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In this paper we consider the (endogenous) choice of mode of behavior within intact households.
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- The only other empirical treatment of this problem of which we are aware is Flinn (2000), which considered the choice of divorced parents to behave efficiently or inefficiently.
Welfare implications regarding the husband and wife. The sustainability of the marriage itself may be influenced by whether efficient payoffs can be attained (though we don’t consider this in what is essentially a static model).
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If the welfare of children is a public good in the household, efficient outcomes result in higher child welfare (see Flinn (2000)).
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If the welfare of children is a public good in the household, efficient outcomes result in higher child welfare (see Flinn (2000)).

If leisure is a private good, moving to efficient outcomes will, in general, result in increases in the labor supply of both spouses. Thus a planner interested in aggregate labor supply would do well to implement policies that promote efficient intrahousehold time allocations.
Two agent households. Preferences of agent $i$ are given by

$$u_i(l_i, K) = \lambda_i \ln l_i + (1 - \lambda_i) \ln K,$$

where $l_i$ is the leisure of spouse $i$ and $K$ is a public good produced in the household.
The Household’s Decision Problem

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  where $l_i$ is the leisure of spouse $i$ and $K$ is a public good produced in the household.

- The household public good produced according to
  
  \[ K = \tau_1^{\delta_1} \tau_2^{\delta_2} M^{1-\delta_1-\delta_2}, \quad \delta_i \in (0, 1), \delta_1 + \delta_2 \leq 1, \]

  where $\delta_i$ is a technology parameter associated with spouse $i$ and total household income is given by

  \[ M = w_1 h_1 + w_2 h_2 + Y, \]

  where $w_i$ is the wage offer of spouse $i$. 

The time constraints of the spouses are given by

\[ h_i + \tau_i + l_i = T, \quad i = 1, 2, \]

where \( h_i \) is time in the market, \( \tau_i \) time in home production, and \( l_i \) leisure (the private good).
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The household’s problem is to set time allocation decisions,

\[ (h_1, \tau_1, h_2, \tau_2) \]

given a choice set described by the parameters

\[ (w_1, Y_1, w_2, Y_2, T), \]

where the price of purchased inputs for the household production technology is set to 1.
Two equation system for each spouse’s reaction function. Let $a_i \equiv (h_i \tau_i)$. For spouse 1,

$$a_1^*(a_2) = \arg \max_{a_1} u_1(a_1, a_2),$$

where the maximization is given nonlabor income $(Y + w_2 h_2)$ and given the time input of agent 2 in household production, $\tau_2$. 
Noncooperative (Inefficient) Behavior

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where the maximization is given nonlabor income $(Y + w_2 h_2)$ and given the time input of agent 2 in household production, $\tau_2$.

- The unique Nash equilibrium is defined by

$$a_1^N = a_1^*(a_2^N)$$
$$a_2^N = a_2^*(a_1^N),$$

with the Nash equilibrium payoff to spouse $i$ given by

$$V_i^N \equiv u_i(a_1^N, a_2^N).$$
We define an efficient outcome using the Benthamite Social Welfare function

\[(a_1^{PO}, a_2^{PO})(\alpha) = \arg \max_{a_1, a_2} \alpha u_1(a_1, a_2) + (1 - \alpha) u_2(a_1, a_2), \quad (1)\]

with \(\alpha \in (0, 1)\).
Efficient Outcomes (No Side Constraints)

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\[(a_{1}^{PO}, a_{2}^{PO})(\alpha) = \arg \max_{a_{1}, a_{2}} \alpha u_{1}(a_{1}, a_{2}) + (1 - \alpha) u_{2}(a_{1}, a_{2})\]

(1)

with \(\alpha \in (0, 1)\).

