum_j \pi_W^j(m)/K$, assumes a value arbitrarily close to the right hand side of (18) for a sufficiently large population of firms.

We use (18) to form an estimator of α jointly with γ , where γ denotes all other model parameters. In particular, we recognize that the wage share of firm revenues is a function of all of the primitive parameters of the model, including α . The idea is to form a concentrated likelihood function in which an estimator for α , written as a function of the parameter vector Γ and an observed value of $\pi_W^j(m)$, is substituted into the “unconditional” likelihood function. From (18) we see that any $\alpha \in (0, 1)$ and $\gamma \in \Gamma$ imply a unique value of the wage share of revenues, which we will now call simply π , thus

$$\pi = \pi(\alpha; \gamma, m).$$

The function π has the following features (for all m):

$$\begin{aligned} \lim_{\alpha \rightarrow 1} \pi(\alpha; \gamma, m) &= 1, \\ \lim_{\alpha \rightarrow 0} \pi(\alpha; \gamma, m) &= \frac{m}{E(\theta|\theta \geq m)} \in (0, 1], \end{aligned}$$

and

$$\frac{\partial \pi(\alpha; \gamma, m)}{\partial \alpha} = \frac{\int_{m - \frac{(1-\alpha)\rho V_n(m)}{\alpha}}^m (\theta - \rho V_n(m)) dG(\theta)}{\int_m \theta dG(\theta)} \in (0, 1).$$

Then $\pi(\alpha; \gamma, m) : [0, 1] \rightarrow [\frac{m}{E(\theta|\theta \geq m)}, 1]$. Furthermore, π is monotone in α . Let the observed value of π be denoted by $\tilde{\pi}$. Then for $\tilde{\pi} \in (\frac{m}{E(\theta|\theta \geq m)}, 1)$ there exists a unique value $\alpha^*(\tilde{\pi}; \gamma, m)$ such that

$$\tilde{\pi} - \pi(\alpha^*(\tilde{\pi}, \gamma, m); \gamma, m) = 0.$$

Thus the condition required for an interior solution for α to exist is that

$$\begin{aligned} \tilde{\pi} &> \frac{m}{E(\theta|\theta \geq m)} \\ \Rightarrow E(\theta|\theta \geq m) &> \frac{m}{\tilde{\pi}}. \end{aligned}$$

This last condition, which involves $E(\theta|\theta \geq m)$, imposes a restriction on the parameter space Γ since a subset of the parameters belonging to γ are those that characterize the distribution of θ , and a subset of these parameters appear in the conditional expectation function $E(\theta|\theta \geq m)$. We have another condition that the estimated parameters must implicitly satisfy, of course, which is that $\rho V_n(m) < m$. We impose this condition directly by working with the parameterization

$$\tilde{\gamma} \equiv (\rho V_n(m), \lambda, \eta, G).$$

Under a lognormality assumption regarding G , the distribution of θ is completely characterized by the two parameters (μ, σ^2) , so that the parameter space for $\tilde{\gamma}$ is given by

$$\tilde{\Gamma}(m) = (-\infty, m) \times R_+^2 \times R \times R_+,$$

since $\mu \in R$ and $\sigma^2 \in R_+$. When we impose the additional condition that $E(\theta|\theta \geq m) > m/\tilde{\pi}$, under the lognormality assumption this implies that

$$\exp(\mu + .5\sigma^2) \frac{\tilde{\Phi}(z(m) - \sigma)}{\tilde{\Phi}(z(m))} > \frac{m}{\tilde{\pi}}, \quad (19)$$

where $z(m) \equiv (\ln(m) - \mu)/\sigma$.²² Given values of m and $\tilde{\pi}$, let the set of values of (μ, σ^2) that satisfy (19) be denoted by $\Omega(m, \tilde{\pi})$. Finally, define the parameter space

$$\Gamma'(m, \tilde{\pi}) = \Gamma(m) \cap \Omega(m, \tilde{\pi}).$$

We form a concentrated likelihood function by substituting our estimator for α into the likelihood (13), which gives us

$$\tilde{L}(\tilde{\gamma}; \tilde{\pi}, m) = L(\tilde{\gamma}, \alpha^*(\tilde{\pi}, \tilde{\gamma}, m)).$$

We have the following result.

Proposition 3 *Assume that $\tilde{\gamma}^0 \in \text{int}(\Gamma'(m, \tilde{\pi}))$ and $\alpha^0 \in (0, 1)$, where $(\tilde{\gamma}^0, \alpha^0)$ are the true population parameter values. Then the maximum likelihood estimators*

$$\begin{aligned}\hat{\gamma} &= \arg \max_{\tilde{\gamma} \in \Gamma'(m, \tilde{\pi})} \ln \tilde{L}(\tilde{\gamma}; m, \tilde{\pi}) \\ \hat{\alpha} &= \alpha^*(\tilde{\pi}, \hat{\gamma}, m)\end{aligned}$$

converge in probability to $\tilde{\gamma}^0$ and α^0 respectively. The asymptotic distribution of $\sqrt{N}(\hat{\gamma} - \tilde{\gamma}^0)$ is normal with mean 0 and covariance matrix $\Sigma_{\tilde{\gamma}}$, which is of full rank. The asymptotic distribution of $\sqrt{N}(\hat{\alpha} - \alpha^0)$ is normal with mean 0 and variance $\sigma_{\hat{\alpha}}^2(\Sigma_{\tilde{\gamma}})$.

Proof: The unconcentrated likelihood function $L(\tilde{\gamma}, \alpha)$ satisfies standard regularity conditions for $\tilde{\gamma}^0 \in \text{int}(\tilde{\Gamma}(m))$ and $\alpha^0 \in (0, 1)$. In particular, the support of the distribution of the data is not a function of $\tilde{\gamma}^0$ and α^0 , though it is a function of the known constant m . The likelihood function is continuously differentiable in both $\tilde{\gamma}$ and α . The implicit function $\alpha^*(\tilde{\pi}, \tilde{\gamma}, m)$ is continuously differentiable in $\tilde{\gamma}$, so the function $\ln L(\tilde{\gamma}, \alpha^*(\tilde{\pi}, \tilde{\gamma}, m)) \equiv \ln \tilde{L}(\tilde{\gamma}; \tilde{\pi}, m)$ is continuously differentiable in $\tilde{\gamma}$ on the space $\Gamma'(m, \tilde{\pi})$. Continuous differentiability of α^* is sufficient for $\text{plim}_{\tilde{\gamma} \rightarrow \tilde{\gamma}^0} \alpha^*(\tilde{\pi}, \tilde{\gamma}, m) = \alpha^0$, and differentiability of $L(\tilde{\gamma}, \alpha)$ in both arguments is sufficient for $\text{plim}_{\alpha \rightarrow \alpha^0} \text{plim}_{N \rightarrow \infty} \arg \max_{\tilde{\gamma} \in \tilde{\Gamma}(m)} \ln L(\tilde{\gamma}, \alpha) = \tilde{\gamma}^0$. Then $\text{plim}_{N \rightarrow \infty} \alpha^*(\tilde{\pi}, \hat{\gamma}, m)$.

Since the concentrated log likelihood function $\ln \tilde{L}(\tilde{\gamma}; \tilde{\pi}, m)$ satisfies all regularity conditions for asymptotic normality, the asymptotic distribution of $\sqrt{N}(\hat{\gamma} - \tilde{\gamma}^0)$ is $N(0, \Sigma_{\tilde{\gamma}})$, where

$$\Sigma_{\tilde{\gamma}} = \left(\frac{\partial^2 \ln \tilde{L}(\tilde{\gamma}; \tilde{\pi}, m)}{\partial \tilde{\gamma} \partial \tilde{\gamma}'} \bigg|_{\tilde{\gamma}=\hat{\gamma}} \right)^{-1}.$$

The asymptotic covariance matrix of $\hat{\alpha}$ is derived using the delta method. The variance of $\hat{\alpha}$ is

$$\text{Var}(\hat{\alpha}) = \frac{\partial \alpha^*(\tilde{\pi}, \tilde{\gamma}, m)}{\partial \tilde{\gamma}} \bigg|_{\tilde{\gamma}=\hat{\gamma}}' \Sigma_{\tilde{\gamma}} \frac{\partial \alpha^*(\tilde{\pi}, \tilde{\gamma}, m)}{\partial \tilde{\gamma}} \bigg|_{\tilde{\gamma}=\hat{\gamma}}.$$

■

As we shall see below, the use of wage share information from a firm or firms in conjunction with the CPS data allows us to obtain precise and credible estimates of all key model parameters. We do face a problem with this information when we attempt to estimate the model for subpopulations, however. Since we do not have wage and productivity information disaggregated by subpopulation, we are forced to assume that this ratio is the same across all subpopulation members.²³ This is a strong assumption that probably largely accounts for the similarities we observe in the estimates of α across subpopulations.

²²See Johnson and Kotz (1970, page 129).

²³Another alternative would be to collect aggregate wage share information by firm, and to estimate the subpopulation-specific share using variability in subpopulation employment rates across firms. The use of such a procedure raises questions regarding the source of the variability in subpopulation employment rates across firms or industrial sectors and also faces the problem that few firms report wages as a separate item in their CSI.

4.4 Estimation of Demand Side Parameters

In this section we discuss the identification and estimation of demand side parameters, which in the case of this highly-stylized model consists of estimating the parameters ψ , the cost of a vacancy, and q , the matching function. We show below that in order to perform policy experiments we also must have a consistent estimator of the distribution of “outside options,” that is, the parameter ζ that indexes the distribution function Q . Identification and estimation of the parameters ζ , q , and ψ builds on the consistent estimators of the parameters we have already considered, namely $\lambda, \eta, G, b, \alpha$. We begin the section by discussing identification of these parameters when only cross-sectional CPS information is available. The second subsection contains a discussion of identification when the CPS information from the months spanning the change in minimum wage rates is utilized. In our case, this information is from the September and the October 1996 CPS.

4.4.1 Cross-Sectional Information Only

Using the maximum likelihood estimator developed above, using cross-sectional CPS information and the labor share of firm revenues, we are able to estimate $\rho V_n(m)$.²⁴ Then we have the following result.

Proposition 4 *ζ is estimable if ρV_o is continuously distributed with c.d.f. R , ζ is a scalar, $R(x, \zeta)$ is monotone in ζ for all x in the support of R , and a consistent estimate of the labor force participation rate is available.*

Proof: Let the consistent estimate of the labor force participation rate be \hat{l} . In the population, we have

$$l = R(\rho V_n(m); \zeta).$$

Since R is monotone in ζ for all x in the support of R , we can write the inverse function

$$\zeta = R^{-1}(\rho V_n(m), l), \quad \rho V_n(m) \in \text{Supp}(R), \quad l \in (0, 1).$$

Because R^{-1} is a known, continuous function, a consistent estimator of ζ is

$$\hat{\zeta} = R^{-1}(\widehat{\rho V_n(m)}, \hat{l})$$

by the invariance property of maximum likelihood estimators. ■

We now move on to consider the estimation of q and ψ . First consider estimation of the matching function q . We have a consistent estimator of the contact rate from the perspective of searchers on the supply side, which is $\hat{\lambda}$. As was the case above, we assume that q is parametric and is characterized by a finite-dimensional parameter vector ω . Then we have

$$\lambda(ul, v; \omega) = q\left(\frac{ul}{v}; \omega\right) \frac{v}{ul}, \tag{20a}$$

where

$$ul = \frac{\eta l}{\eta + \tilde{G}(r)\lambda(ul, v; \omega)}.$$

Now note that consistent estimates of η, λ, l, u are available from the CPS data augmented with (other CPS) data on the labor market participation rate l . If data on the steady state vacancy rate

²⁴Under an assumption regarding the value of the discount rate ρ we are able to infer the value of b - hence its inclusion in the list of estimable parameters in the previous paragraph.

is available, v , then a consistent estimate of ω can be obtained solely by using (20a) in the case that ω is scalar.

