Econometrics I

Fall 2000

Assignment 3

Today's Date: September 27
Due Date: October 3

Please show all of your work and clearly indicate your final response to each question.

1. If X and Y have joint probability density function (p.d.f.) given by

$$f_{X,Y}(x, y) = 2\chi[x \in [0, y]]\chi[y \in [0, 1]],$$

   (a) Find $COV(X, Y)$
   (b) Find the conditional p.d.f. of Y given $X = x$.

2. The joint p.d.f. of X and Y is given by

$$f_{X,Y}(x, y) = 3(x + y)\chi[x + y \in [0, 1]]\chi[x \in [0, 1]]\chi[y \in [0, 1]].$$

   (a) Find the marginal density of X.
   (b) Find $P(X + Y < .5)$
   (c) Find $E(Y|X = x)$
   (d) Find $COV(X, Y)$.

3. Let Y be a random variable having a Poisson distribution with parameter λ.Assume that the conditional distribution of X given $Y = y$ is binomial with parameters y and p. Find the (marginal) distribution of X, if $X = 0$ when $Y = 0$.

4. Find the correlation coefficient ρ in the following three settings.

   (a) Let X_1, X_2, and X_3 be uncorrelated random variables with common variance σ^2. Find the correlation coefficient between X_1+X_2 and $X_2 + X_3$.
   (b) Let X_1 and X_2 be uncorrelated random variables. Find the correlation coefficient between $X_1 + X_2$ and $X_2 - X_1$ in terms of $VAR(X_1)$ and $VAR(X_2)$.

(c) Let X_1, X_2, and X_3 be independently distributed random variables with common mean μ and common variance σ^2. Find the correlation coefficient between $X_2 - X_1$ and $X_3 - X_1$.

5. Let $X_1, ..., X_n$ be independently and identically distributed (i.i.d.) random variables with common p.d.f. given by

$$f_X(x) = x^{-2}\chi[x > 1].$$

Set $Y = \min[X_1, ..., X_n]$. Does $E(X_1)$ exist? If so, find it. Does $E(Y)$ exist? If so, find it.

6. If $X_1, ..., X_n$ are independently Poisson-distributed random variables, show that the conditional distribution of X_1 given the sum $X_1 + ... + X_n$ is binomial.

7. If X has a normal distribution with mean μ and variance σ^2, find the p.d.f., mean, and variance of the random variable $Y = \exp(X)$.

8. Let X have a uniform distribution on $(0, 1)$. Find the density of $Y = X^{-1}$. Does $E(Y)$ exist? If so, find it.

9. Let X_1 and X_2 be i.i.d. with common density

$$f_X(x) = \lambda \exp(-\lambda x)\chi[x > 0].$$

Let $Y_1 = X_1/X_2$ and $Y_2 = X_1 + X_2$. Find the joint density of Y_1, Y_2, and find the marginal densities of Y_1 and Y_2.