

Labor Economics II
Assignment 4
Spring 2005

Due Date: April 5

Assume that spouses who inhabit the same household each have utility functions given by

$$u_i(l_i, K) = \alpha_i \ln(l_i) + (1 - \alpha_i) \ln K, \quad i = 1, 2,$$

where l_i is the leisure of spouse i , α_i is the leisure weight, and K is a public good produced in the household. The household public good production function is given by

$$\begin{aligned} K &= \tau_1^{\delta_1} \tau_2^{\delta_2} M^{\delta_3}, \\ \delta_1 + \delta_2 + \delta_3 &= 1 \text{ (CRS)} \end{aligned}$$

where τ_i is the time input of spouse i and M are total expenditures on a composite good purchased in the market which has price 1. Each spouse has a wage rate of w_i and both have a total time endowment of T . Thus

$$T = l_i + \tau_i + h_i, \quad i = 1, 2,$$

and

$$M = (y_1 + y_2) + w_1 h_1 + w_2 h_2,$$

where h_i is the labor supply of spouse i and y_i is the nonlabor income of spouse i . In responding to the following questions, assume the following values:

Parameter	Value
α_1	.3
α_2	.4
T	126
w_1	6
w_2	5
δ_1	.15
δ_2	.25
y_1	50
y_2	40

1. Find the noncooperative household equilibrium, characterized in terms of $(h_1, h_2, \tau_1, \tau_2)$. Is the equilibrium unique (answer this in general, i.e., for any set of permissible parameter values)? Is the equilibrium only a function of $Y = y_1 + y_2$, or does it depend on the nonlabor income distribution as well?
2. Assume that the household members are able to cooperate, and that the household equilibrium is determined using a Nash bargaining objective in which the spouses have equal “bargaining” power, with the outside option of each given by their value in the noncooperative equilibrium. Find the equilibrium time allocations in this case. Does the outcome depend on the distribution (y_1, y_2) , or only on the sum Y ?
3. Same as question 2, but with a different outside option. If parents cannot agree to cooperate, they divorce. Since the household production function requires separate male and female time inputs, if they divorce they must hire someone of the opposite sex to provide the missing time input. Let the price of the type 1 time input be 3 and the price of the type 2 time input be 4. First solve each spouse’s optimization problem when living alone. Using these values as the outside option, determine the Nash bargaining solution and compare to your solution to question 2. In particular, does the NB solution using the divorce state values as the outside option depend on the distribution (y_1, y_2) ?