Knowledge of the function $q(\cdot; \omega)$ is sufficient to enable estimation of the cost of vacancy parameter ψ . Recall that the free entry condition can be written

$$\psi = q(ul/v; \omega) \tilde{G}(r) J(\rho V_n(m), m, G, \rho, \eta, \alpha), \quad (21)$$

where J is the expected value of a filled job. All of the arguments in J are either directly observable (m), are assumed known (ρ), or can be consistently estimated from the CPS data augmented with the labor share information. Furthermore, if ul is estimable, v is observed, and ω known or consistently estimated, then all terms of the expression on the right hand side of (21) can be consistently estimated, thereby directly producing a consistent estimator of ψ .

Unfortunately, information on v cannot be assumed to be (directly) available. Empirical studies of the matching function, particularly those conducted using U.S. data, typically use a vacancy index constructed from help wanted advertising (see Petrongolo and Pissarides (2001) for a survey of these studies). Without information on v , the parameter ω cannot be consistently estimated and therefore ψ cannot be consistently estimated either. With cross-sectional CPS information and a labor share measure, ψ can only be estimated under the following stringent conditions.

Proposition 5 *Augmented cross-sectional CPS information can be used to consistently estimate ψ if and only if the matching function contains no unknown parameters and is monotone in κ .*

Proof: We only demonstrate sufficiency since the previous discussion makes necessity clear. Since a consistent estimate of ul is available, if $q(\cdot; \omega)$ is known then (20a) can be used to recover v given that q is monotone in its sole argument κ (which is monotone in v). Given a consistent estimator of v , and a known ω , all of the terms on the right hand side of (21) can be consistently estimated as well as their product. ■

In summary, using cross-sectional information from the CPS and labor share data we can recover all of the parameters required to perform policy evaluation under the assumption of endogenous contact rates only if we assume that the matching function contains no unknown parameters. This stringent assumption is required due to the absence of credible measures of vacancy rates.

4.4.2 Multiple CPS Cross-Sections

We now consider identification issues when using the CPS Outgoing Rotation Group samples from two months, $t < t'$, in which we assume a steady state equilibrium (approximately) holds and where the minimum wages are different. Let the earlier minimum wage be less than the later one, and denote the minimum wage in period t by m_t . All primitive parameters are assumed to be identical in the two months.

In terms of the matching function, we limit attention to the case in which it is Cobb-Douglas, in particular,

$$\begin{aligned} M(ul, v; \omega) &= (ul)^\omega v^{1-\omega}, \quad \omega \in (0, 1), \\ \Rightarrow q\left(\frac{ul}{v}; \omega\right) &= \left(\frac{ul}{v}\right)^\omega. \end{aligned}$$

Pragmatically speaking, this is not a severe restriction since M is assumed to be CRS and because we know that it will be difficult to precisely estimate more than one parameter characterizing M .

Under the assumption that the cost of a vacancy is a primitive parameter, for an estimator of ω to be permissible requires that it assume a value in the open unit interval. The required condition,

stated in terms of a restriction on functions of consistent estimators of model parameters, as follows.

Condition C: Let $\hat{\lambda}_s$, \hat{J}_s , and $\widehat{\tilde{G}(m_s)}$ be consistent estimators of the rate of arrival of contacts, the value of expected value of a filled job, and the probability of drawing a match value greater than the minimum wage, respectively, in the equilibrium associated with minimum wage m_s . Then two estimated equilibria are coherent if

$$\hat{\lambda}_t^{\frac{-\omega}{1-\omega}} \widehat{\tilde{G}(m_t)} \hat{J}_t = \hat{\lambda}_{t'}^{\frac{-\omega}{1-\omega}} \widehat{\tilde{G}(m_{t'})} \hat{J}_{t'}$$

for some $\omega \in (0, 1)$.

Condition C is required if our estimates of the primitive parameters of the model can be used to form an estimator of the Cobb-Douglas parameter ω that lies in the parameter space $(0, 1)$. This condition is a minimal one in the sense that there are other restrictions on parameters that are implied by the model that are not imposed or checked in forming estimators of the demand side parameters. Model estimates may violate Condition C for a variety of reasons. Perhaps most obviously, even given the correct specification of all other model components (an unlikely event), we may have misspecified the matching function M . Another problem is that the model structure forces us to consider equilibria not too temporally distant. Anticipation of the impending minimum wage change after time t could have led the economy out of a steady state equilibrium at time t . At the same time, the economy may not have reached its new steady state equilibrium at the time of the next measurement, t' . As we will see below, one of the greatest impediments to the satisfaction of Condition C is the small changes in the minimum wage rates and the small proportion of individuals impacted.

Proposition 6 *Given access to two equilibrium outcomes, the Cobb-Douglas parameter ω and the vacancy cost parameter ψ are estimable if Condition C is satisfied.*

Proof: From the augmented CPS data we have consistent estimates of λ_t within each equilibrium, as well as having consistent estimates of $(ul)_t$. Under the Cobb-Douglas assumption we can write

$$\begin{aligned} \lambda_s &= \left(\frac{(ul)_s}{v_s} \right)^{\omega-1} \\ \Rightarrow v_s &= \lambda_s^{\frac{1}{1-\omega}} (ul)_s, \quad s = t, t'. \end{aligned}$$

Substituting this expression into the free entry condition in period s , we have

$$\psi = \lambda_s^{\frac{-\omega}{1-\omega}} \tilde{G}(m_s) J_s, \tag{22}$$

where J_s is the equilibrium value of a filled vacancy under m_s . Then we have

$$\lambda_t^{\frac{-\omega}{1-\omega}} \tilde{G}(m_t) J_t = \lambda_{t'}^{\frac{-\omega}{1-\omega}} \tilde{G}(m_{t'}) J_{t'}$$

at the two sets of equilibrium values. From this we have

$$\omega = \frac{A_{t,t'}}{1 + A_{t,t'}}$$

where

$$A_{t,t'} \equiv \frac{\ln \tilde{G}(m_{t'}) - \ln \tilde{G}(m_t) + \ln J_{t'} - \ln J_t}{\ln \lambda_{t'} - \ln \lambda_t}. \quad (23)$$

Define $\hat{A}_{t,t'}$ as the estimator of $A_{t,t'}$ obtained by replacing all of the quantities on the right hand side of (23) with consistent estimators of them. Under Condition C, the estimator $\hat{A}_{t,t'} > 0 \Rightarrow \hat{\omega} \equiv \hat{A}_{t,t'}/(1 + \hat{A}_{t,t'}) \in (0, 1)$ as required. Given this permissible and consistent estimator of ω , we use the consistent estimators of the relevant equilibrium values as well as the consistent estimate of ω , $\hat{\omega}$, to form the consistent estimator

$$\hat{\psi} = \hat{\lambda}_s^{\frac{-\omega}{1-\omega}} \widehat{\tilde{G}(m_s)} \hat{J}_s, \quad s = t \text{ or } t',$$

since $\hat{\psi}$ is identical whether t or t' values are used. ■

To summarize the results of this section, we have shown that when we have data from two CPS surveys in which data outcomes are generated from steady state equilibrium behavior associated with two values of the minimum wage, a Cobb-Douglas matching function parameter and the cost of a vacancy can be consistently estimated if estimates of a function of other primitive parameters at the two equilibrium values satisfy a condition that restricts the estimator of the Cobb-Douglas parameter to the unit interval. Though we can never expect the observations in the two CPS surveys to be exactly generated by two distinct steady state equilibria, the hope is that the approximation may be good enough to yield a credible estimate of the matching function parameter.

5 Empirical Results

The data used contain information on 16 to 24 year olds [inclusive] from the Outgoing Rotation Group (ORG) monthly Current Population Survey (CPS) samples. We focused attention on this age group since minimum wage workers are disproportionately young; our feeling was that if minimum wage effects on welfare were determined to be small for this group of participants, they are likely to be even more insignificant for older individuals. The fact that the sample is restricted to this age group should be borne in mind when interpreting the parameter estimates, and particularly when the welfare exercise results are reported.

The months used in the analysis are September, 1996; February and August 1997; and January, 1998. As we made clear in the previous section, identification of matching function parameters clearly depends on our having access to CPS data at more than two points in time. In September 1997 we assume that the market was in a steady state equilibrium at the minimum wage \$4.25 which was due to expire at the end of the month. When moving to the new steady state equilibrium associated with the minimum wage of \$4.75, there will be some transitional dynamics. To avoid capturing these effects, we will perform the “pooled” estimation exercises using data from 5 months after the change, when we hope the market has nearly reached its new steady state equilibrium. We cannot allow for even more separation in time between the “before and after” observations since (1) the minimum wage changes once again on September 1, 1997 and (2) the model specification allows for no changes in the price level (the change in the Consumer Price Index between September 1996 and February 1997 was about 1.1 percent).

To examine the sensitivity of our estimates to the particular months chosen, we reestimate the model when the subsequent minimum wage change occurs at the beginning of September, 1997. Unfortunately, the month before the change is August, a period when many students find temporary employment. This phenomenon is particularly at odds with the steady state equilibrium basis of our econometric model. The post-change month used in the pooled estimation exercise is

January 1998, which also may exhibit some transitory labor market behavior in conjunction with the holiday period. For these reasons, we will not devote as much attention to the estimates generated using these months. Nevertheless, it is encouraging to note that the estimates of most primitive parameters are stable across these two periods of minimum wage change.

Before discussing model estimates it is necessary to point out a few restrictions that were imposed on the sample information. For reasons cited above, classical measurement error in wages could not be introduced. That means that the model is not consistent with observed wages below the minimum wage. We dealt with this problem in two different ways. The first method involved rounding all wages below the minimum up to the minimum wage, which adds around 0.05 of probability mass at m . Under the second method, observations with w less than m were simply deleted from the sample. We found that estimates of the primitive parameters did not vary much across the sample information modified in these two ways, and report estimates from the first method.

Estimates of primitive parameters, and particularly the bargaining power parameter α , were found to be highly sensitive to outliers in the wage data. As a result, we rounded all wage observations greater than \$30 per hour down to \$30. This affected only about 10 cases in any given month. No other modifications of duration or wage information were made.

Our first attempt to estimate values of primitive parameters imposes a variety of normalizations (i.e., identifying restrictions) on the model. While the model is theoretically identified under any of the distributional assumptions we have made, as we will see below identification is tenuous at best. In Table 2 we report estimates of a subset of parameters of the complete model assuming that the matching distribution is lognormal, Gamma, or χ^2 (the χ^2 is a Gamma distribution with 2 degrees of freedom). Because the χ^2 is a special case of a Gamma, we also report estimates from a specification in which G is assumed to be χ^2 but α is estimated as a free parameter. In all other specifications reported in Table 2 we have restricted α to equal 0.5 - the symmetric Nash bargaining case.