- By varying \(\alpha\) we trace out the Pareto frontier,

\[(u_{1}(a_{1}^{PO}(\alpha), a_{2}^{PO}(\alpha)), u_{2}(a_{1}^{PO}(\alpha), a_{2}^{PO}(\alpha))), 0 < \alpha < 1.\]
Efficient Outcomes (No Side Constraints)

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(a_{1}^{PO}, a_{2}^{PO})(\alpha) = \arg \max_{a_{1}, a_{2}} \alpha u_{1}(a_{1}, a_{2}) + (1 - \alpha) u_{2}(a_{1}, a_{2}), \quad (1)
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- By varying \(\alpha\) we trace out the Pareto frontier,

\[
(u_{1}(a_{1}^{PO}(\alpha), a_{2}^{PO}(\alpha)), u_{2}(a_{1}^{PO}(\alpha), a_{2}^{PO}(\alpha))), \quad 0 < \alpha < 1.
\]

- Then why would any household choose not to implement the cooperative solution?
Efficiency is a nice property for a household time allocation to have, but efficient outcomes may be “unreasonable” on other dimensions.
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(Constrained) Efficient Outcomes

- Efficiency is a nice property for a household time allocation to have, but efficient outcomes may be “unreasonable” on other dimensions.
- Efficient outcomes require particular types of coordination, trust, etc. to implement, as was especially convincingly discussed by Lundberg and Pollak (1993).
- Nash equilibrium actions are unique in our example and don’t require coordination (being best responses). They seem like a natural side constraint on the efficient outcomes set.
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Nash equilibrium actions are unique in our example and don’t require coordination (being best responses). They seem like a natural side constraint on the efficient outcomes set.

Let $\alpha_0$ denote a “notional” Pareto weight, which might be culturally determined, for example.
Define critical values

\[ \alpha(V_1^N) : u_1(a_1^{PO}(\alpha(V_1^N)), a_2^{PO}(\alpha(V_1^N))) = V_1^N \]

\[ \bar{\alpha}(V_2^N) : u_2(a_1^{PO}(\bar{\alpha}(V_2^N)), a_2^{PO}(\bar{\alpha}(V_2^N))) = V_2^N \]
(Constrained) Efficient Outcomes 2

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\]

\[
\underline{\alpha}(V_2^N) : u_2(a_1^{PO}(\underline{\alpha}(V_2^N)), a_2^{PO}(\underline{\alpha}(V_2^N))) = V_2^N
\]

- The \(CPO(\alpha_0)\) actions are:

\[
\{a_1^{CPO}(\alpha_0), a_2^{CPO}(\alpha_0)\} =
\begin{cases}
\{a_1^{PO}(\alpha_0), a_2^{PO}(\alpha_0)\} & \text{if } \underline{\alpha}(V_1^N) \leq \alpha_0 \leq \bar{\alpha}(V_2^N) \\
\{a_1^{PO}(\underline{\alpha}(V_1^N)), a_2^{PO}(\underline{\alpha}(V_1^N))\} & \text{if } \alpha_0 < \underline{\alpha}(V_1^N) \\
\{a_1^{PO}(\bar{\alpha}(V_2^N)), a_2^{PO}(\bar{\alpha}(V_2^N))\} & \text{if } \bar{\alpha}(V_2^N) < \alpha_0
\end{cases}
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Define critical values

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\alpha(V_1^N) & : u_1(a_1^{PO}(\alpha(V_1^N)), a_2^{PO}(\alpha(V_1^N))) = V_1^N \\
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\end{align*} \]

The \( CPO(\alpha_0) \) actions are:

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\{ a_1^{CPO}(\alpha_0), a_2^{CPO}(\alpha_0) \} &= \\
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\{ a_1^{PO}(\alpha(V_1^N)), a_2^{PO}(\alpha(V_1^N)) \} & \quad \text{if } \alpha_0 < \alpha(V_1^N) \\
\{ a_1^{PO}(\bar{\alpha}(V_2^N)), a_2^{PO}(\bar{\alpha}(V_2^N)) \} & \quad \text{if } \bar{\alpha}(V_2^N) < \alpha_0
\end{align*} \]

