

Household Choices and Child Development

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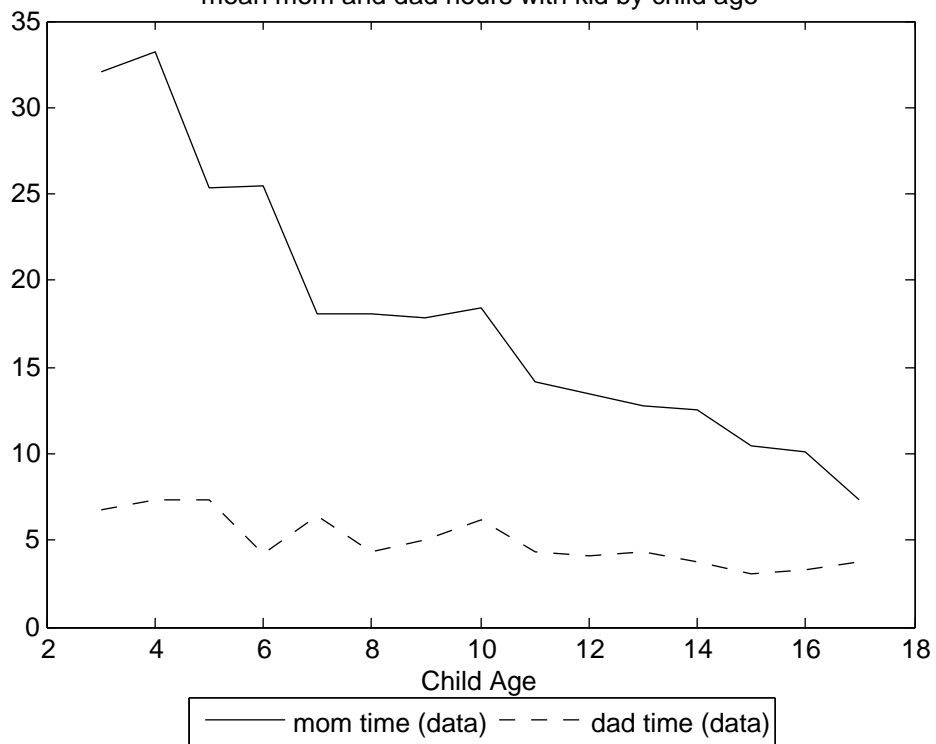
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- Increased labor market participation of mothers, particularly those with young children
- What are the implications for child development and welfare?
- Economic theory does not yield unambiguous predictions
- Empirical studies often have produced conflicting results, as well

Some Notable Empirical Research

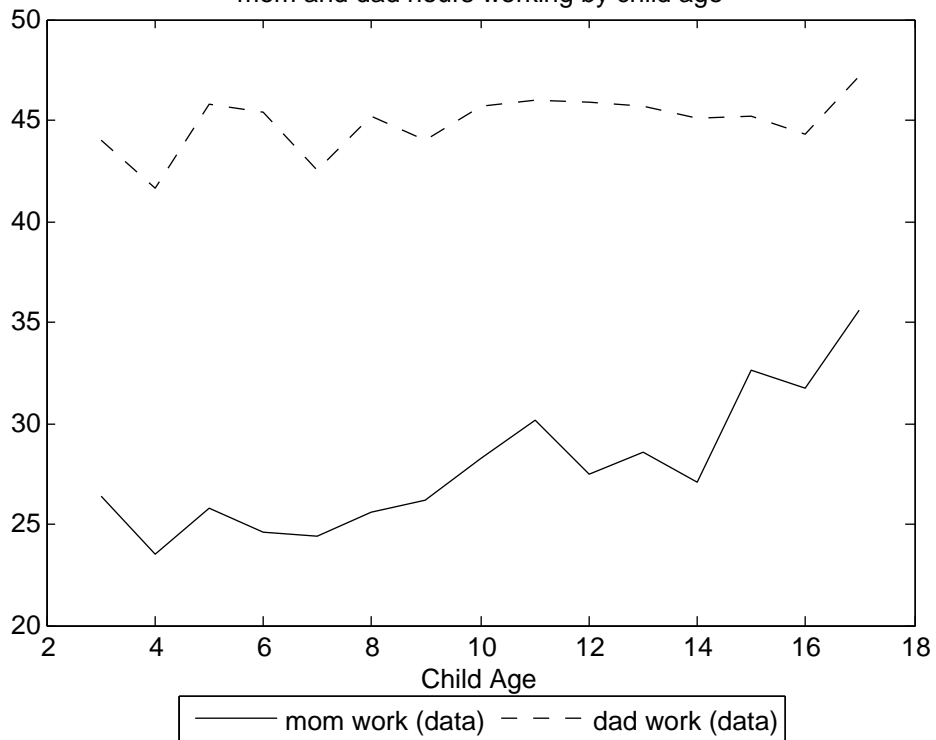
- Todd and Wolpin (2003) - Value added approach to child quality production, emphasis on schooling inputs
- Cunha and Heckman (2008) - Multiple indicator approach to child investment and outcomes
- Bernal (2008) - Single mothers work and child outcomes
- Brown and Flinn (2006) - Model of family law effects on fertility, divorce, and child investment

Figure 1: Average Hours with Children for Mothers and Fathers by Child Age
mean mom and dad hours with kid by child age