Table 2 contains model estimates from single-month CPS samples in the months prior to the minimum wage changes to \$4.75 and \$5.15. The top panel contains estimates from the ORG September 1996 sample of 16-24 year olds. The first column contains estimates from the symmetric Nash bargaining lognormal matching distribution specification. For this estimation exercise we have pooled all observations (that is, no matter what their differences in terms of observable characteristics) and assumed that all sample members operate within a homogeneous environment. Estimates in the top panel indicate that the average \ln match value draw in the population is 2.186 with a standard deviation of 0.511 (the implied mean of θ in levels is 10.141). The average time between contacts for searchers is $1/.316$, slightly more than 3 months. Since the acceptance value for a match draw under a binding minimum wage is m , the estimated average length of an unemployment spell is $1/(\lambda\widehat{G}(m)) = 1/.293 = 3.417$ months, which is identical to the sample average. The estimated dissolution rate of jobs is 0.031, implying that the average length of a job is about 32 months. Such an estimate is reasonable considering the youth of the sample members and the degree of turnover in the U.S. labor market. Finally, the “implicit” reservation wage is estimated to be 3.003. In performing the estimation this value is always constrained to be less than the minimum wage in effect at the time.

The second column in the top panel of Table 2 contains estimates obtained under the Gamma specification of the matching distribution where once again symmetric Nash bargaining has been assumed. There are few changes in the estimated values of any of the rate parameters. The estimates of the parameters characterizing G change markedly of course, since they are not comparable across the two specifications. The estimate of the implicit reservation wage is higher than under the

lognormal specification, though not markedly so. Since the same number of parameters have been estimated, the likelihood function value indicates a somewhat worse fit under the Gamma than under the lognormal specification.

The last two columns of the top panel report estimates under the χ^2 assumption on G with and without the symmetric Nash bargaining restriction. When α is restricted to equal 0.5 the rate parameters are little changed with respect to those reported in the first two columns. Naturally, the estimates of the parameters characterizing G are different than those reported in columns 1 and 2. The specification in column 3 is in fact a special case of the one reported in column 2, and we see that the estimate of the scale parameter ζ_2 reported in column 2 is significantly different from the value ‘2’ associated with the χ^2 . Given the restriction that $\alpha = .5$ we would overwhelmingly reject the χ^2 restriction that $\zeta_2 = 2$ and preference would be given to the unrestricted Gamma specification.

Because the parameter ζ_2 is a scale parameter, and it is the presence of a scale parameter that significantly complicates the estimation of the bargaining power parameter α , we have attempted to estimate α under the χ^2 assumption. The results appear in the fourth column. We notice that there are substantial differences between the estimates in columns 3 and 4 as a result of treating α as a free parameter. Most interestingly, we note that the estimate of α is 0.880, which indicates that employees have substantially more bargaining power than employers. This result is surprising, particularly given the fact that the sample is so young and contains many students. Though the estimate of α is relatively precise, the reason is the strong distributional assumption we have made in column 4.

The bottom panel of Table 2 reports estimates of the same specifications using the CPS data from August 1997, the month before the minimum wage was changed from \$4.75 to \$5.15. Obviously there was some degree of inflation in the 11 month period separating these two months, so we might expect some of the parameter estimates to reflect this phenomenon. Comparing estimates in column 1, we see the estimated lognormal parameters are very similar across the two months. The substantially lower unemployment rate in August 1997 produces a much higher estimate of the contact rate parameter λ . The implicit reservation value is higher in August 1997, but due to the minimum wage difference there is no reason to expect equality.

Parameter estimates are more variable in columns 2 and 3. The estimates of λ remain higher, and the parameter estimates of the gamma distribution exhibit more variability across months than was the case under the lognormal assumption. The estimated implicit reservation wage is also greater in August 1997 under the Gamma distribution assumption. While α is identified theoretically given the χ^2 assumption, given the configuration of the data in August 1997 the log likelihood function was essentially monotone in α , and the estimation procedure did not converge. This illustrates how tenuous identification is using strictly functional form assumptions on the matching distribution.

In Section 4.3 we described an estimator that utilized the same CPS data that has been employed to this point and one additional piece of demand side information, the ratio of total wages paid to firm revenues. It goes without saying that our model is highly stylized, and that in reality there is no single value of this ratio when looking across firms in the economy at any point in time. We have opted to use the value (or a reasonable approximation to it) from one large U.S. corporation, McDonald’s. This firm has been selected for a number of reasons. First, it is the largest private sector employer in the U.S., so that the law of large numbers results which we relied upon in Section 4.3 are appropriate. Second, this firm has a disproportionate number of employees from the age group in which we are interested, those between the ages of 16 and 24. Moreover, a large number of its employees are paid at or near minimum wage level. Third, McDonald’s is one of the few large employers in this sector that reports sufficiently disaggregated information in

its Consolidated Statement of Income (CSI) to allow computation of the number in which we are interested.

In obtaining a value for the wage share at McDonald’s we have used information from the company’s 1996 CSI, which covers a period that includes the survey month of the CPS we are using (September 1996). For purposes of homogeneity we have restricted attention to revenues and costs associated with restaurants owned by the company (i.e., we have excluded franchises). We computed our value of “net” revenue by subtracting “costs of food and packaging” plus “occupancy and other operating expenses” from total sales by Company-operated restaurants.” These two expense items are essentially treated as fixed costs. The numerator of the ratio is given by “payroll and other employee benefits.” We found the value of this ratio to be 0.576 in 1996. It is worthwhile to note that the ratio seems to be relatively stable over time. For example, using the same computational method the ratio was equal to 0.568 in 1995 and 0.560 in 1994.²⁵

Table 3 contains estimates of the model with the profit information included. All model specifications assume a lognormal matching distribution. The first column of the top panel contains the estimates for the entire sample with those employed at a wage less than 4.25 excluded. The addition of the demand side information has resulted in virtually no change in the sampling distribution associated with λ and η , as was to be expected from the identification discussion provided above. Conversely, we notice a very large change in the estimates of the matching distribution parameters and the bargaining power parameter. The point estimates of μ and σ are 2.301 and 0.528, respectively, and their standard errors of 0.036 and 0.020 indicate that they are precisely estimated. The point estimate of the implicit reservation wage is 3.093 and its estimated standard error is 0.146.

Of most interest to us is the estimate of the bargaining power parameter. The point estimate of α is 0.424 and the estimated standard error of α is 0.007. With this level of precision it is straightforward to reject the null hypothesis of symmetric Nash bargaining. The estimate clearly indicates that this class of worker is at a disadvantage with respect to extracting match surplus. The use of the profit information adds a large amount of information regarding the estimation of the size of the pie to be divided and has resulted in what we consider a reasonable estimate given our modeling assumptions.

There now exists a few other estimates of the bargaining power parameter obtained using disaggregated data. Cahuc et al. (2003) obtain a much lower estimate of the bargaining power parameter, as do Dey and Flinn (2005). The main difference in the modeling approaches taken in those papers with the one followed here is in the allowance for on-the-job search. Both papers allow for bargaining between competing firms for an individual’s services when a current employee of one meets another potential employer. With relatively frequent bidding wars this mechanism suffices to transfer much of the match surplus to employees even with a low value of α .²⁶ Generally speaking, it appears that bargaining models that allow for on-the-job search and inter-firm competition for labor services produce quite low estimates of the bargaining power parameter.²⁷ Models without on-the-job search don’t have this transfer mechanism and must rely on larger values of α to fit the observed wage distributions.

The other columns in the top panel of Table 3 demonstrate the relative stability of the α

²⁵We use the same figure when estimating the model using the 1997 data. The wage share of revenues is virtually identical in these two years.

²⁶In fact, in Postel-Vinay and Robin (2002) employees are assumed to have *no* bargaining power, but due to the Bertrand game played between competing firms the model is able to fit the data quite well.

²⁷Yashiv (2003) is an exception. Using time series data from Israel, he finds a low estimate of α in a model without on-the-job search. The fact that his result is obtained using data gathered at a much higher level of aggregation and that α is identified from time series variation makes comparison with the other studies difficult.

estimates across demographic groups. As was discussed in the Section 4.3, this near constancy is largely an artifact of our estimation procedure. Since productivity and wage information is not available by demographic group in the company’s CSI we have had to assume that the same ratio applied to all groups. This is the key piece of identifying information in determining $\hat{\alpha}$, and as a result near constancy of these estimates is almost guaranteed.²⁸

Even if the group-specific values of $\hat{\alpha}$ are relatively similar, because of the high degree of precision with which they are estimated there do exist statistically significant differences between them. For example, youths who are black or Hispanic are estimated to have a slightly lower level of bargaining power than others (0.383 versus 0.437). They also have a slightly lower estimated value of the mean and standard deviation of the logarithm of the match value. When we divide the sample along gender lines, we find that males are estimated to have a slightly higher degree of bargaining power than females, as well as having higher values of μ and σ .

The estimates in the bottom panel of Table 3 are broadly consistent with those in the top panel. As noted previously, due to the lower aggregate unemployment rate during August 1997, $\hat{\lambda}$ is substantially higher using this sample. The estimate of $\rho V_n(4.75)$ is significantly greater than the estimate of $\rho V_n(4.25)$, which is due not only to “real” minimum wage impacts but also to inflation. Inflation may also be contributing to the differences in the estimates of the match distribution parameters, particularly σ , in the top and bottom panels. Estimates of α are substantially lower in August 1997 than in September 1996. This is partially produced by the change in the shape of the estimated match distribution. It may also be due to composition effects in that the August sample includes more seasonal workers than the September one does, and it may be that this group of workers have less bargaining power than their less seasonal counterparts. We don’t want to make too much of this point, since it is at odds with our assumption of a homogeneous population facing fixed primitive parameters. The differences we saw across demographic groups in the top panel are broadly mirrored in the estimates obtained using the August 1997 data.

In Table 4 we use the model to directly determine whether or not the minimum wage increase that occurred at the end of September 1996 was beneficial in the sense of our first welfare measure, the value of unemployed search. In this analysis we only estimate the model for the total sample, and we begin by combining the September 1996 ORG-CPS sample with an analogous one from the February 1997 survey. Given the sample design these two surveys do not include the same individuals. In combining the surveys we assigned wages as follows. For individuals who earned less than \$4.25 in September 1996, we assigned them a wage of \$4.25, the then current minimum. Similarly, we assigned all those making less than \$4.75 an hour in February 1997 a wage rate of \$4.75.

Before turning to the results we must present a word of caution in terms of the interpretation of the estimates. Strictly speaking, the consistency of our estimator in this case depends on the search environment remaining stationary in the wake of the minimum wage change, so that all “primitive” parameters assume the same values in September 1996 and February 1997. There are a number of reasons why this assumption may be questionable; we shall consider a limited test of it below.

For our inferences to be valid requires constancy of environmental parameters. In the absence of endogenous contact rates and a labor market participation decision, no other assumptions are required to obtain valid estimates of primitive parameters and the value of search before and after the minimum wage change as long as the implicit reservation wage after the minimum wage change is no larger than the initial minimum wage rate. In terms of our application, we require $\rho V_n(4.75) < 4.25$. If this is the case, then even under the old minimum wage match values in

²⁸Cahuc et al (2003) are able to estimate group-specific values of α given there access to employer-employee matched data and their assumption that match values are the product of the idiosyncratic productivity values of an employer and an employee.

the interval $[4.25, 4.75)$ will be accepted since working at the wage associated with such a θ given $m = 4.25$ has a higher value than waiting to search under the new minimum wage. Point estimates from the model indicate that this restriction is not unreasonable.