The restriction is manifested in a restricted connected set of Pareto weights and actions associated with them.
In this case, household behavior is endogenously selected.
Endogenous Interaction

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- The choice is between (inefficient) static Nash Equilibrium allocations and (efficient) Constrainted Pareto Optimal allocations.
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Consider the case in which the same static household game is played infinitely often.
Endogenous Interaction

- In this case, household behavior is endogenously selected.
- The choice is between (inefficient) static Nash Equilibrium allocations and (efficient) Constrainted Pareto Optimal allocations.
- Consider the case in which the same static household game is played infinitely often.
- Assume that couples play a grim trigger strategy with the punishment phase being perpetual Nash equilibrium play. Let $a_i(t)$ denote the choice of action of spouse $i$ in period $t$. Then each spouse uses the rule

$$a_{EI_i}^i(t) = \begin{cases} a_{PO_i}^i(\alpha) & \text{if } a_{i'}(s) = a_{PO_i}^i(\alpha), \ s = 1, \ldots, t-1 \\ a_{NE_i}^i & \text{if } a_{i'}(s) \neq a_{EI_i}^i & \text{for any } s = 1, \ldots, t-1 \end{cases}$$

in the equilibrium, which is conditional on a Pareto weight $\alpha$. 

Implementability is considered using Folk Theorem motivations.
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Each spouse’s objective is to maximize the present discounted value of their sequence of payoffs given the state variables characterizing the household and the history of past actions of the spouses.
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To determine whether or not cooperation is an equilibrium outcome, define the value of spouse 1 cheating on the cooperative agreement given that spouse 2 does not by

$$P_1^R(\alpha) = V_1^R(\alpha) + \beta \frac{V_1^{NE}}{1 - \beta},$$

where $V_1^R(\alpha) = \max_{a_1} u_1(a_1, a_2^{PO}(\alpha))$.  

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$$P^R_1(\alpha) = V^R_1(\alpha) + \beta \frac{V^NE_1}{1 - \beta},$$

(3)

where $V^R_1(\alpha) = \max\limits_{a_1} u_1(a_1, a^PO_2(\alpha))$.

If the spouse chooses to implement the cooperative outcome (and it is assumed that spouse 2 chooses $a^PO_2(\alpha)$), then the payoff from this action is

$$P^E_1(\alpha) = \frac{V^PO_1(\alpha)}{1 - \beta}.$$
Then spouse 1 is indifferent between reneging and implementing the efficient allocation when

\[ P_{1E}^{E}(\alpha) = P_{1R}^{R}(\alpha) \]

\[ V_{1PO}^{PO}(\alpha) = V_{1R}^{R}(\alpha) + \beta \frac{V_{1NE}^{NE}}{1 - \beta}. \]  \hspace{1cm} (4)
Then spouse 1 is indifferent between reneging and implementing the efficient allocation when

\[ P_1^E(\alpha) = P_1^R(\alpha) \]

\[ \Rightarrow \frac{V_1^{PO}(\alpha)}{1 - \beta} = V_1^R(\alpha) + \beta \frac{V_1^{NE}}{1 - \beta}. \]  

(4)

The discount factor \( \beta \) is not a determinant of stage game payoffs, so we can look for a critical value of the discount factor at which the equality (4) holds. This critical value is given by

\[ \beta_1^*(\alpha) = \frac{V_1^R(\alpha) - V_1^{PO}(\alpha)}{V_1^R(\alpha) - V_1^N}. \]