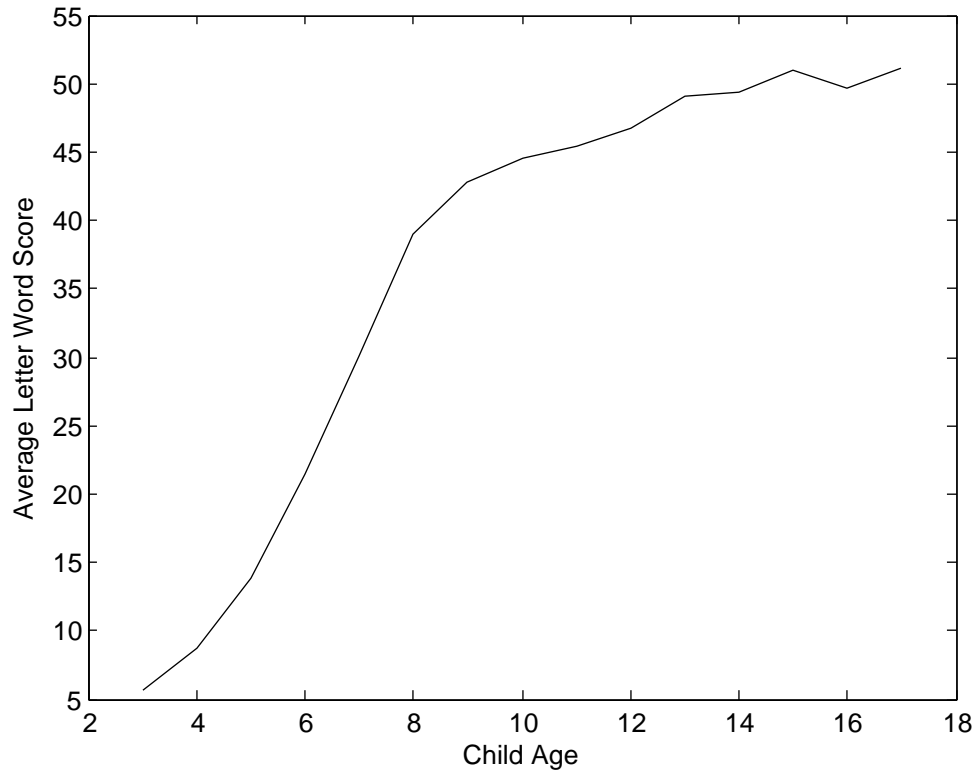
Notes:

Figure 2: Average Hours Working for Mothers and Fathers by Child Age
mom and dad hours working by child age



Notes:

Figure 3: Average Letter Work Score by Child Age



Notes:

- Begins with birth of child, development process lasts T periods (years)
- Household utility

$$u(l_{1t}, l_{2t}, c_t, k_t) = \alpha_1 \ln l_{1t} + \alpha_2 \ln l_{2t} + \alpha_3 \ln c_t + \alpha_4 \ln k_t, \quad \sum_j \alpha_j = 1$$

- Child quality production

$$k_{t+1} = f(k_t, \tau_{1t}, \tau_{2t}, e_t) = R_t \tau_{1t}^{\delta_{1t}} \tau_{2t}^{\delta_{2t}} e_t^{\delta_{3t}} k_t^{\delta_{4t}}$$

where $R_t > 0$ is a scaling factor (TFP).

- In period t

$$V_t(S_t) = \max_{l_{1t}, \tau_{1t}, l_{2t}, \tau_{2t}, e_t} u(l_{1t}, l_{2t}, c_t, k_t) + \beta V_{t+1}(S_{t+1}),$$

$$S_t = (k_t \ w_{1t} \ w_{2t} \ l_t)$$

and β ($\in [0, 1)$) is the discount factor.

- The state variable vector at the birth of the child are the initial conditions of the problem, $S_1 = (k_1 \ w_{11} \ w_{21} \ l_1)$. Constraint set in period t

$$TT = l_{jt} + h_{jt} + \tau_{jt}, \quad j = 1, 2.$$

$$c_t + e_t = w_{1t}h_{1t} + w_{2t}h_{2t} + l_t.$$

In period $T + 1$ the child development period ends. We *assume* that

$$V_{T+1}(w_{1,T+1}, w_{2,T+1}, l_{T+1}; k_{T+1}) = \tilde{V}_{T+1}(w_{1,T+1}, w_{2,T+1}, l_{T+1}) + \psi \alpha_4 \ln k_{T+1}$$

where

$$\begin{aligned} \tilde{V}_{T+1}(w_{1,T+1}, w_{2,T+1}, l_{T+1}) = & \\ & \max_{l_{1,T+1}, l_{2,T+1}} \alpha_1 \ln l_{1,T+1} + \alpha_2 \ln l_{2,T+1} \\ & + \alpha_3 \ln(w_{1,T+1}(TT - l_{1,T+1}) + w_{2,T+1}(TT - l_{2,T+1}) + l_{T+1}) \\ & + \beta \tilde{V}_{T+2}(w_{1,T+2}, w_{2,T+2}, l_{T+2}). \end{aligned}$$

- The problem becomes stationary after T

$$\tilde{V}_{T+s}(\tilde{S}_{T+s}) = \tilde{V}(\tilde{S}_{T+s}), \quad s = 1, 2, \dots,$$

where $\tilde{S}_t = (w_{1t} \ w_{2t} \ I_t)$.

- We have assumed that the “terminal” value of child quality in period $T + 1$ is given by $\psi \alpha_4 \ln k_{T+1}$.
- If we assumed child quality remained the same $T + 1, \dots$, then

$$\psi = (1 - \beta)^{-1}.$$

Terminal Valuation - 3

- With no saving or borrowing allowed, the period s ($\in T + 1, \dots$) optimization problem is

$$\max_{l_{1,s}, l_{2,s}} \alpha_1 \ln l_{1,s} + \alpha_2 \ln l_{2,s} + \alpha_3 \ln(w_{1,s}(TT - l_{1,s}) + w_{2,s}(TT - l_{2,s}) + l_s).$$

- We can write

$$\begin{aligned} V_T(w_{1T}, w_{2T}, l_T, k_T) = & \max_{l_{1T}, \tau_{1T}, l_{2T}, \tau_{2T}, e_T} \alpha_1 \ln l_{1T} + \alpha_2 \ln l_{2T} + \alpha_3 \ln c_T \\ & + \psi \alpha_4 \{ \delta_{1T} \ln \tau_{1T} + \delta_{2T} \ln \tau_{2T} + \delta_{3T} \ln e_T \} \\ & + Q_T(k_T, \tilde{S}_{T+1}), \end{aligned}$$

where $Q_T(k_T, \tilde{S}_{T+1})$ is not a function of any current period choices.

