Introduction to Econometrics

Fall 2004

Assignment 6

Today's Date: 11/2/2004
Due Date: 11/8/2004

Please show all of your work and clearly indicate your final response to each question.

1. (Review) A random variable Y is normally distributed with mean μ and variance σ_y^2 in a population. You have access to a random sample of size N from the population. Denote the sample draws by $\{y_1, ..., y_N\}$.

   1. Define the least squares estimator of μ as

      $$\tilde{\mu} = \arg\min_{\mu} \sum_{i=1}^{N} (y_i - \mu)^2.$$

      Show that this estimator of μ is unbiased. Derive the standard error associated with this estimator.

   2. Define the maximum likelihood estimator of μ, which you can denote by $\hat{\mu}$. How are the $\tilde{\mu}$ and $\hat{\mu}$ related?

   3. Add regressors to this model. Instead of every population member have a common mean value of Y, let the mean for individual i be given by $X_i\beta$, where X_i is a $1 \times K$ vector of observed characteristics and β is a $K \times 1$ (unknown parameter vector). Compare the least squares and maximum likelihood estimators of β in this case.

2. A dependent variable y has a uniform distribution on the interval $[0, a]$, so that the density (p.d.f.) of y is given by

   $$f(y) = \frac{1}{a}\chi[y \in [0, a]].$$

   You have access to a random sample of 5 draws from this distribution, and your goal is to estimate the unknown parameter a. The observations from the random sample are $\{2, 1, 5, 3, 8\}$.

   1. Derive the maximum likelihood estimator of a, which we will denote by $\hat{a}$. What is the maximum likelihood estimate of a from this sample?

2. Consider an alternative estimator for a, which is

$$\tilde{a} = 2\overline{y}_N,$$

where $\overline{y}_N$ is the sample mean based on N observations. Find the value of $\tilde{a}$ from this sample.

3. Determine whether $\tilde{a}$ and $\hat{a}$ are unbiased. Based on this criterion alone, which one would you favor?

4. Assuming that the variance of $\hat{a}$ is a decreasing function of sample size (which it is), are both estimators consistent?

3. Let the difference in utility between choosing an action a and not choosing it to individual i be given by

$$U_i^* = \beta x_i + \varepsilon_i,$$

where the random variable ε_i is uniformly distributed on the interval [-.5,.5] for all individuals i, x_i denotes an observable characteristic individual i, and β is an unknown parameter. The individual chooses the action a if the net utility from doing so is positive, that is,

$$d_i = \begin{cases} 1 & \text{if and only if} \quad U_i^* > 0 \\ 0 & \text{if and only if} \quad U_i^* \leq 0 \end{cases}.$$

1. Derive the probability that individual i with characteristics x_i will choose action a.

2. Say that you have access to a random sample of observations on $\{d_i, x_i\}_{i=1}^N$. Derive the maximum likelihood estimator of β.