Note that if \( V_1^{PO}(\alpha) > V_1^N \), then \( V_1^R(\alpha) > V_1^{PO}(\alpha) \), and \( V_1^R(\alpha) - V_1^{PO}(\alpha) < V_1^R(\alpha) - V_1^N \), so that \( \beta_1^*(\alpha) \in (0, 1) \). Clearly, an exactly symmetric analysis can be used to determine a critical discount factor for the spouse 2, \( \beta_2^*(\alpha) \). We have the following result.
**Proposition** Under a grim trigger strategy and given constrained Pareto optimal actions \((a_1^E, a_2^E)(\alpha)\), the household implements the efficient outcome if and only if 
\[ \beta \geq \max\{\beta_1^*(\alpha), \beta_2^*(\alpha)\}, \]
where \(\beta\) is the common household discount factor and \(\alpha\) is the given Pareto weight.
• \textit{Proposition} Under a grim trigger strategy and given constrained Pareto optimal actions \((a_1^E, a_2^E)(\alpha)\), the household implements the efficient outcome if and only if \(\beta \geq \max\{\beta_1^*(\alpha), \beta_2^*(\alpha)\}\), where \(\beta\) is the common household discount factor and \(\alpha\) is the given Pareto weight.

• After determining the Nash bargaining solution to the household’s problem given a value of \(\alpha\), using (1), we can then see if they are implementable.
The spouses can easily improve on this outcome, even when required to use grim trigger strategies.
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Define

$$\beta^{**} = \min_{\alpha} \max \{ \beta_1^*(\alpha), \beta_2^*(\alpha) \}.$$
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Then if \( \beta \geq \beta^{**} \), the household will implement an efficient solution.
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Then if $\beta \geq \beta^{**}$, the household will implement an efficient solution.

Since the punishment for deviation is Nash equilibrium payoffs forever, the value given to each spouse must be at least the amount they would get under NE.
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Therefore the period values must be at least those attained under CPO, which was a restriction on the set of feasible Pareto weights to $\alpha \in [\alpha(V_1^N), \bar{\alpha}(V_2^N)]$. 

\[ \text{Daniela Del Boca, Christopher Flinn (Institute) Endogenous Household Interaction February 2009 14 / 27} \]
At $\beta = \beta^{**}$, there exists exactly one value of $\alpha$ consistent with implementation, which is

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$$\alpha^{**} = \arg \min_{\alpha} \max\{\beta^*_1(\alpha), \beta^*_2(\alpha)\}.$$ 

For $\beta > \beta^{**}$, there exists a connected set of values of $\alpha$ consistent with implementation, which are given by

$$[\alpha(V^N, \beta), \bar{\alpha}(V^N, \beta)],$$

with

$$[\alpha(V^N, \beta), \bar{\alpha}(V^N, \beta)] \subset [\alpha(V^N, \beta'), \bar{\alpha}(V^N, \beta')], \quad \beta < \beta'.$$
Let the notional Pareto weight be given by $\alpha_0$. Then the EI choices are given by

$$\{a_1^{EI}(\alpha_0), a_2^{EI}(\alpha_0)\} =$$

$$\begin{cases} 
\{a_1^{NE}, a_2^{NE}\} & \text{if } \beta < \beta^{**} \\
\{a_1^{PO}(\alpha_0), a_2^{PO}(\alpha_0)\} & \text{if } \alpha(\mathcal{V}^N, \beta) \leq \alpha_0 \leq \overline{\alpha}(\mathcal{V}^N, \beta), \\
\{a_1^{PO}(\alpha(\mathcal{V}^N, \beta)), a_2^{PO}(\alpha(\mathcal{V}^N, \beta))\} & \text{if } \alpha_0 < \alpha(\mathcal{V}^N, \beta), \beta \geq \beta \\
\{a_1^{PO}(\overline{\alpha}(\mathcal{V}^N, \beta)), a_2^{PO}(\overline{\alpha}(\mathcal{V}_2^N))\} & \text{if } \overline{\alpha}(\mathcal{V}^N, \beta) < \alpha_0, \beta \geq \beta 
\end{cases}$$
Figure 1
Critical $\beta$ Values
Figure 2
Pareto Frontier and Admissible Solutions
We draw a sample from the PSID for the year 2005.
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We have information on all of the dependent outcomes of the model
Estimation of the Model - Data