By using information from a sample collected four to five months after the minimum wage change, it is hoped that we can ignore transitional dynamics in terms of the unemployment duration distribution. While it is straightforward to adjust the likelihood function for transitional dynamics given exogenous λ and a fixed labor force, this is decidedly not the case under endogenous contact rates and participation decisions. Thus we have chosen a period long enough to hope that most transitional dynamics are absent and short enough to hope that price level changes can be ignored.

We estimated the model under three specifications. In specification *A*, it was assumed that all primitive parameters remained fixed between September and February, except for b , and that the rate of contacts between searchers and firms remained fixed. We say that b is allowed to change since we are also imposing constancy in the value of search across the two months, and since the minimum wage changed and all other parameters are fixed, unless b changes this cannot be the case under the model structure. Specification *B* allows the value of search to change between periods, but imposes constancy of the contact rate λ . Under specification *C*, both the contact rate and the value of search are allowed to change after imposition of the new minimum wage.

Comparisons between the estimates under the three specifications reveal few significant differences. Holding the contact rate fixed, the value of search does seem to have increased slightly between September and February, with the likelihood ratio statistic having a p value of 0.118 under the null of no change. Comparing columns *B* and *C*, there is absolutely no indication that the rate of contacts changed over the two months. On the basis of these results we cannot reject the hypothesis that λ is a primitive parameter, in the sense that it was seemingly invariant to the change in the minimum wage. We will come back to this point below, since assessing the policy impacts of the minimum wage will depend critically on estimates of α and the determination of λ .

We repeated the same exercise for the minimum wage change of September 1, 1997. We pooled monthly data from August 1997 and January 1998 in the estimation, and recoded all positive August wages less than \$4.75 to \$4.75 and all positive January wages less than \$5.15 to \$5.15. As mentioned above, results from this exercise are particularly troublesome to interpret due to seasonal employment in August and, to a lesser extent, in January. This is evidenced by the marked drop in the labor market participation rate between August and January, a drop that is not consistent with the small change in the value of search (which determines the participation rate within the model).

Once again, the estimates vary little across specifications. While the point estimate of the implicit reservation wage did increase, the p value associated of the test statistic is only 0.215. There is no indication that the contact rate λ changed between the two months. Thus the results we obtain are very consistent across the two pooled samples. While the value of search seems to have increased to some degree, the change is at best marginally significant. There is no indication that the parameter λ is endogenous, *at least over the the range of values of m observed in this time period.*

We now move on to obtain estimates of the other parameters that characterize the model, namely, the flow cost of a vacancy, ψ , the Cobb-Douglas matching parameter (estimable in principle when we have access to observations of the steady state equilibrium outcomes under two minimum wages), and parameter(s) characterizing the distribution of outside (the labor market) values. In forming the estimates of the identifiable primitive parameters, we will begin with those that use data from a single cross-section. In this case, we showed that a one-parameter outside option value distribution could be identified along with the cost of a vacancy, but the matching distribution could

have no unknown parameters. We then utilize the pooled cross-sectional information to consider the estimation of the same parameters, with the addition of the single unknown parameter from the Cobb-Douglas matching function. All estimation results are contained in Table 6.

The first column of Table 6 contains estimates of the parameters ψ and ζ , which is the parameter of the negative exponential distribution of out of the labor force valuations. Since the data used are from the single month of September 1996, a known matching function must be assumed. We use the CRS form

$$q\left(\frac{ul}{v}\right) = v(1 - \exp(-ul/v)). \quad (24)$$

We will compute the vacancy rate v and then the elasticity of the matching function evaluated at the September 1996 equilibrium values of u , l , and v .

The estimated vacancy rate in September was 0.020, which should be compared with the unemployment rate (in the population) of 0.062. The matching function elasticity, with respect to the unemployment rate, is only 0.147. The estimated flow cost of a vacancy is 130.759. Finally, the estimated parameter value of the negative exponential distribution of outside values is 0.0326.

We repeated the same exercise using the August 1997 data, the main purpose being to assess the variability of the estimates of primitive parameters over time. The vacancy rate is estimated to be 0.030 in August, which is a value 50 percent greater than what was inferred 11 months earlier. This is an outcome variable, of course, so that there is no presumption that it should be constant after a minimum wage change. The flow vacancy cost estimate declines a bit to 105.025. This is supposed to be a primitive parameter, so its value should not change over time. Given the extremely indirect way in which this parameter is estimated, the decline in the estimate of about 20 percent does not seem particularly worrisome. We should also bear in mind that we are considering data from a summer month in which seasonal employment is a major issue for the age group we are considering.

The matching function elasticity is found to be substantially higher in August 1997 than in September 1996, moving from 0.147 to 0.333. It is slightly smaller than the estimated bargaining power parameter obtained using data from that month, the value of which was 0.375. These estimates imply that the Hosios condition was approximately satisfied; this will have implications for the welfare analysis conducted below. The estimate of the parameter characterizing the distribution of outside values is little changed from the value obtained using the September 1996 data.

We then turned to estimation of the model parameterizations that were available to us using observations on outcomes associated with two distinct steady state equilibria. We first tried to implement the estimation procedure using the September 1996 and February 1997 pooled data. This was ultimately unsuccessful since the point estimates of the parameters did not satisfy Condition C of Section 4.4.2 (that is, the implied estimate of the Cobb-Douglas weight attached to unemployed search was outside the unit interval). We then attempted estimation using the pooled data from August 1997 and January 1998. In this case Condition C was satisfied and we could obtain estimated values of parameters that were contained in the appropriate parameter space.

The estimated vacancy rates in August 1997 and January 1998 are very similar. Using the pooled information and a different matching function, the vacancy rate for August is estimated to be 0.024 as opposed to 0.030. The largest difference between the estimates of primitive parameters obtained from the single samples and the pooled sample is in the flow cost of a vacancy. That declines all of the way to 2.46, which is a huge drop from the single sample estimates of over 100. We should bear in mind, however, that the estimator of ψ is quite different in the two contexts, that $\hat{\psi}$ is very much a “backed out” parameter estimate, and that the matching functions are different.

With the pooled data we were able to estimate a one-parameter matching function. The elasticity of the this function with respect to the search input from the supply side of the market is

the Cobb-Douglas parameter ω . We found $\hat{\omega} = 0.196$, which lies between the elasticities computed under the matching function used with the single month data. The similarities in the estimates is moderately reassuring.

We do not report an estimate of the parameter(s) of the distribution of the outside options. Since the estimated value of search increased between the August and January, the model implies that the participation rate increased. Instead, it fell by six percent. This drop was in large part due to students returning to school from seasonal employment. We note that even if the participation rate had increased, a two parameter distribution of outside options would have been necessary to fit the data, in general. Thus the distributional assumptions made of necessity will depend on the number of months that are pooled, this limiting comparability of estimates across different sampling schemes.

6 Policy Experiments

In this section we use the model estimates obtained under different model specifications to determine the welfare, and unemployment, impacts of the minimum wage. Specifically, we will plot the equilibrium steady state distribution of the population across labor market states as a function of the minimum wage; the size of the employed population will be equal to the size of the set of firms with filled vacancies. We will also plot the average steady state values associated with the states of OLF, unemployment, employment, and having a filled vacancy. Finally, we will plot the values of the two aggregate welfare measures, differentiated by whether they include the OLF subpopulation.

The results of the policy experiments we conduct depend critically on the assumptions we make regarding the model. We first consider the impact of minimum wages on labor market outcomes and welfare under the assumption that *contact rates are invariant with respect to changes in m* . We take the results of this exercise seriously since the empirical evidence we have presented is consistent with this assumption. From the results in Tables 4 and 5 we concluded that there was no evidence that the contact rate λ differed before and after the two minimum wage changes that occurred in the late 1990s. This finding is consistent with λ being a constant, though it is also consistent with λ being determined endogenously. The small changes in the minimum wage, and the relatively small number of matches directly impacted, make finding powerful tests of endogeneity of λ extremely difficult.

Most of our results are presented in graphical form. In Figure 5 we plot the size of the sets of individuals and firms occupying the various labor market states and the average welfare level in the state as a function of the minimum wage *when λ is held fixed at the estimated value of 0.309*. Figure 5.a contains the plots of the sizes of the various states. We see that the size of the employed population actually increases by a small amount with increases in the minimum wage up to approximately 8 dollars an hour, after which it begins to decrease relatively rapidly. The size of the OLF population declines as the minimum wage increases up to the point at which the value of unemployed search begins to decrease. The unemployed population grows steadily in the minimum wage. Due to the participation margin, we see that for minimum wage changes up through approximately 8 dollars an hour both unemployment and employment register gains.

Figure 5.b plots the changes in the average value of occupying the state with changes in m . The average welfare values in the unemployed and employed states are single-peaked in m . The value of m that maximizes the value of the unemployed is \$8.29. The value of m that maximizes the welfare of labor market participants is \$8.66. The average welfare of firm owners with filled vacancies is monotonically increasing in m , through the truncation effect and the fact that the threat point of workers ($\rho V_n(m)$) is not rising at a fast enough rate to offset the positive selection impact of an

increasing m .

In Figure 6 we plot the results from the endogenous contact rate case, and the conclusions we draw are strikingly different. From Figure 6.a we see that the size of the employed population declines rapidly in the minimum wage, as does the total number of employed and unemployed individuals. The decreases in the number of employed individuals is already significant even for an m of \$5. In Figure 6.b we see that the value of unemployed search is monotonically decreasing in m ; if the criterion for choosing m is to maximize the value of search, which also maximizes the participation rate in the market, then $m = 0$ is the solution to the planner’s problem. The average welfare of employed individuals increases in the minimum wage up to \$3.33, at which point it begins to decrease in m . The average welfare of firms is monotonically increasing in m ; this is the case due to the constant erosion in the worker’s threat point as m increases in combination with the truncation effect. When we form the aggregate (labor market) welfare measure, the end result is a single-peaked function of m that has its maximum at \$3.36.

The results of the policy experiment are summarized in Table 7. Column 1 contains the “baseline” values in the reference month of September 1996. Under our model, whether with endogenous contact rates or not, the aggregate labor market welfare measure had a value of 613.974. Column 2 contains the results from the model in which λ is fixed. As noted above, the aggregate labor market welfare maximizing value of m is \$8.66 in this case, which is slightly more than double the baseline value. At this value of m there is significantly more participation in the labor market by individuals on the supply side; the OLF rate decreases 13 percent from the baseline value. There is a marked increase in the unemployment rate of labor market participants, on the order of 48 percent. Nonetheless, aggregate labor market welfare increases by 24.2 percent.

Under endogenous contact rates (the last column of Table 7), the optimal minimum wage is 20 percent less than the baseline value. At this value of m , the market participation rate is slightly greater than at the baseline, and the unemployment rate among participants decreases by about 5 percent. Aggregate labor market welfare is less than 2 percent greater than in the baseline. Thus this large decrease in the minimum wage translates into relatively small effects on outcomes and welfare under this model.

Given such a large discrepancy in the policy implications depending on the assumption made regarding the determination of contact rates, it is natural to ask which results should be given more credence (if either)? The model with endogenous contact rates has the advantage of being a general equilibrium one, even if extremely stylized. The main disadvantage of that model, from an empirical perspective, is the necessity of utilizing a simple “black box” matching technology (with no unknown parameters) to close it. Because identification requirements severely limit the range of matching technologies we can consider, we cannot properly assess the sensitivity of model estimates and policy implications to variations in the specification of this function.