- Factors of production:

$$\tau_{1t} = (TT - h_{1t}) \frac{\varphi_{1t}}{\alpha_1 + \varphi_{1t}}$$

$$\tau_{2t} = (TT - h_{2t}) \frac{\varphi_{2t}}{\alpha_2 + \varphi_{2t}}$$

$$e_t = (w_{1t}h_{1t} + w_{2t}h_{2t} + l_t) \frac{\varphi_{3t}}{\alpha_3 + \varphi_{3t}},$$

where

$$\varphi_{jt} = \beta \delta_{jt} \eta_{t+1}, \quad j = 1, 2, 3.$$

- The sequence $\{\eta_t\}_{t=1}^{T+1}$ is defined (backwards-) recursively as

$$\begin{aligned}\eta_{T+1} &= \psi\alpha_4 \\ \eta_T &= \alpha_4 + \beta\delta_{4,T}\eta_{T+1} \\ &\vdots \\ \eta_t &= \alpha_4 + \beta\delta_{4,t}\eta_{t+1} \\ &\vdots \\ \eta_1 &= \alpha_4 + \beta\delta_{4,2}\eta_2.\end{aligned}$$

η_t is the period t marginal utility of (log) child quality to the household: $\eta_t = \partial V(S_t) / \partial \ln k_t$.

- Hours choices: Define

$$A_{1t} = A_1(w_{1t}, l_t, \alpha_1, \alpha_3, \delta_{1t}, \delta_{3t}, \beta, \eta_{t+1})$$

$$A_{2t} = A_2(w_{1t}, w_{2t}, \alpha_1, \alpha_3, \delta_{1t}, \delta_{3t}, \beta, \eta_{t+1})$$

$$B_{1t} = B_1(w_{2t}, l_t, \alpha_2, \alpha_3, \delta_{2t}, \delta_{3t}, \beta, \eta_{t+1})$$

$$B_{2t} = B_2(w_{1t}, w_{2t}, \alpha_2, \alpha_3, \delta_{2t}, \delta_{3t}, \beta, \eta_{t+1}).$$

- Then

$$(h_{1t}^*, h_{2t}^*) = \begin{cases} (0, 0) & \text{if } A_{1t} \leq 0 \text{ and } B_{1t} \leq 0 \\ (A_{1t}, 0) & \text{if } A_{1t} - A_{2t}B_{1t} > 0 \text{ and } B_{1t} - B_{2t}A_{1t} < 0 \\ (0, B_{1t}) & \text{if } A_{1t} - A_{2t}B_{1t} < 0 \text{ and } B_{1t} - B_{2t}A_{1t} > 0 \\ (\hat{h}_{1t}, \hat{h}_{2t}) & \text{if } A_{1t} - A_{2t}B_{1t} \geq 0 \text{ and } B_{1t} - B_{2t}A_{1t} \geq 0 \end{cases}$$

Characteristics of Decision Rules

- Decision rules independent of shocks to production.
- Due to the assumption of no saving or borrowing, decisions independent of future paths of wages and nonlabor income
- The dynamics are captured solely through

$$\{\eta_t\}_{t=2}^{T+1}$$

- Makes solving the problem extremely fast, even with 5 endogenous variables in each period.

- Production function varies with age

$$\delta_{jt} = \exp(\gamma_{j0} + \gamma_{j1}t), \quad j = 1, \dots, 4; t = 1, \dots, T.$$

- Similarly, we assume that TFP, R_t , is given by

$$R_t = \exp(\gamma_{00} + \gamma_{01}t + \varepsilon_t),$$

where ε_t is i.i.d. $N(0, \sigma_R^2)$. Thus the production process is characterized by 11 parameters, $(\{\gamma_{j0}, \gamma_{j1}\}_{j=1}^5, \sigma_R)$.

Econometric Issues - Specification 2

- Heterogeneous household preferences $G(\alpha; \theta)$
- Let the 3×1 vector v be normally distributed with

$$v \sim N(\mu_\alpha, \Sigma_\alpha),$$

and define

$$\alpha_1 = D^{-1} \exp(v_1)$$

$$\alpha_2 = D^{-1} \exp(v_2)$$

$$\alpha_3 = D^{-1} \exp(v_3)$$

$$\alpha_4 = D^{-1}$$

- Wage process:

$$\begin{bmatrix} \ln w_{1,t} \\ \ln w_{2,t} \end{bmatrix} = \begin{bmatrix} \mu_{1,t} \\ \mu_{2,t} \end{bmatrix} + \begin{bmatrix} \omega_{1,t} \\ \omega_{2,t} \end{bmatrix},$$

where

$$\begin{bmatrix} \omega_{1,t} \\ \omega_{2,t} \end{bmatrix} = \begin{bmatrix} \rho_1 \omega_{1,t-1} \\ \rho_2 \omega_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix},$$

and where

$$\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \stackrel{i.i.d.}{\sim} N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} \right), \quad t = 2, 3, \dots$$

Specification 4.

- Initial wage disturbance draws taken from SS distribution
- Nonlabor income process: Autogressive Tobit structure

$$I_t^* = \mu_{3,t} + \omega_{3,t},$$

be the latent nonlabor income in period t , with a mean given by $\mu_{3,t}$ and with $\omega_{3,t} = \rho_3 \omega_{3,t-1} + \varepsilon_{3,t}$, where $\varepsilon_{3,t} \stackrel{i.i.d.}{\sim} N(0, \sigma_{33})$, $t = 2, 3, \dots$

- The actual nonlabor income process is given by

$$I_t = \max(0, I_t^*), \text{ for all } t.$$

- We treat child quality as a latent variable (k_t^*) with

$$k_t = \min(57, k_t^*)$$

- If all variables appearing in the model were available every year in the PSID, we show that identification of all model parameters is straightforward.
- Large gaps in observations on child outcomes and investments requires us to use simulation estimators.
- We found the MSM estimator performed well in Monte Carlo studies, and in our empirical application we obtained reasonable estimates of model parameters.

- Given initial conditions (observed child quality in the first wave and draws of wage and nonlabor income shocks from SS distributions, we simulate paths of exogenous and endogenous variables over the development cycle.
- PSID regular interviews were conducted in 1997, 1999, 2001, 2003, etc. CDS was conducted in 1997 and 2002-3. Using these data, we form sample moment information, M_N
- Primitive parameters contained in vector Ω .
- Using primitive parameter vector Ω , NS simulation draws per household, we form $NS \times N$ sample paths, and from these “data” compute analogous sample characteristics, $\tilde{M}_{NS,N}(\Omega)$



$$\hat{\Omega}_{NS,N,W} = \arg \min_{\Omega} (M_N - \tilde{M}_{NS,N}(\Omega))' W (M_N - \tilde{M}_{NS,N}(\Omega)),$$

- Moments include:
 - mean and s.d. for each child age of:
 - child test score
 - hours of work of mothers and fathers
 - hours with the child spent by mothers and fathers (individually)
 - autocorrelation in wages, correlations in wages across parents.

