

3. Two firms inhabit an industry. Firm A's production technology is given by $q_A = \min[2x, y]$ and firm B's production technology is given by $q_B = \min[x, 2y]$, where x and y are two factors of production. If the price of factor x is 2 and the price of factor y is 1, then firm B will be a monopolist under Bertrand competition.

4. A firm with a production function given by $q = 4x^5y^6$, where x and y are two factors of production, faces factor prices $\pi_x = 2$ and $\pi_y = 3$. Its average cost of production is increasing in q .

5. A monopolist sells its output in two geographically distinct markets. In market A the demand function for its output is given by $q_A = 20 - p_A$ and in market B the demand function for its output is given by $q_B = 20 - 2p_B$ [it charges two different prices and supplies different quantities to the two markets]. The firm's cost function is given by $C(q_A, q_B) = C(q_A + q_B)$. The price charged in market A will be lower than the price charged in market B .

6. Let two firms with 0 costs of production compete as Bertrand competitors in an output market. There is a proposal made with one that they act as a monopolist and simply split the monopoly profits. Such an arrangement can always be sustained since under the “noncooperative” alternative to collusion each firm earns 0 profits.

7. Let there exist N symmetric firms that share an industry, and assume that each firm has an average cost of production equal to 2. Let output decisions be determined in Cournot equilibrium and let the Cournot equilibrium price be given by $p^*(N)$. The equilibrium price is decreasing in the number of firms, and $p^*(N = \infty) = 2$.

Part II. Problems.

Answer each part of each of the following problems. Remember to show all of your work.

8. (14 points) 100 individuals live in a market and consume two goods x and y . The utility function of each individual is given by $U(x, y) = .25 \ln(x) + .75 \ln(y)$. Each individual faces the same prices of the two goods, p_x and p_y , and each has an income of $I = \exp(1)$, which is approximately equal to 2.718. A number of potential firms each with a total cost function given by $C(q) = \exp(q)$ exist that can enter the industry.

(a) Find the market demand function for the good x .

(b) Find the long run competitive equilibrium price for the good x .

(c) Find the number of firms in the industry in long run equilibrium..

9. (16 points) Two firms share an industry in which they compete as Bertrand competitors. The average cost of production is 2 for both firms, and the demand function for industry output is given by $Q = 100 - p$, where p is the lowest of the two prices charged by the firms.

(a) What is the equilibrium price and the total amount of output produced by the two firms? Can you determine how much each firm produces in equilibrium? Why or why not?

(b) One of the firms, let's say firm A , has the opportunity to purchase a new technology that will lower its marginal cost of production to 0. If firm A purchases the technology, what will the new equilibrium price be? How much output will be produced?

(c) If the technology costs 180, should firm A buy it?

(d) If both firms can purchase the technology for a price of 180 (not only firm A), will either or both buy it? Why?.

10. (16 points) Five firms “share” an industry; the demand for the industry’s output is given by $Q = 100 - p$, where p is the common price charged by all firms. Each firm has a total cost function given by $TC(q) = 50 + 10q + 2.5q^2$.

(a) Find the short-run supply function of each firm.

(b) Find the short-run competitive equilibrium price and output level of each firm.

(c) Does each firm earn profits in the short run? What are they?

(d) Does a long run competitive equilibrium exist in this market? If so, compare the long run equilibrium price with the short-run equilibrium price from part (b).

11. (12 points) Two dry cleaning establishments are located on a residential block which has length 1. Firm A is located $1/3$ units from the left end of the line while firm B is located on the far right end of the block [its “address” would be 1]. All individuals living on the block purchase exactly one unit of dry cleaning services from one of the two firms, and individual consumers are uniformly distributed along the block. If a consumer who lives a distance x from firm 1 goes to firm 1, he pays $p_1 + x$, where p_1 is the price charged by firm 1. Similarly, if the individual lives a distance y from firm 2, he pays a total price of $p_2 + y$ if he goes there. Given that each firm is a profit-maximizer and that the marginal cost of production is 0 for both, what are the equilibrium prices charged by the two firms?