The exogenous contact rate specification undoubtedly has more of a partial equilibrium flavor. It seems reasonable to suppose that contact rates will vary with sufficiently large increases in the minimum wage, but of course the same can be said about the dismissal rate, bargaining power, and virtually all of the “primitive” parameters. Given the small changes in the minimum wage that were observed, and the relatively small number of matches that were directly impacted by these movements, identification of changes in parameters of an econometric model (be it “structural” or “reduced form”) is extremely difficult. Our formal econometric tests for a change in the contact rate for searching individuals did not lead us to conclude the contact rate was endogenous *with respect to this small policy intervention*. This does not mean that it is valid to conclude that the contact rate would be unaffected by a sufficiently large change in the minimum wage. In particular, at the “optimal” minimum wage rate of \$8.66 it may well be the case that the contact rate will be different than the estimated value of λ for September 1996. However, given the range of policy

choices in the data, we have no way to assess the accuracy of model predictions for such high minimum wage rates.

7 Conclusion

We have formulated simple partial and general equilibrium models of wage determination and labor market dynamics that carry implications broadly in accord with the empirical findings of other researchers who have worked with disaggregated data. By simply appending a side constraint to the standard Nash bargaining problem, we were able to generate accepted wage distributions with a mass point at the minimum wage and a continuous density of wages to the right of m . The model is parsimoniously specified in that the equilibrium wage and unemployment spell distributions are characterized in terms of a small set of primitive parameters.

Often studies of minimum wage impacts on the labor market have focused exclusively on unemployment, employment, and participation rates. Besides looking at the effect of minimum wages on observable labor market outcomes such as these, we are able to assess their welfare effects as well. We look at the effect of minimum wages on the total welfare of labor market participants on both the supply and demand sides. Using the matching function formulation along with Nash bargaining between workers and firms, the results of Hosios (1990) inform us that an efficient labor market equilibrium requires that the elasticity of the matching function with respect to the size of the set of searchers be equal to the share of the surplus they receive. While macroeconomic models of search equilibrium have postulated the presence of a social planner that could ensure satisfaction of this condition, we do not. In the absence of the satisfaction of this condition, the minimum wage, or a maximum wage for that matter, can be viewed as a crude instrument that moves the labor market in the direction of an efficient allocation. Our estimates of the bargaining power parameter, although significantly less than .5, are significantly larger than our estimates of the match function elasticity and as a result yield an optimal minimum wage rate less than the then current value of \$4.25. Our empirical work did, however, cast some doubt on the applicability of the matching function framework in these data. In particular, we could not reject the null hypothesis that the contact rate parameter was invariant with respect to minimum wage changes. If we fix the contact rate at its estimated (equilibrium) value in September 1996, we find a welfare-maximizing minimum wage rate of over \$8 per hour. We are unable to say more regarding which estimate is more credible given the restrictions imposed by our modeling assumptions and the data available to us.

In terms of useful generalizations of the model, the most critical appears to be allowing for on-the-job (OTJ) search. It is well-known that over one-half of all job spells terminate with the direct transition into another job in the U.S.; the proportion is even higher for young labor market participants. In terms of the impact of OTJ in our model, it is not difficult to show that when OTJ search is possible, and firms compete for a worker by making alternating offers until the firm with the lowest match value offers a wage equal to that value (see Dey and Flinn (2005), or Postel-Vinay and Robin (2002) for a different but related renegotiation method), only the first job in any consecutive sequence of jobs can possibly pay the minimum wage. As a result, workers spend a lower proportion of their labor market careers being paid the minimum wage, which leads us to suspect that minimum wages will have less impact on the value of labor market careers than in the no OTJ search case analyzed here. To analyze this case requires information not available in the CPS, so the final verdict of the impact of minimum wages in the presence of OTJ search must await a fresh analysis using different data.²⁹

²⁹James Mabli and I are currently engaged in such an analysis using Survey of Income and Program Participation

The minimum wage is a crude, blunt tool, the use of which results in some deadweight loss. As we saw through examples and simulations based on estimated parameters, large increases in unemployment can result from significant increases in the minimum wage. On the positive side, the minimum wage is easy to understand, implement, and enforce. Its role within the asymmetric Nash bargaining model was particularly clear. Given low bargaining power, which in conjunction with other unfavorable values of labor market parameters gives searchers low threat points, the worker's share of the match value is small. Within a model of the labor market in which the contact rate is fixed, the social planner is able to increase aggregate welfare, and the welfare of those on the supply side of the market in particular, by setting a high minimum wage rate, thus effectively giving a larger share of the surplus to the supply side of the market. In a model with endogenous contact rates, the minimum wage is particularly important in helping to align the incentives of individuals and firms to make decisions consistent with socially optimal levels of search and vacancy creation. In this view, the minimum wage becomes a tool to increase aggregate welfare and is not merely a redistribution device.

(SIPP) data, using which event history job spell data can be constructed.

Table 1
Descriptive Statistics CPS-ORG
Individuals Aged 16-24
(Standard Deviations in Parentheses)

<i>Characteristic</i>	<i>9/96</i>	<i>2/97</i>	<i>8/97</i>	<i>1/98</i>
<i>N</i>	3199	3236	3117	3150
N_o/N	0.365	0.369	0.307	0.375
$N_u/(N_u + N_e)$	0.097	0.119	0.082	0.104
Female	0.505	0.522	0.518	0.510
Black and Hispanic	0.252	0.248	0.274	0.254
t_u	3.411 (4.973)	2.889 (3.230)	2.612 (1.006)	2.515 (3.576)
w	6.932 ¹ (3.480 ¹)	7.016 (3.874)	7.254 (4.361)	7.513 (4.178)
$\chi[w = 4.25 e]$	0.053	0.022	0.013	0.002
$\chi[w = 4.50 e]$	0.020	0.012	0.006	0.002
$\chi[w = 4.75 e]$	0.024	0.063	0.047	0.007
$\chi[w = 5.00 e]$	0.125	0.118	0.101	0.053
$\chi[w = 5.15 e]$	0.002	0.002	0.010	0.075
$\chi[w = 5.25 e]$	0.023	0.039	0.029	0.050
$\chi[w = 5.50 e]$	0.035	0.046	0.056	0.054
$\chi[w = 6.00 e]$	0.083	0.092	0.092	0.092
$\chi[w = 7.00 e]$	0.062	0.061	0.069	0.060
$\chi[w = 8.00 e]$	0.047	0.046	0.052	0.046
$\chi[w = 9.00 e]$	0.019	0.027	0.023	0.031
$\chi[w = 10.00 e]$	0.031	0.028	0.024	0.032
Total at These Values	0.524	0.556	0.522	0.505
$\chi[w < m e]$	0.054	0.052	0.054	0.041

Note:

One extremely large wage observation was deleted when computing the descriptive statistics for this month.

Table 2
Model Estimates with Normalizations
September 1996 and August 1997
CPS-ORG

Parameter	Matching Distribution			
	Log Normal	Gamma	χ^2	χ^2
September 1996 ($N = 2022$)				
λ	0.316 (0.023)	0.388 (0.033)	0.304 (0.022)	0.469 (0.059)
η	0.031 (0.003)	0.031 (0.003)	0.031 (0.003)	0.031 (0.003)
ζ_1	2.186 (0.034)	2.234 (0.236)	5.523 (0.156)	2.923 (0.266)
ζ_2	0.511 (0.019)	3.803 (0.267)	2 -	2 -
$\rho V_n(m)$	3.003 (0.151)	3.329 (0.091)	2.125 (0.234)	0.563 (1.760)
α	0.500 -	0.500 -	0.500 -	0.880 (0.063)
$\ln L$	-5066.737	-5076.6078	-5130.121	-5078.772
August 1997 ($N = 2151$)				
λ	0.490 (0.042)	0.853 (0.104)	0.411 (0.031)	-
η	0.034 (0.004)	0.034 (0.004)	0.034 (0.004)	-
ζ_1	2.014 (0.046)	1.080 (0.142)	5.290 (0.138)	-
ζ_2	0.590 (0.024)	5.361 (0.326)	2 -	-
$\rho V_n(m)$	3.811 (0.088)	3.971 (0.058)	2.839 (0.187)	-
α	0.500	0.500	0.500	-
$\ln L$	-5180.461	-5187.254	-5298.928	D.N.C.

Table 3
Model Estimates with Profit Information
September 1996 CPS-ORG

Parameter	Demographic Group				
	All	Males	Females	White Non-H	Black or H
September 1996					
λ	0.309 (0.023)	0.278 (0.027)	0.356 (0.039)	0.328 (0.030)	0.300 (0.040)
η	0.031 (0.003)	0.030 (0.004)	0.034 (0.005)	0.026 (0.003)	0.055 (0.010)
μ	2.301 (0.036)	2.342 (0.044)	2.273 (0.057)	2.331 (0.040)	2.150 (0.092)
σ	0.528 (0.020)	0.554 (0.026)	0.479 (0.028)	0.504 (0.021)	0.627 (0.053)
$\rho V_n(m)$	3.093 (0.146)	3.313 (0.173)	2.798 (0.273)	2.901 (0.202)	3.572 (0.165)
α	0.424 (0.007)	0.427 (0.006)	0.429 (0.013)	0.437 (0.0122)	0.383 (0.106)
N	2022	1049	973	1612	410
$\ln L$	-5065.676	-2739.338	-2582.245	-3972.758	-1072.138
August 1997					
λ	0.447 (0.036)	0.397 (0.043)	0.516 (0.063)	0.554 (0.056)	0.341 (0.046)
η	0.034 (0.004)	0.032 (0.005)	0.037 (0.006)	0.033 (0.005)	0.047 (0.008)
μ	2.218 (0.043)	2.318 (0.047)	2.104 (0.078)	2.252 (0.047)	2.167 (0.089)
σ	0.622 (0.024)	0.594 (0.028)	0.646 (0.040)	0.617 (0.026)	0.581 (0.048)
$\rho V_n(m)$	3.944 (0.080)	3.981 (0.116)	3.960 (0.104)	3.857 (0.103)	4.11 (0.136)
α	0.375 (0.006)	0.388 (0.006)	0.355 (0.007)	0.387 (0.005)	0.336 (0.009)
N	2151	1112	1039	1650	501
$\ln L$	-5179.436	-2772.805	-2379.684	-3960.016	-1188.202

Table 4
Model Estimates with Profit Information
September 1996 and February 1997
 $N = 4059$

Parameter	<i>A</i>	<i>B</i>	<i>C</i>
$\lambda(4.25)$	0.355 (0.018)	0.356 (0.018)	0.353 (0.022)
$\lambda(4.75)$	-		0.404 (0.022)
η	0.039 (0.003)	0.039 (0.003)	0.039 (0.003)
μ	2.253 (0.028)	2.244 (0.029)	2.243 (0.029)
σ	0.575 (0.015)	0.578 (0.016)	0.579 (0.016)
$\rho V_n(4.25)$	3.415 (0.079)	3.382 (0.083)	3.383 (0.083)
$\rho V_n(4.75)$	-	3.510 (0.096)	3.511 (0.096)
α	0.403 (0.003)	0.404 (0.003)	0.404 (0.003)
$\ln L$	-10118.373	-10117.152	-10117.124
<i>LR</i> Test		2.442	0.056
<i>p</i> value		0.118	0.813

Table 5
Model Estimates with Profit Information
August 1997 and January 1998
 $N = 4111$

