Introduction to Econometrics

Fall 2004

Assignment 9

Today's Date: 11/29/2004
Due Date: 12/6/2004

Please show all of your work and clearly indicate your final response to each question.

1. You want to estimate the following model (with a lagged dependent variable):

$$y_t = \alpha y_{t-1} + \varepsilon_t,$$

where $E(\varepsilon_t) = 0$ for all t. You have access to observations on y from time 0 through time T. Define the OLS estimator of α as

$$\hat{\alpha} = \frac{\sum_{t=1}^{T} y_t y_{t-1}}{\sum_{t=0}^{T-1} y_{t-1}^2}.$$

Determine whether $\hat{\alpha}$ is a consistent estimator under the following distributional assumptions on ε. In all cases you should assume that the ε process is stationary.

   1. ε is *i.i.d.* with variance σ_ε^2.
   2. ε is a 1st order moving average process, that is,

   $$\varepsilon_t = u_t + \delta u_{t-1},$$

   where u is *i.i.d.* with mean 0 and variance σ_u^2.
   3. The process ε is given by

   $$\varepsilon_t = \rho \varepsilon_{t-2} + u_t,$$

   where u is *i.i.d.* with mean 0 and variance σ_u^2.

2. You want to estimate the model

$$y_t = X_t \beta + \varepsilon_t, \ t = 1, 2, ...$$

where X_t is a purely exogenous stationary stochastic process (i.e., it is independent of ε_t at all leads and lags) and β is a conformable parameter vector that you wish to estimate. The ε_t are independently distributed over time, with the mean and variance of ε_t being 0 and at, respectively. The unknown parameter a is strictly positive. You have access to a sample of observations $\{y_s, X_s\}_{s=1}^T$, with T large enough to permit estimation of β but finite.

1. Is the OLS estimator of β, denoted by $\hat{\beta}$, unbiased? Is it consistent?
2. Say you regress $y_t/\sqrt{t}$ on $X_t/\sqrt{t}$. Call the OLS estimator in this case $\tilde{\beta}$. Is $\tilde{\beta}$ unbiased? Is it consistent?
3. Compare the estimators $\hat{\beta}$ and $\tilde{\beta}$. Which would you prefer and why?

3. W 11.3
4. W 11.7
5. W 12.1
6. W 12.3