- Data taken from the PSID and Child Development Supplement (detailed information on up to 2 children per household)
- Child quality is measured by the raw score (not age normed) of the Letter-Word Identification subtest of the Woodcock-Johnson Achievement test
- The CDS collects detailed time diary and expenditure information in addition to the aptitude-ability tests administered.
- We use one child by household, the oldest one in the appropriate age range for inclusion in the CDS (0-12 years).
- After excluding cases with missing data, etc., $N = 491$.

Table 1: **Descriptive Statistics**

Variable	1997		2002-03	
	Mean	(Std.)	Mean	(Std.)
Child's age	7.45045	(2.839872)		
Father's age	38.43372	(6.305459)		
Mother's age	36.12484	(5.606386)		
Father's education	13.25997	(2.776811)		
Mother's education	13.30116	(2.600934)		
Father's time with child	5.577536	(8.48059)	4.255899	(7.304067)
Mother's time with child	22.33422	(16.94523)	12.95109	(12.64746)
Letter Word raw score	29.38867	(16.35427)	47.19691	(6.304849)

Notes: Sample of
Source: PSID.

Table 2: Descriptive Statistics for Labor Supply, Wages, and Income

Variable	1996		1998		2000	
	Mean	(Std.)	Mean	(Std.)	Mean	(Std.)
Mother's work hours per week	25.75	(18.59)	25.89	(17.91)	26.79	(17.81)
Father's work hours per week	44.43	(10.01)	45.49	(10.83)	45.31	(11.32)
Father's hourly wage	20.71	(13.40)	22.03	(14.45)	23.99	(16.11)
Mother's hourly wage	10.05	(10.03)	11.13	(10.61)	11.30	(10.25)
Weekly non-labor income	66.90	(135.08)	–	–	98.63	(166.24)

Notes: Sample of
Source: PSID.

- $T = 16$
- $\beta = 0.95$
- All other parameters freely estimated.
- Specification 2 of model has

$$\delta_{jt} = \exp(\gamma_{j0} + \gamma_{j1}t + \gamma_{j2}s_j), \quad j = 1, 2,$$

Table 3: Preference Parameter Estimates

	(1)	(2)
Mean of α_1	0.1058 ()	0.1028 ()
Mean of α_2	0.2181 ()	0.2201 ()
Mean of α_3	0.3394 ()	0.3374 ()
Mean of α_4	0.3367 ()	0.339 ()
Std. of α_1	0.0050 ()	0.0242 ()
Std. of α_2	0.0417 ()	0.0445 ()
Std. of α_3	0.0509 ()	0.0553 ()
Std. of α_4	0.0309 ()	0.0380 ()
Correlation of α_1 and α_2	-0.0466 ()	0.9484 ()
Correlation of α_1 and α_3	-0.6241 ()	-0.7063 ()
Correlation of α_2 and α_3	-0.7215 ()	-0.8794 ()
ψ (Terminal Payoff to Child Quality)	8.3595 ()	7.7165 ()

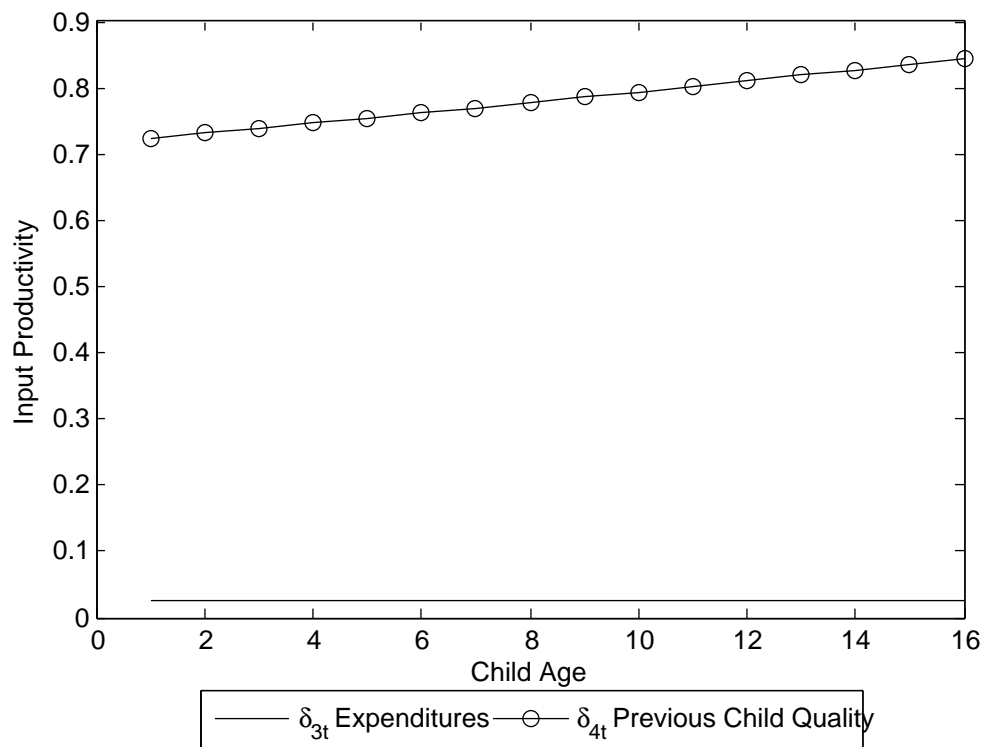
Notes: Model 1 does not include parental education in the production function. Model 2 includes parental education.