Parameter	<i>A</i>	<i>B</i>	<i>C</i>
$\lambda(4.75)$	0.488 (0.028)	0.491 (0.029)	0.506 (0.035)
$\lambda(5.15)$	-		0.476 (0.033)
η	0.040 (0.003)	0.040 (0.003)	0.040 (0.003)
μ	2.165 (0.038)	2.157 (0.039)	2.158 (0.038)
σ	0.670 (0.020)	0.672 (0.020)	0.672 (0.020)
$\rho V_n(4.75)$	4.056 (0.055)	4.033 (0.059)	4.032 (0.059)
$\rho V_n(5.15)$	-	4.118 (0.072)	4.116 (0.071)
α	0.370 (0.005)	0.371 (0.006)	0.371 (0.005)
$\ln L$	-10007.255	-10006.487	-10006.155
<i>LR</i> Test		1.536	0.664
<i>p</i> value		0.215	0.415

Table 6
Point Estimates of Remaining Parameters

<i>Characteristic</i>	<i>Single CPS Samples</i>	<i>Pooled Sample</i>
	<i>9/96</i>	<i>8/97</i>
		<i>8/97 and 1/98</i>
Vacancy Rates		
v_t	0.020	0.030
v_t'		0.024
		0.026
Flow Vacancy Cost		
ψ	128.960	105.025
Match Elasticity ⁽¹⁾	0.147	0.333
		0.196
Outside Option		
Distribution Parameter		
$\zeta^{(2)}$	0.326	0.299
		-

Notes:

1. This was computed from the relevant matching function for the sample. Using only cross-sectional information, the matching function was $v(1 - \exp(-u/v))$. A CRS Cobb-Douglas matching function was used for the pooled data case, so that the elasticity is equal to the Cobb-Douglas parameter associated with the size of the set of unemployed searchers.
2. The participation rate parameter was not estimated for the pooled data case, since the decline in the participation rate is inconsistent with the increase in the estimate of unemployed search under our assumption of a stable distribution of outside option values.

Table 7
Policy Experiments
Based on Estimates from September 1996 CPS-ORG Data
(Proportionate Change with Respect to Baseline)

<i>Outcome</i>	<i>Baseline</i>	<i>Given Optimal m</i>	
		<i>Fixed λ</i>	<i>Endogenous λ</i>
<i>m</i>	4.25	8.66 (+1.038)	3.36 (-.209)
OLF Proportion	0.365	0.318 (-0.129)	0.358 (-0.019)
Unemployment Rate	0.096	0.142 (+0.479)	0.092 (-.042)
Aggregate Labor	613.974	762.408	624.509
Market Welfare		(+.242)	(+0.017)

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Figure 1.a
CPS Wages
September 1996

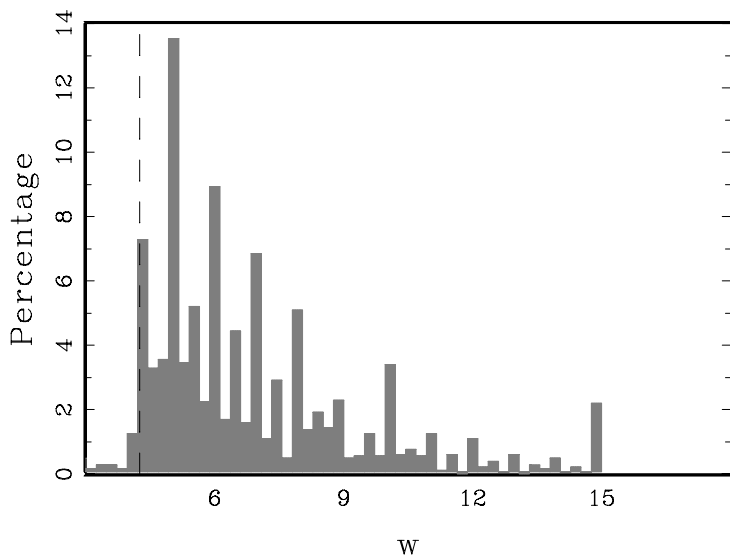


Figure 1.b
CPS Wages
February 1997

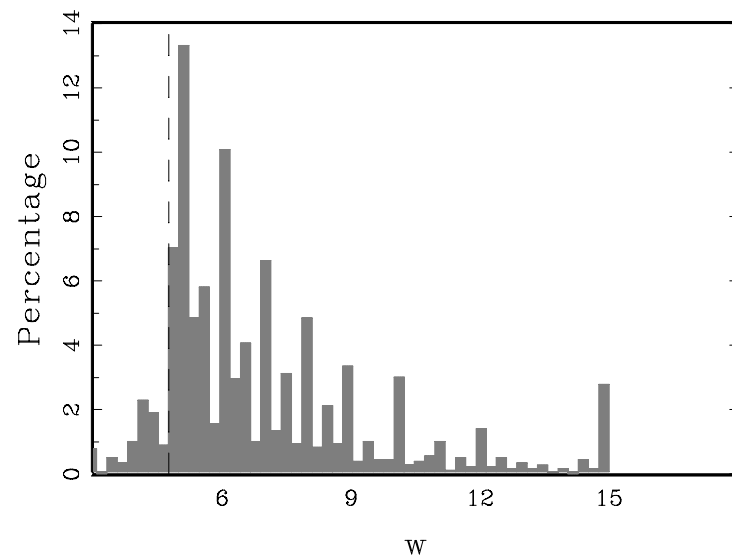


Figure 1.c
Simulated Wages
September 1996

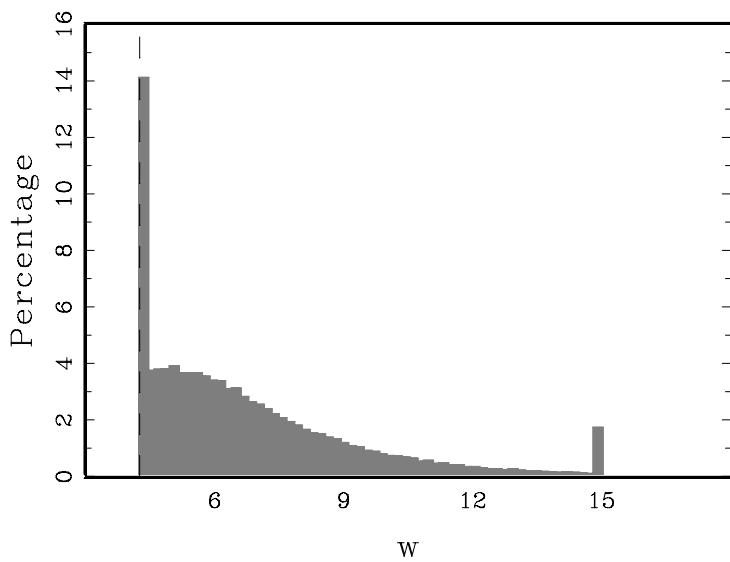


Figure 1.d
Simulated Wages
February 1997

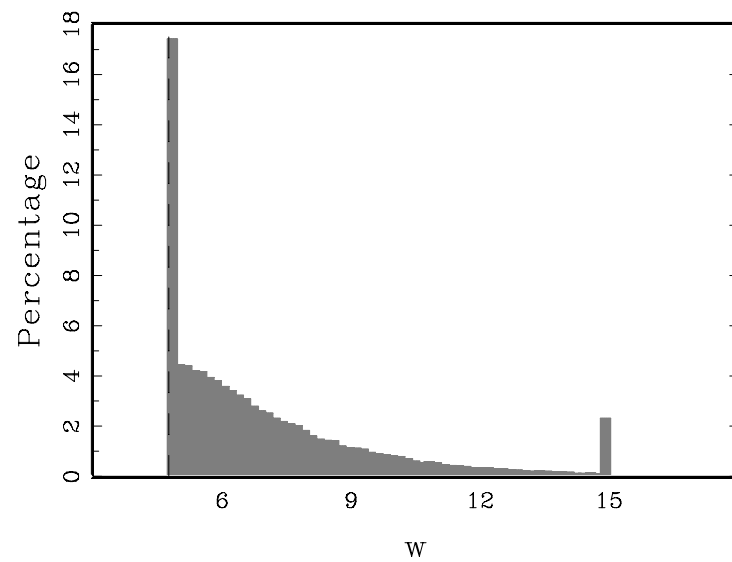


Figure 2.a
CPS Durations
September 1996

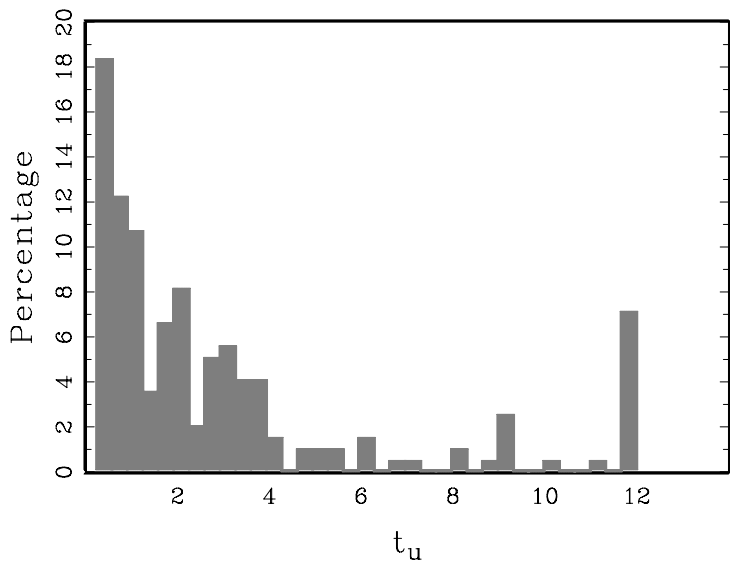


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CPS Durations
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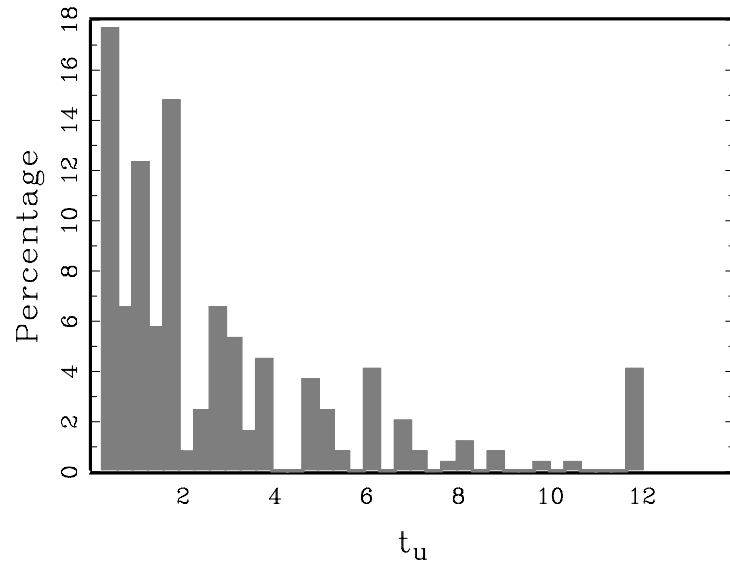