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We have information on all of the dependent outcomes of the model:
  - Labor market time and time spent in housework for both spouses.
We also have (average) hourly wage rates when the spouse is employed.
Given the assumptions of the model, there is income-pooling in terms of nonlabor income. We only use total household nonlabor income, computed as a residual.
Selection Criteria

- Married households, with both spouses between the ages of 30-49
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- No HH nonlabor income greater than $1000 per week
- No child less than 7 year of age in HH
## Table 1
### PSID 2005 Sample
#### Means and (Standard Deviations)

\[ N = 823 \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Husband</th>
<th>Wife</th>
</tr>
</thead>
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<tr>
<td>( h )</td>
<td>45.706</td>
<td>38.588</td>
</tr>
<tr>
<td></td>
<td>(8.546)</td>
<td>(10.512)</td>
</tr>
<tr>
<td>( \tau )</td>
<td>7.787</td>
<td>14.920</td>
</tr>
<tr>
<td></td>
<td>(6.418)</td>
<td>(9.428)</td>
</tr>
<tr>
<td>( w )</td>
<td>22.009</td>
<td>15.823</td>
</tr>
<tr>
<td></td>
<td>(13.626)</td>
<td>(9.327)</td>
</tr>
<tr>
<td>( Y )</td>
<td>118.151</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(182.526)</td>
<td></td>
</tr>
</tbody>
</table>
Table 2
Correlation Matrix of Observables

<table>
<thead>
<tr>
<th></th>
<th>$h_1$</th>
<th>$\tau_1$</th>
<th>$h_2$</th>
<th>$\tau_2$</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1$</td>
<td>1.000</td>
<td>-0.017</td>
<td>0.093</td>
<td>0.060</td>
<td>0.029</td>
<td>-0.004</td>
<td>0.084</td>
</tr>
<tr>
<td>$\tau_1$</td>
<td>1.000</td>
<td>0.081</td>
<td>0.321</td>
<td>-0.031</td>
<td>-0.026</td>
<td>0.024</td>
<td></td>
</tr>
<tr>
<td>$h_2$</td>
<td>1.000</td>
<td>-0.189</td>
<td>-0.132</td>
<td>0.084</td>
<td>0.011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>1.000</td>
<td>-0.018</td>
<td>-0.137</td>
<td>0.066</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_1$</td>
<td>1.000</td>
<td>0.294</td>
<td>0.115</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_2$</td>
<td>1.000</td>
<td>0.026</td>
<td></td>
<td></td>
<td></td>
<td></td>
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We observe $X = (h_1, \tau_1, h_2, \tau_2; w_1, w_2, Y)$. Our objective is to estimate parameters characterizing the population distributions of $S = (\lambda_1, \lambda_2, \delta_1, \delta_2), \alpha, \text{ and } \beta$. 

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We spend some time considering identification issues.
Households are described by the complete state variable vector

\[ S = (\lambda_1 \ \lambda_2 \ \delta_1 \ \delta_2 \ w_1 \ w_2 \ \ Y), \]

with the first four elements unobserved \((S_U)\) and the last three observed \((S_O)\).
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If so, \(\hat{F}_S(B)\) is the empirical distribution function of \((S_U(A, S_O; B) S_O)\).
Under our functional form assumptions, $\hat{F}_S(NE)$ is identified.

Similarly, $\hat{F}_S(PO_\alpha)$ is nonparametrically identified conditional on a value of $\alpha$.

For certain values of $\alpha$ and $(SOA)$, the inverse does not map into $(0,1)$ for $\lambda_1$ and/or $\lambda_2$.