Table 4: **Technology Parameter Estimates**

γ_{00} (TFP R intercept)	0.2106	0.2113
	()	()
γ_{01} (TFP R slope)	0.0003	0.0012
	()	()
σ_R (Std. of Child Quality Shock)	0.0031	0.0008
	()	()
γ_{10} (Mother's Time intercept)	-1.9948	-2.0115
	()	()
γ_{11} (Mother's Time slope)	-0.0981	-0.0990
	()	()
γ_{12} (Mother's Time education slope)	-	0.0026
		()
γ_{20} (Father's Time intercept)	-1.7948	-1.8143
	()	()
γ_{21} (Father's Time slope)	-0.0704	-0.0707
	()	()
γ_{22} (Father's Time education slope)	-	0.0000
		()
γ_{30} (Child Expenditures intercept)	-3.6741	-3.7037
	()	()
γ_{31} (Child Expenditures slope)	-0.0011	-0.0003
	()	()
γ_{40} (Last Period's Child Quality intercept)	-0.3221	-0.3238
	()	()
γ_{41} (Last Period's Child Quality slope)	0.0101	0.0103
	()	()

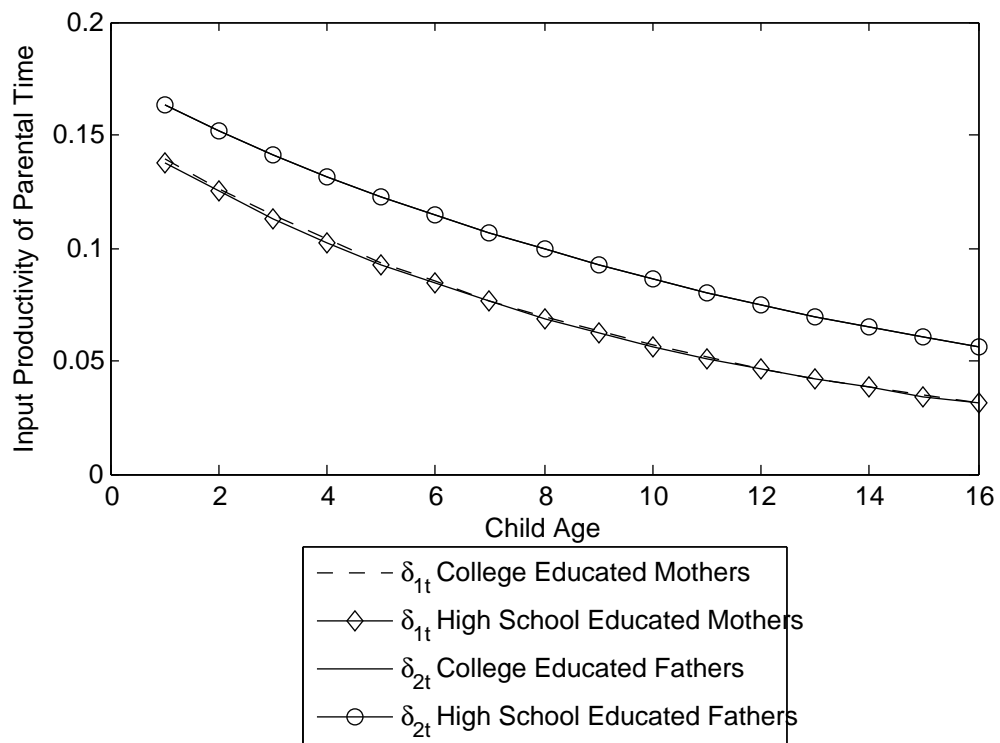
Notes: Model 1 does not include parental education in the production function. Model 2 includes parental education.

Figure 4: Estimated Child Development Parameters by Child Age



Notes:

Figure 5: Estimated Child Development Parameters by Child Age



Notes:

Table 5: **Wage and Income Parameter Estimates**

	(1)	(2)
Mother's Log Wage Offer		
μ_1^0 (Intercept)	-0.4465	-0.3656
	()	()
μ_1^1 (Mother's Schooling)	0.1555	0.1510
	()	()
μ_1^2 (Mother's Age)	0.0107	0.0102
	()	()
μ_1^3 (Mother's Age Sq x 1000)	-0.0601	-0.0648
	()	()
σ_1 (Standard Deviation of Innovation)	0.2153	0.2179
	()	()
ρ_1 (Auto-Correlation)	-0.5072	-0.4976
	()	()
ρ_{12} (Correlation with Father's Wage Shock)	0.7595	0.7519
	()	()
Father's Log Wage Offer		
μ_2^0 (Intercept)	0.6560	0.6417
	()	()
μ_2^1 (Father's Schooling)	0.1643	0.1627
	()	()
μ_2^2 (Father's Age)	-0.0009	-0.0002
	()	()
μ_2^3 (Father's Age Sq x 1000)	-0.0059	-0.0114
	()	()
σ_2 (Standard Deviation of Innovation)	0.7018	0.7312
	()	()
ρ_2 (Auto-Correlation)	-0.6221	-0.5875
	()	()

Notes: Model 1 does not include parental education in the production function. Model 2 includes parental education.

Table 6: **Wage and Income Parameter Estimates (con't)**

	(1)	(2)
	Latent Non-Labor Income	
μ_3^0 (Intercept)	-92.5490	-92.5490
	()	()
σ_3 (Standard Deviation of Innovation)	289.2813	289.2813
	()	()
ρ_3 (Auto-Correlation)	0.3591	0.3591
	()	()