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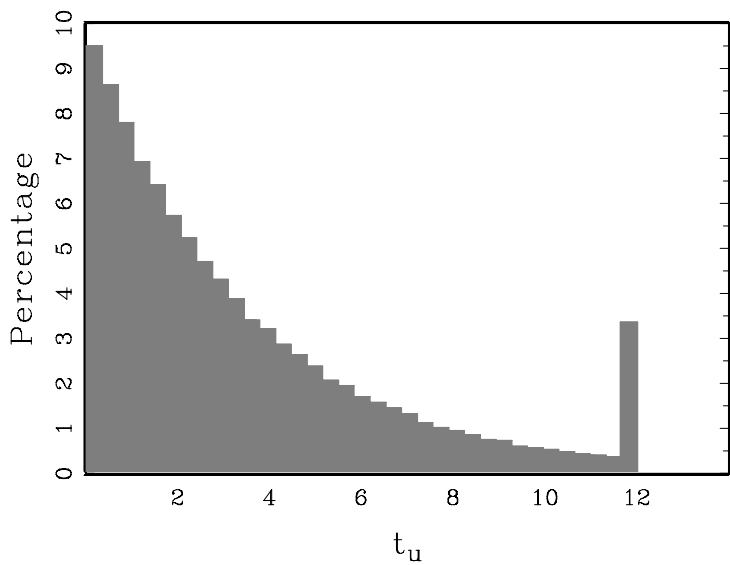


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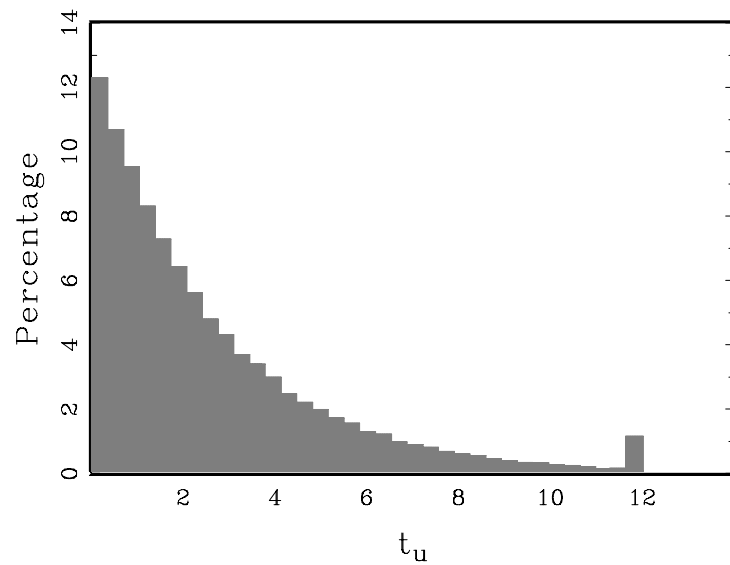


Figure 3.a
Match Value Density

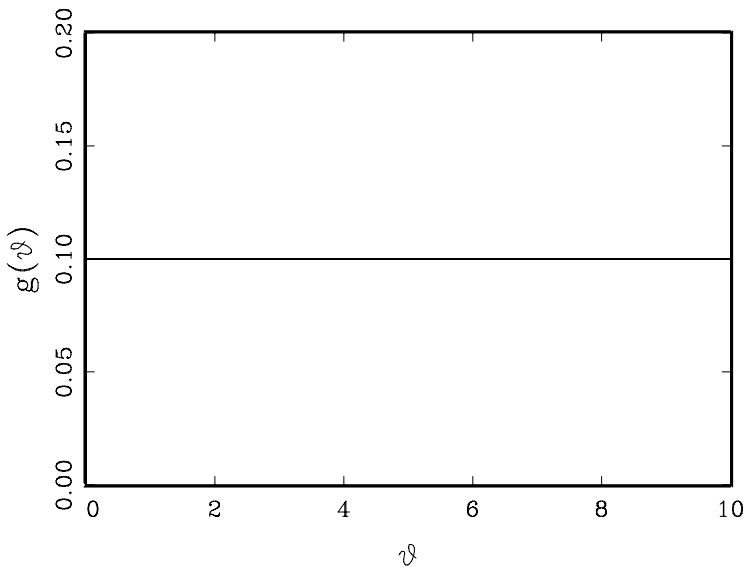


Figure 3.b
Equilibrium Wage Function

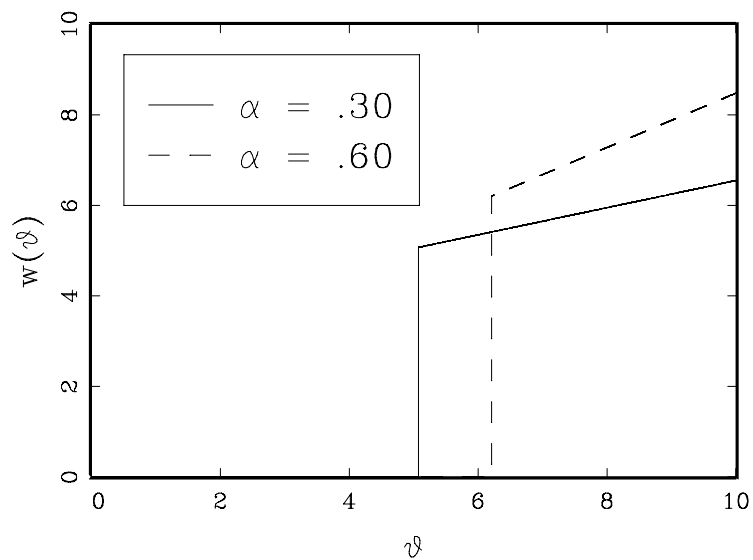


Figure 3.c
Wage Density
 $\alpha = .30$

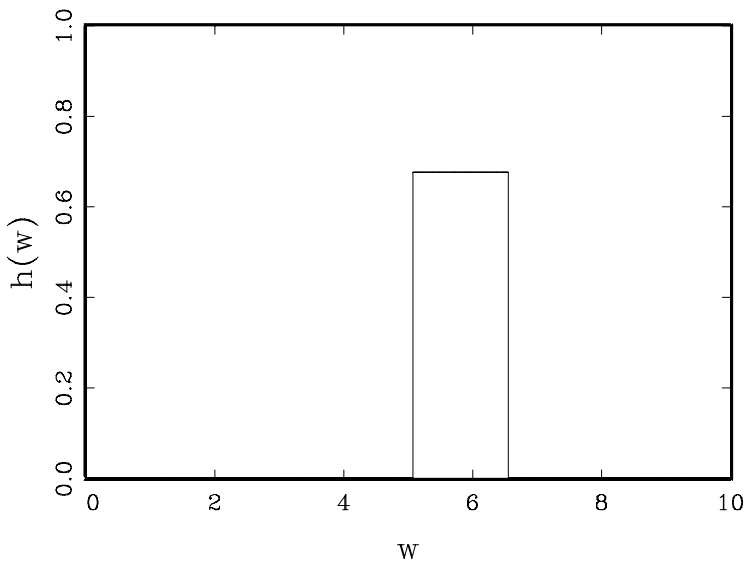


Figure 3.d
Wage Density
 $\alpha = .60$

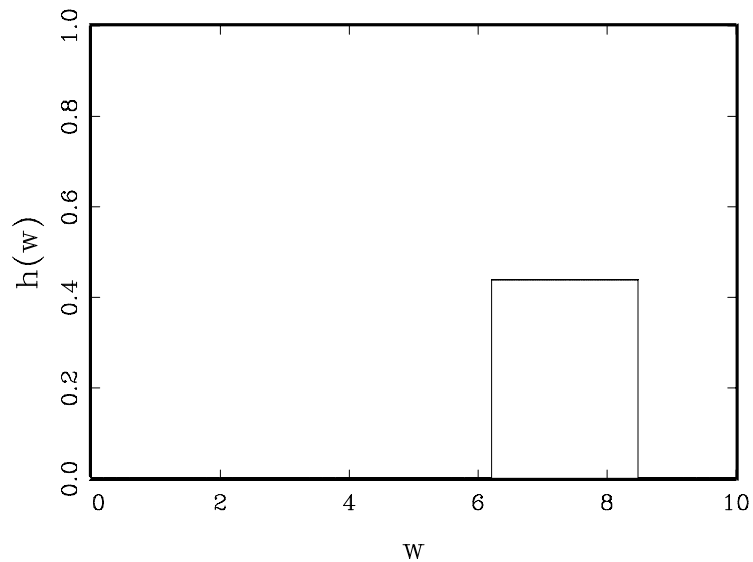


Figure 4.a
Match Value C.D.F.