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Let $x \sim MVN(\mu, \Sigma)$, where $x$ is 4-dimensional (14 free parameters).
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Then

\[
\begin{align*}
\lambda_1 &= \logit(x_1) \\
\lambda_2 &= \logit(x_2) \\
\delta_1 &= \exp(x_3)/(1 + \exp(x_3) + \exp(x_4)) \\
\delta_2 &= \exp(x_4)/(1 + \exp(x_3) + \exp(x_4))
\end{align*}
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\end{align*}
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We treated $\beta$ as fixed. The homogeneous specification of $\beta$ performed better than the heterogeneous specification we tried.
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\]

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Given the identifications issues with respect to \( \alpha \), we kept it fixed at 0.5 throughout, even though theoretically identified under the parametric specification.
Model is estimated using Method of Simulated Moments
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Chose 23 sample characteristics to fit, \( m \).
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\hat{\Omega} = \text{arg min} (m - M(\Omega))' W (m - M(\Omega)).
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$$\hat{\Omega} = \arg \min (m - M(\Omega))'W(m - M(\Omega)).$$

We have only taken 100 draws from the distributions for each observation, and bootstrap standard errors ($R = 25$) have been computed but are not presented.
### Table A.1

*Moments Used in the MSM Estimator*

<table>
<thead>
<tr>
<th>Sample Characteristic</th>
<th>Sample Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average $h_1$</td>
<td>45.706</td>
</tr>
<tr>
<td>Average $h_2$</td>
<td>38.588</td>
</tr>
<tr>
<td>Average $\tau_1$</td>
<td>7.787</td>
</tr>
<tr>
<td>Average $\tau_2$</td>
<td>14.920</td>
</tr>
<tr>
<td>St. Dev. $h_1$</td>
<td>8.546</td>
</tr>
<tr>
<td>St. Dev. $h_2$</td>
<td>10.512</td>
</tr>
<tr>
<td>Average $(h_1 \times h_2)$</td>
<td>1772.074</td>
</tr>
<tr>
<td>Average $(h_1 \times Y)$</td>
<td>5530.755</td>
</tr>
<tr>
<td>Average $(h_2 \times Y)$</td>
<td>4580.739</td>
</tr>
<tr>
<td>Average $(h_1 \times w_1)$</td>
<td>1009.293</td>
</tr>
<tr>
<td>Average $(h_2 \times w_2)$</td>
<td>618.773</td>
</tr>
<tr>
<td>Average $(\tau_1 \times Y)$</td>
<td>947.620</td>
</tr>
<tr>
<td>Average $(\tau_2 \times Y)$</td>
<td>1876.535</td>
</tr>
<tr>
<td>St. Dev. $\tau_1$</td>
<td>6.418</td>
</tr>
<tr>
<td>St. Dev. $\tau_2$</td>
<td>9.423</td>
</tr>
<tr>
<td>Average $(h_1 \times w_2)$</td>
<td>722.880</td>
</tr>
<tr>
<td>Average $(h_2 \times w_1)$</td>
<td>830.367</td>
</tr>
<tr>
<td>Average $(h_1 \geq 40)$</td>
<td>0.955</td>
</tr>
<tr>
<td>Average $(h_2 \geq 40)$</td>
<td>0.694</td>
</tr>
<tr>
<td>Average $(25 \leq h_1 &lt; 40)$</td>
<td>0.0346</td>
</tr>
<tr>
<td>Average $(25 \leq h_2 &lt; 40)$</td>
<td>0.210</td>
</tr>
</tbody>
</table>
The nonparametric estimators of $F_S(B)$ all fit equally well for all $B$ for which the inverse exists. Thus cannot compare model ‘fit’ for these specifications.
Focus of Interest in Estimates