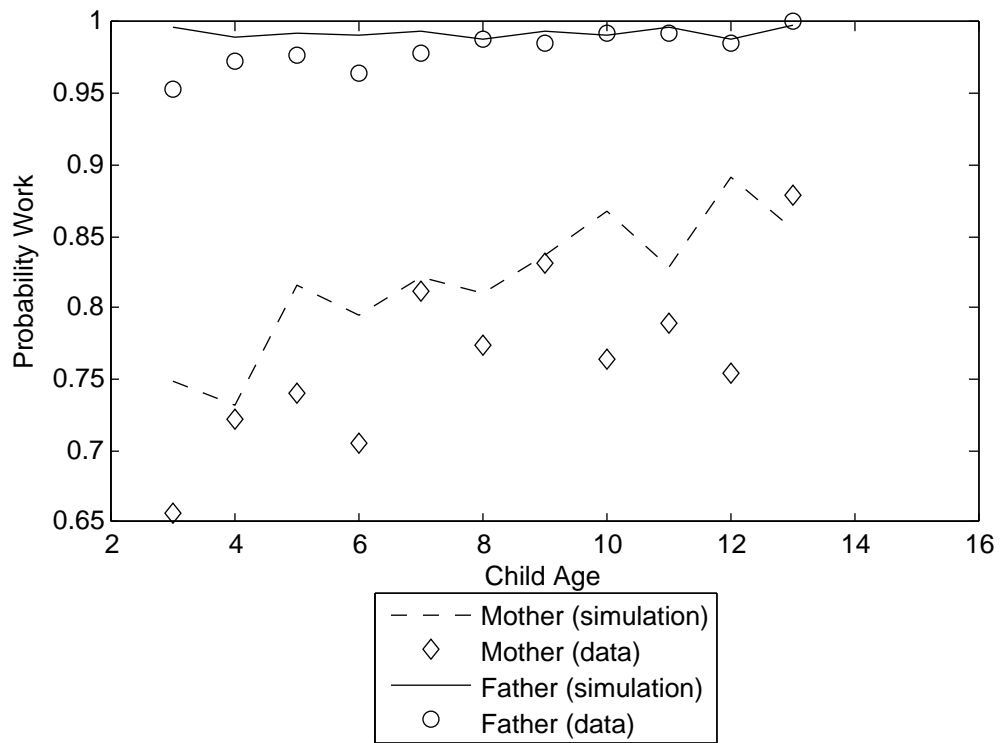
Notes: Model 1 does not include parental education in the production function. Model 2 includes parental education.

Table 7: **Sample Fit for Wages and Income**

	Data	Simulated
Avg. Mother's Wage	10.8278	10.4955
Std. Mother's Wage	10.3097	9.1187
Avg. Father's Wage	22.2421	21.5814
Std. Father's Wage	14.7498	17.3501
Avg. Non-Labor Income	82.7660	82.7801
Std. Non-Labor Income	150.6585	150.1142

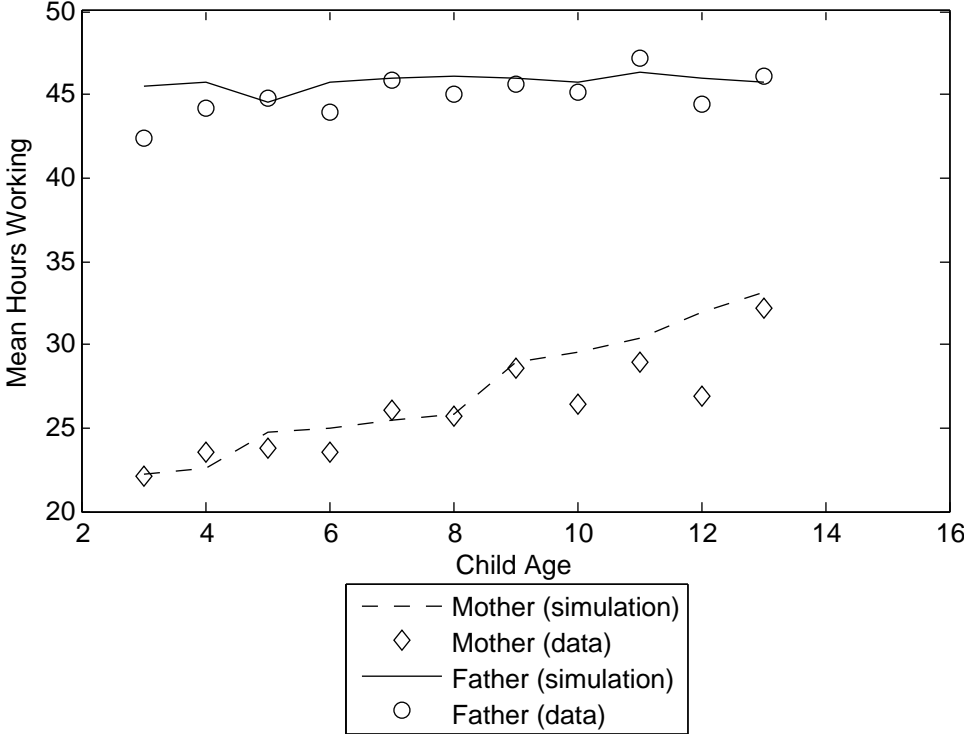
Notes: Sample of

Figure 6: Employment Probabilities for Mothers and Fathers by Child Age



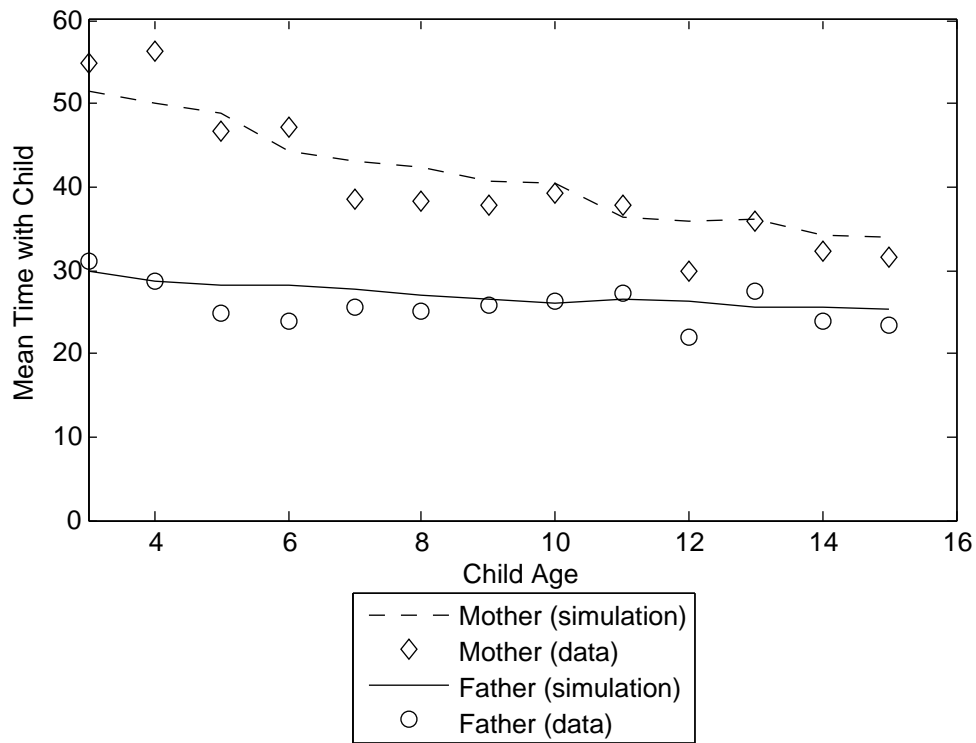
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Figure 7: Average Hours Working for Mothers and Fathers by Child Age



