Labor Economics
Spring 2004
Final Examination

Instructions: Please answer all of the following questions. In answering the questions be sure to show all of your work and clearly indicate your final response. While you have 48 hours to complete the exam, I am not looking for lengthy responses, but rather succinct, well-organized ones. If you feel that a question is ambiguous, state your interpretation of it before responding. If you are taking the exam at JHU, please turn in your answers to the Department Office by noon on Friday. Good luck.

1. An economy exists for six periods. In the first 5 periods of its existence, a new cohort enters. Cohort members live for two periods; i.e., those born in period 1 are the sole “inhabitants” of the economy in its first period of existence and those born in period 5 are the sole “inhabitants” of the economy in its last period of existence (period 6). The number of individuals born in each cohort $i$ is given by $n_i$, $i = 1, ..., 5$. The sequence of cohort sizes is perfectly known to all agents alive at any time during the economy’s existence (i.e., perfect foresight). When born, each cohort member has one unit of human capital. In her first period of life, a cohort member can increase her second period human capital stock by attending school. Given that her available time in each period is equal to 1, let $s$ denote the proportion of her first period of life that she spends in school, and assume that her second period level of human capital will be given by $2s$ (note that if she invests nothing in the first period she will have no human capital in the second). The objective of individuals born in period $t$ is to maximize

$$r_t h_t(1-s_t) + r_{t+1}2s_t,$$

where $s_t$ is the time in school spent by a member of cohort $t$.

The total amount of human capital in the market in period 1 is $H_1 = n_1(1-s_1)$, the total amount of human capital in period 6 is $H_6 = n_5(2s_5)$, and the total amount in all other periods is

$$H_t = n_t(1-s_t) + n_{t-1}2s_{t-1}, \quad t = 2, ..., 5.$$

(a) Assume that the rental rate on human capital in period $t$ is given by $r(H_t) = 2 - H_t$. Derive the schooling investment decision rule for individuals born in period $t$ as a function of the cohort size sequence $n_1, ..., n_5$.

(b) Assume that $n_1 = ... = n_5 = N$. If one exists, find a value of $N$ such that $s_1^* = .5$, that is, find a common cohort size such that individuals born in the first period will spend one-half of their first period of life in school. [You should not worry about the “units” problem, i.e., a value of $N$ not equal to an integer is perfectly valid.]
2. We discussed several search-theoretic, equilibrium models of job turnover, where the term “equilibrium” implies the wage distribution is endogenous. Some of these models allowed for Nash bargaining between workers and firms, while others were based on wage posting, that is, firms chose a wage offer to make to all potential employees they encountered and did not alter it. Construct two simple, continuous time models, one of wage posting and one of Nash bargaining that are similar in as many dimensions as possible. For example, in each assume unemployed individuals receive a flow utility of $b$, let the rate of arrival of wage offers to unemployed searchers be $\lambda_u$ and let the contact rate for employed searchers be $\lambda_e$, let the rate of exogenous termination of matches be given by $\eta$, etc.

(a) Compare the turnover decisions in your two models, and characterize steady state labor market dynamics at the individual level.

(b) Describe the implications of the models for steady state wage distributions. Which produces more “realistic” implications in your opinion?

(c) Discuss the sustainability of the two models. In particular, do firms in a wage posting equilibrium have an incentive to deviate from the constant wage they offer?

3. The following three questions concern static labor supply behavior in a household. The first part refers to the case in which there exists a household utility function (i.e., the “unitary” modeling framework), while the second asks you to employ a cooperative bargaining setup. The third asks for a comparison between the approaches.

(a) A household has the utility function given by

$$u(c, l_h, l_w) = \alpha_1 \ln(c) + \alpha_2 \ln(l_h) + (1 - \alpha_1 - \alpha_2) \ln(l_w),$$

where $c$ is total household consumption of a market good with price $p_c$, $l_h$ is the leisure of the husband, $l_w$ is the leisure of the wife, and the $\alpha$’s satisfy the usual restrictions. Each spouse has a time endowment of 1. Each spouse has a nonlabor income of $y_s = 1$, $s = h, w$, and each faces a wage offer $w_s$, $s = h, w$. Determine the labor supply policy of the household. Do labor market participation decisions exhibit a critical value property?
(b) Each household member has a utility function given by

\[ u_p(c_p, l_p, \theta) = \beta_{1,p} \ln(c_p) + \beta_{2,p} \ln(l_p) + (1 - \beta_{1,p} - \beta_{2,p}) \theta, \quad p = h, w, \]

and each individual has a nonlabor income level of \( y_s = 1 \), the cost of private consumption is the same for each individual and is equal to \( p_c \), \( \theta \) is a marriage-specific “match” value, and the \( \beta'_p \)'s satisfy the usual restrictions. The next best option (outside of the marriage) is living alone, in which case the match value is equal 0. Given that the spouses engage in cooperative behavior and that symmetric Nash bargaining is used to divide the surplus from the marriage, characterize labor supply decisions in this setting. As before, pay particular attention to labor market participation decisions.

(c) Which framework do you prefer from a theoretical perspective? Why? Say you were to estimate a static model of family labor supply. Which setup would you use (assuming that you have to choose between these two)? List any important differences in the two models in terms of empirical implications and data requirements. In particular, what types of data are required to estimate \( \alpha_1 \) and \( \alpha_2 \) (or distributions of these values in the population) under the unitary assumption. What types of data are required to estimate the \( \beta' \)'s (or the distribution of the \( \beta' \)'s assuming population heterogeneity) under the nonunitary setup?