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- Thus, EI nests CPO and NE through $\beta$. 
Table 3
Estimates of Primitive Parameters
Means and (Standard Deviations) of Fixed Effects Distributions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Behavioral Specification</th>
<th>NE</th>
<th>PO</th>
<th>CPO</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_1 )</td>
<td></td>
<td>0.369</td>
<td>0.580</td>
<td>0.531</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.095)</td>
<td>(0.166)</td>
<td>(0.106)</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td></td>
<td>0.302</td>
<td>0.430</td>
<td>0.456</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.102)</td>
<td>(0.158)</td>
<td>(0.120)</td>
</tr>
<tr>
<td>( \delta_1 )</td>
<td></td>
<td>0.075</td>
<td>0.075</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.057)</td>
<td>(0.057)</td>
<td>(0.057)</td>
</tr>
<tr>
<td>( \delta_2 )</td>
<td></td>
<td>0.106</td>
<td>0.106</td>
<td>0.106</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.066)</td>
<td>(0.066)</td>
<td>(0.066)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td></td>
<td>0.500</td>
<td>0.500</td>
<td></td>
</tr>
</tbody>
</table>
Table 4
Estimates of Primitive Parameter Moments
Flexible Parametric Specification
Means and (Standard Deviations)

<table>
<thead>
<tr>
<th></th>
<th>NE</th>
<th>PO</th>
<th>CPO</th>
<th>EI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>0.355</td>
<td>0.474</td>
<td>0.489</td>
<td>0.484</td>
</tr>
<tr>
<td></td>
<td>(0.247)</td>
<td>(0.088)</td>
<td>(0.197)</td>
<td>(0.094)</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.315</td>
<td>0.432</td>
<td>0.495</td>
<td>0.472</td>
</tr>
<tr>
<td></td>
<td>(0.196)</td>
<td>(0.112)</td>
<td>(0.229)</td>
<td>(0.137)</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.138</td>
<td>0.065</td>
<td>0.074</td>
<td>0.067</td>
</tr>
<tr>
<td></td>
<td>(0.158)</td>
<td>(0.050)</td>
<td>(0.070)</td>
<td>(0.065)</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.178</td>
<td>0.152</td>
<td>0.099</td>
<td>0.121</td>
</tr>
<tr>
<td></td>
<td>(0.216)</td>
<td>(0.162)</td>
<td>(0.029)</td>
<td>(0.100)</td>
</tr>
<tr>
<td>$\beta$</td>
<td></td>
<td></td>
<td></td>
<td>0.522</td>
</tr>
<tr>
<td>$\alpha$ (Actual)</td>
<td>0.500</td>
<td>0.509</td>
<td>0.528</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(--)</td>
</tr>
</tbody>
</table>

Proportion PF

|      | 0  | 1  | 1  | 0.941 |

Proportion $\alpha = 0.5$

|      | 0.591 | 0.092 |

Distance Measure

|      | 4897.747 | 5014.291 | 4209.456 | 3991.784 |
Preferred specification is EI.
Interpretation of Results

- Preferred specification is EI.
- Discount factor estimate is low, at $\hat{\beta} = 0.522$. 
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At this value, about 94 percent of households on Pareto frontier.

However, only about 10 percent have an *ex post* value of the Pareto weight equal to the *ex ante* value.
Figure 3.a
Distribution of $\alpha$
Constrained Pareto Optimal Specification
(Excludes Cases with $\alpha = 0.5$)

Figure 3.b
Distribution of $\alpha$
Endogenous Interaction Specification
(Excludes Efficient Cases with $\alpha = 0.5$)
Figure 4.a
Spousal Preference Parameters
Nash Equilibrium

Figure 4.b
Spousal Productivity Parameters
Nash Equilibrium

Figure 4.c
Husbands' Primitive Parameters
Nash Equilibrium

Figure 4.d
Wives' Primitive Parameters
Nash Equilibrium
Conclusions

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- It would be important to extend the constraint set to one more appropriate for a dynamic setting, as various authors have recently begun to do.
- Our estimates indicate considerable variability in household outcomes and behavioral patterns.
- A companion paper uses some of these findings to investigate their impact on sorting in the marriage market.