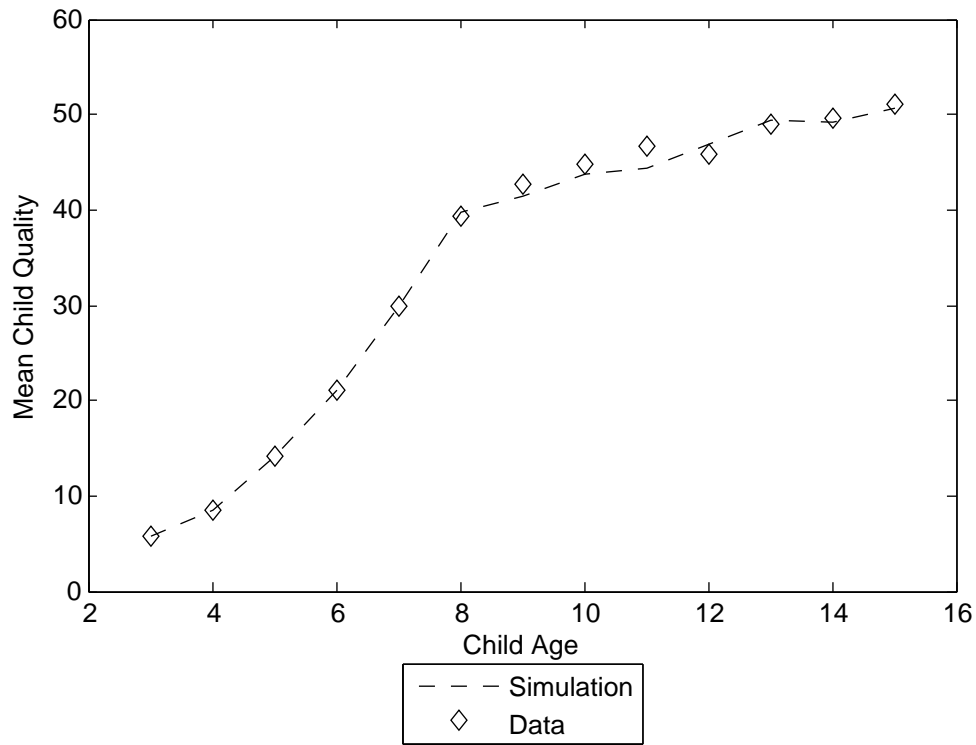
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Figure 8: Average Hours with Children for Mothers and Fathers by Child Age



Notes:

Figure 9: Average Letter Work Score by Child Age



Notes:

Comparative Statics Exercises

- Increase simulated wages by 10 percent in each period for mothers and fathers separately
- Look at impact on household investment behavior, final child outcomes
- Net effects not large, reflecting counterbalancing effects on labor supply, time spent with the child, and expenditures on the child

Table 8: **Comparative Statics**

	Level at Baseline	Mother's Wage Increase	10 % Increase in Father's Wage
Mean Child Quality at Age 16	53.7174	0.008	0.096
Mean Hours Work (Mother)	30.9604	1.379	-1.329
Mean Hours Work (Father)	45.0087	-0.423	0.425
Mean Time w/ Child (Mother)	38.8619	-0.530	0.511
Mean Time w/ Child (Father)	26.6258	0.286	-0.287
Mean Child Expenditures	166.9252	0.289	0.695
Mean Household Consumption	1,273.30	0.288	0.696

Input Allocation Exercise

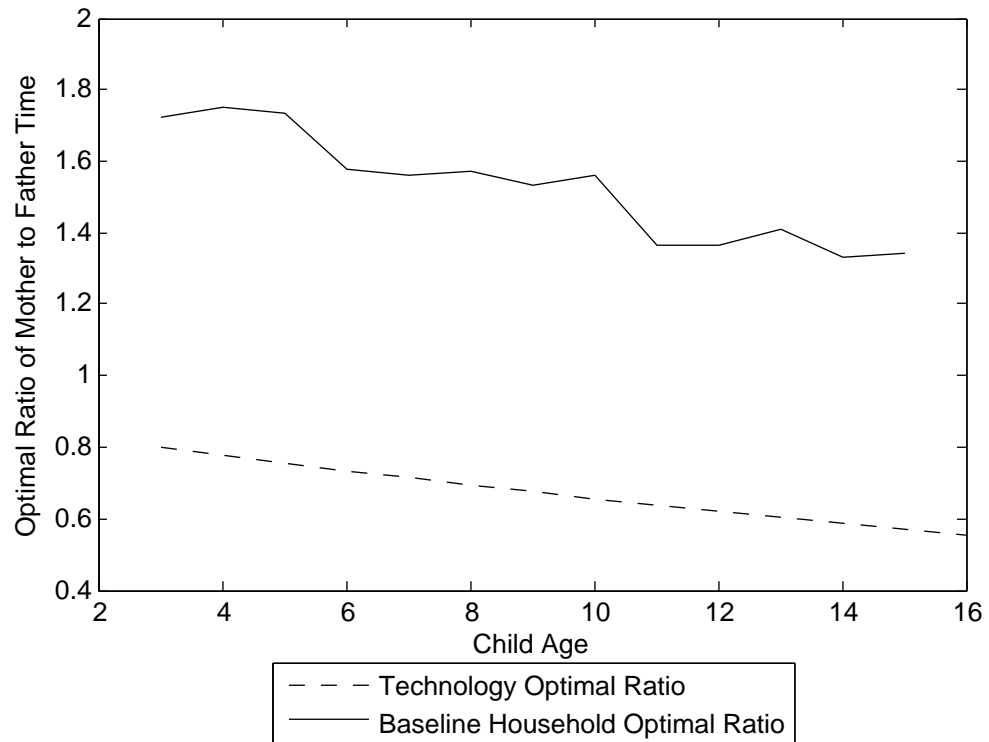
- If household only cared about child quality ($\alpha_1 = \alpha_2 = \alpha_3 = 0$ for all households)
- $\alpha_4 = 0$ (no investment in children), to look at parental labor supply behavior
- Technologically optimal allocations (ratios of inputs)

