

**Labor Economics II**  
**Assignment 1**  
**Spring 2006**

*Due Date: February 2*

In the Salop model, individuals search for a job in different “labor markets,” where each labor market is characterized by a different probability of getting a job offer in a period of search. Once an offer is received in a given market, the individual must either take it, move on to another market to continue search, or stop searching altogether.

Consider a situation in which there exists 10 separate markets. In each market the wage offer distribution ( $F$ ) is the same, as is the cost of search  $c$ . The markets only differ in the probability of getting a wage offer in each period of search. In market 1, the offer probability is .1, in market 2 the offer probability is .2, etc. Thus

$$p_m = .1m, \quad m = 1, \dots, 10,$$

where  $m$  indicates the market number. As Salop demonstrates, given that the individual searches at all, she should begin by searching in the best market, which is the one in which the offer probability is highest. If she doesn't receive an acceptable offer in that market, she should consider searching in the next best market. This process continues until all the markets have been exhausted, or until the value of nonparticipation is greater than the value of search in the best remaining market.

Perhaps the easiest way to solve this problem is to use backwards recursion, thus we first consider what happens in the worst market the individual faces [which will be the last one in which she could search]. Now in the worst market, the probability of getting an offer is .1. If an offer is received and not accepted in this market, the individual must stop searching altogether since there will be no markets left in which to search. Thus the value of not accepting an offer in this case is equal to the value of nonparticipation for the rest of the individual's life, which we will normalize to 0 [i.e.,  $V_0 = 0$ ]. Then write

$$V_1 = -c + \beta[.1 \int \max[\frac{w}{1-\beta}, 0] dF(w) + .9V_0].$$

Given values of  $c$ ,  $\beta$ , and  $F$ , this expression can be solved for  $V_1$ . If the expression is less than 0, then the individual will not search in the first market, and the value of  $V_1$  should be set to the value of nonparticipation [which is 0].

The value of searching in the second market is similarly

$$V_2 = -c + \beta[.2 \int \max[\frac{w}{1-\beta}, V_1] dF(w) + .8V_1].$$

Once again, if  $V_2$  is less than 0, the individual would never search in the second market, and  $V_2$  should be reset to the value of nonparticipation, which is 0.

This process is repeated until we get to the 10<sup>th</sup> market. The value of search here is

$$V_{10} = -c + \beta \int \max\left[\frac{w}{1-\beta}, V_9\right] dF(w).$$

You are to write a program (e.g., MATLAB, GAUSS, FORTRAN) to solve the above problem when  $c = 1$  and  $\beta = .8$ . Further assume that  $w$  is distributed according to a uniform on the interval  $[0,10]$ , that is,

$$\begin{aligned} F(w) &= \frac{w}{10}, \quad w \in [0, 10] \\ f(w) &= .1, \quad w \in [0, 10] \end{aligned}$$

1. Solve for the reservation wage sequence  $w_{10}^*, w_9^*, \dots$  for all the markets in which the agent would actually search. For all such markets, compute the probability of getting an acceptable offer, as well as the expected value of an accepted wage offer in market  $s$  when an individual (optimally) searches in market  $s$ .
2. Compute the conditional probability of leaving unemployment after  $t$  periods given unemployment through  $t$  periods [i.e., the discrete time hazard function] for this model when  $t = 1, 2, 3$ . Provide a characterization of the hazard for any arbitrary  $t$ ,  $t = 1, 2, \dots$ .
3. Say the number of markets was countably infinite and that in each the probability of receiving an offer in a period was  $.8$  [i.e.,  $p_m = .9$ ,  $m = 1, 2, \dots$ ]. Solve for the reservation wage in this case [continue to assume that  $\beta = .8$  and  $c = 1$ ], the discrete time hazard, and the expectation of the *accepted* wage distribution.
4. Consider a simple generalization of the discrete time model. Let the times between offers be negatively exponentially distributed, so that the time to contacting the  $j^{\text{th}}$  firm in the search process is given by  $t_j$ , which has a p.d.f. given by

$$g_j(t) = \lambda_j \exp(-\lambda_j t), \quad \lambda_j > 0 \quad \forall j,$$

and without loss of generality assume that  $\lambda_1 = .9 > \lambda_2 = .8 > \dots > \lambda_{10} = 0$ . Replace the discrete time discount factor  $\beta$  with a rate of time preference parameter  $\rho > 0$ . Let the wage offer distribution continue to be fixed at a known  $F$ , and let the cost of search remain at  $c = 1$ , but interpreted as a rate. Find a value of  $\rho$  such that no individual will search at more than 5 firms.

5. Does the model in (4) produce the “discouraged worker” phenomenon? If not, think of how it might be generalized to do so.