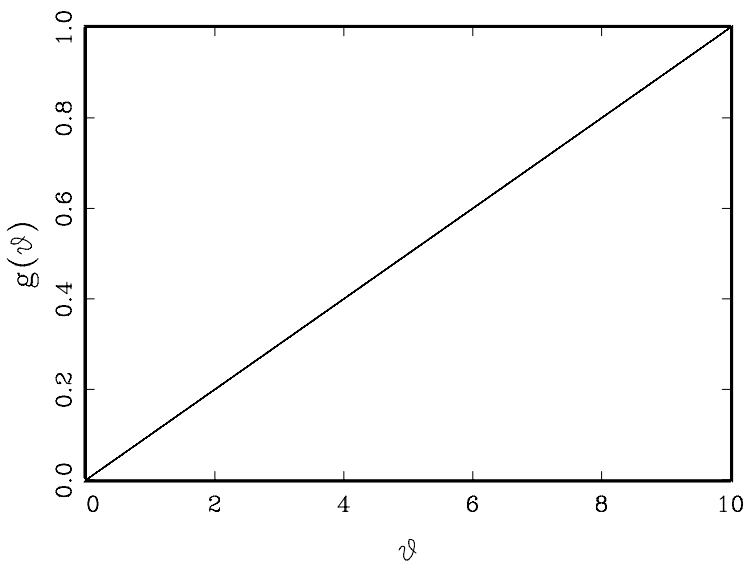


Figure 4.b
Equilibrium Wage Functions

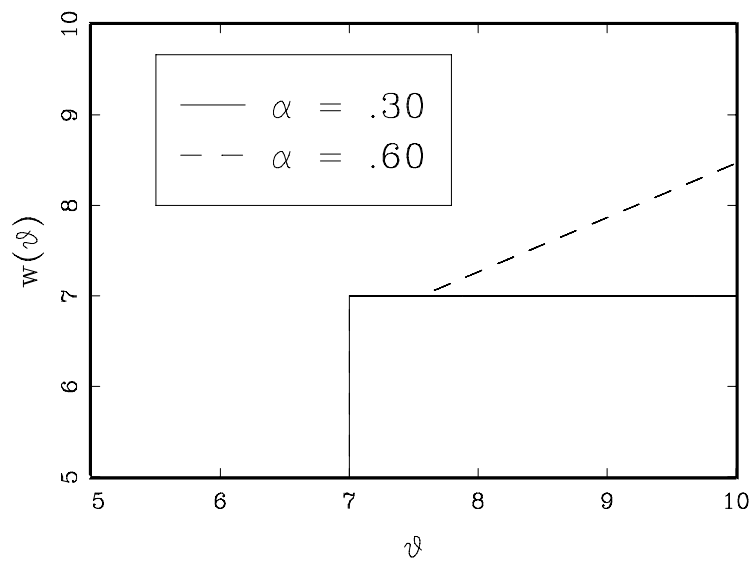


Figure 4.c
Wage C.D.F.
 $\alpha = .30$

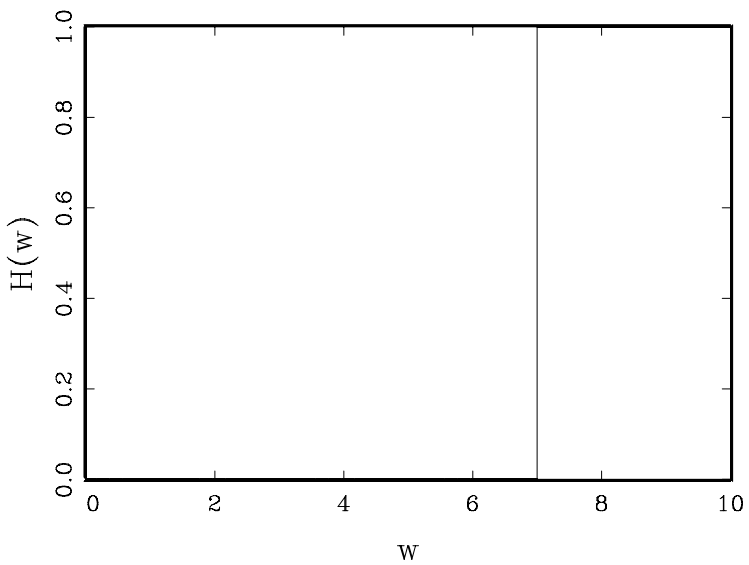


Figure 4.d
Wage C.D.F.
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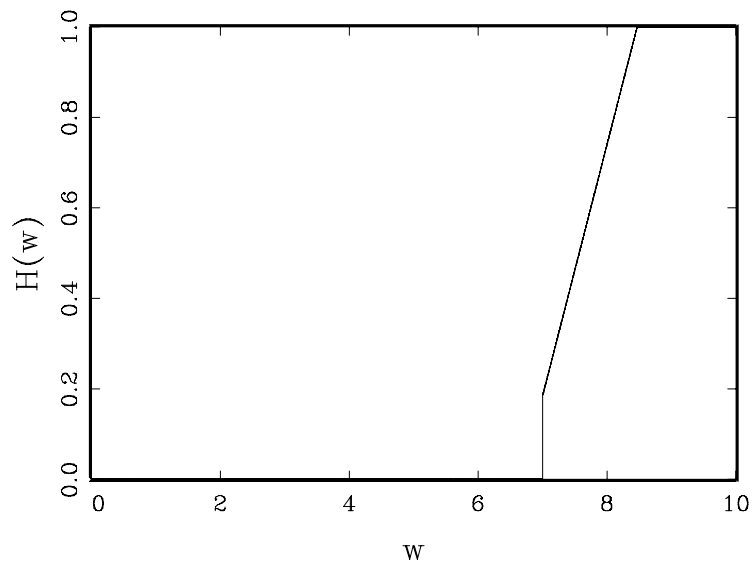


Figure 5.a
Exogenous Contact Rate
Group Size

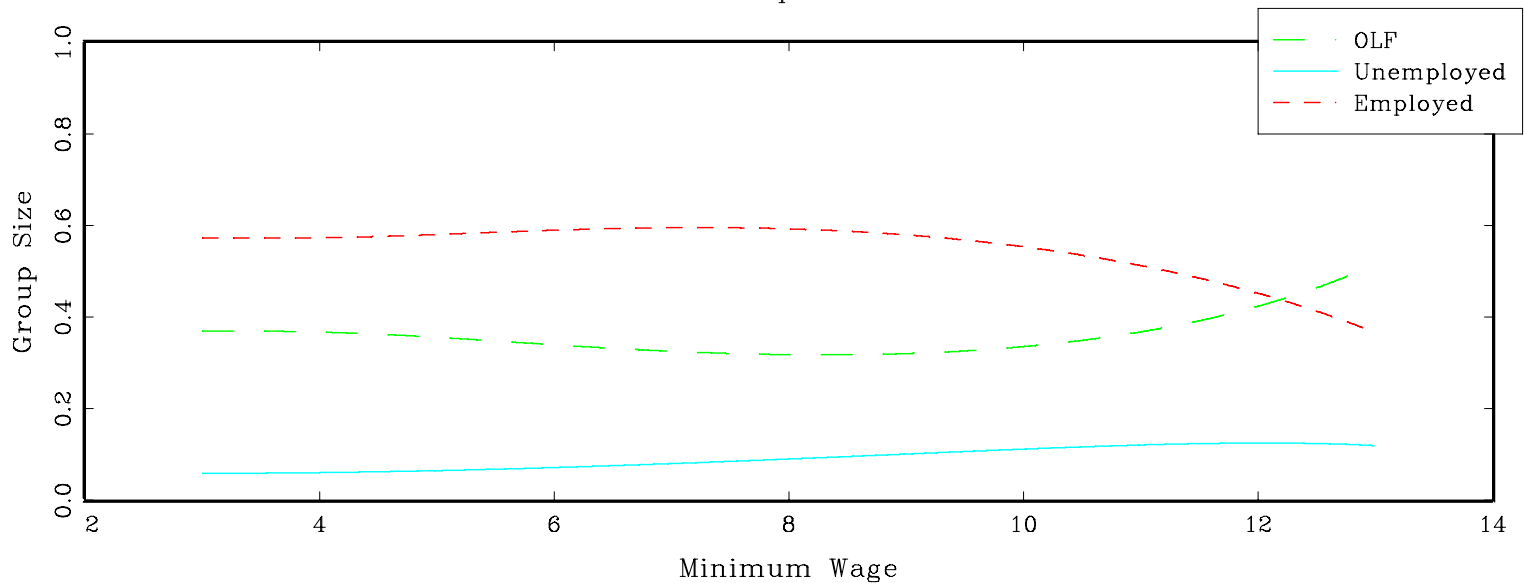


Figure 5.b
Exogenous Contact Rate
Average Welfare

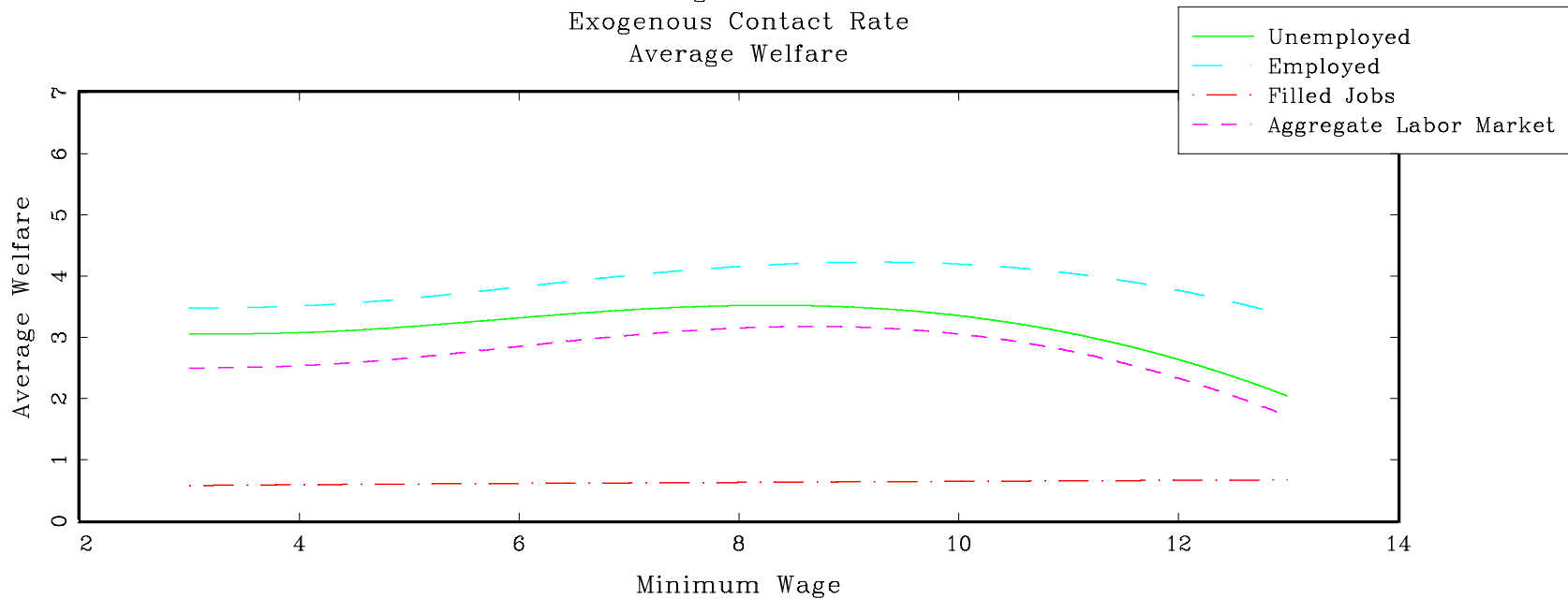


Figure 6.a
Endogenous Contact Rate
Group Size

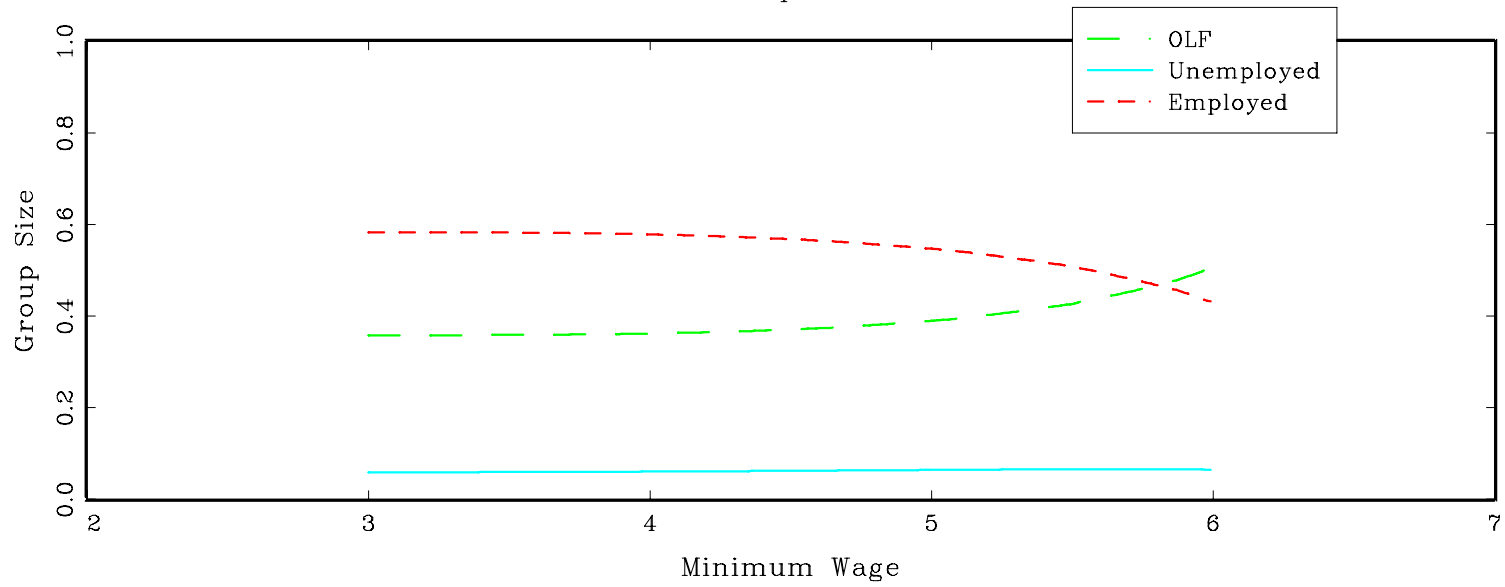


Figure 6.b
Endogenous Contact Rate
Average Welfare