Table 9: **Optimal Decisions with Alternative Preferences**

	Level at Baseline	Child Quality Maximizing Preferences	Selfish Parent Preferences
Mean Child Quality at Age 16	53.6935	121.9042	0
Mean Hours Work (Mother)	31.1472	6.2011	49.9173
Mean Hours Work (Father)	45.0536	18.5512	54.1575
Mean Time w/ Child (Mother)	38.7207	105.7989	0
Mean Time w/ Child (Father)	26.5843	93.4488	0
Mean Child Expenditures	167.0617	600.1822	0
Mean Household Consumption	1,270.60	0	1,809.40

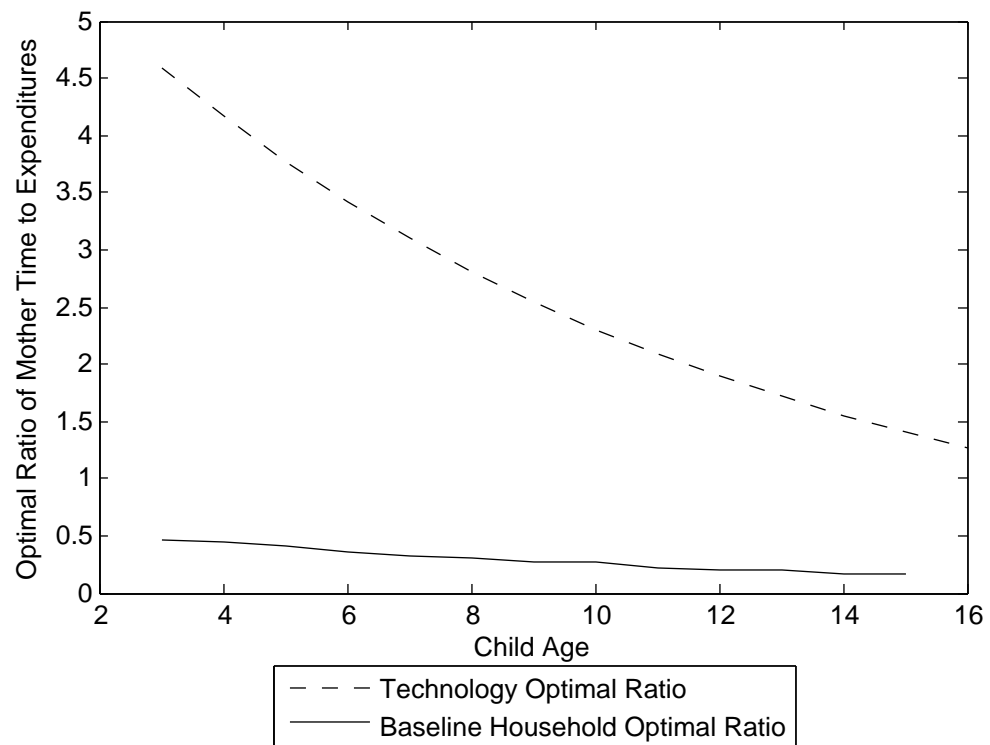
Notes: Child Quality Maximizing Preferences set preference weight on parental leisure and consumption to 0: $\alpha_1 = \alpha_2 = \alpha_3 = 0$. Under these preferences, the household then maximizes the level child quality, and consumption $c_t = 0$ for all t . Selfish Parent Preferences set $\alpha_4 = 0$, and the household puts no weight on child quality. With these preferences, $\tau_{1t} = \tau_{2t} = e_t = 0$ for all t . Mean Child Quality at Age 16 is the terminal latent child quality level (i.e. the level at the start of period $t = 17$, k_{17}). Note that with the test score ceiling of $\bar{k} = 57$, the *measured* score would be truncated. We impose this test score ceiling in the estimation to match the data generating process. Here we report the latent, non-truncated scores in the policy simulations. Latent child quality reported here can therefore exceed \bar{k} .

Figure 10: Optimal Ratio of Mother's and Father's Time with Child under Different Modeling Assumptions



Notes:

Figure 11: Optimal Ratio of Mother's Time with Child and Child Expenditures under Different Modeling Assumptions



Notes:

- Public Schooling - Households receive \$192 per week in nonlabor income, with the constraint that \$192 is spent on e
- Restrictions on labor supply - French case, no parent may work more than 35 hours a week.

Table 10: **Policy Simulations**

	Experiment	(1)	(2)	(3)
	Level at Baseline	Child Expend. Floor of $e \geq \$192$ and Inc. Transf. of \$192	Income Transfer of \$ 192 Only	Hours Ceiling $h_{1t} \leq 35$ and $h_{2t} \leq 35$
Mean Child Quality at Age 16	53.6935	58.0148	56.6422	56.0630
Mean Hours Work (Mother)	31.1472	28.6156	26.1458	29.8259
Mean Hours Work (Father)	45.0536	41.7178	38.7838	33.7879
Mean Time w/ Child (Mother)	38.7207	39.4077	40.5746	38.8941
Mean Time w/ Child (Father)	26.5843	27.7023	28.8464	31.1688
Mean Child Expenditures	167.0617	236.1468	180.2980	134.6063
Mean Household Consumption	1,270.60	1,318.70	1,341.40	990.30

Notes: Experiment 1 gives each household an income transfer of \$192 each week and forces the household to spend at least \$192 on child goods. Experiment 2 gives each household the same \$192 income transfer each week but leaves the household unrestricted in how the household spends the extra income. Experiment 3 imposes a labor supply ceiling on the parents and forces mothers and fathers to work 35 or fewer hours each week. Mean Child Quality at Age 16 is the terminal latent child quality level (i.e. the level at the start of period $t = 17$, k_{17}). Note that with the test score ceiling of $\bar{k} = 57$, the *measured* score would be truncated. We impose this test score ceiling in the estimation to match the data generating process. Here we report the latent, non-truncated scores in the policy simulations.