

Introduction to Econometrics

Fall 2004

Assignment 3

Today's Date: 9/27/2004

Due Date: 10/4/2004

Please show all of your work and clearly indicate your final response to each question.

1. You want to estimate the following bivariate linear regression

$$\ln w = \beta_0 + \beta_1 s + \varepsilon,$$

where $\ln w$ is the logarithm of the wage rate and s is a variable equal to 1 if the individual has completed more than high school and equals 0 if not. The random variable ε is independently and identically distributed in the population with mean 0 and variance σ_ε^2 . You have access to a sample of size 200 drawn randomly from the population.

Define

$$X = \begin{bmatrix} 1 & s_1 \\ 1 & s_2 \\ \vdots & \vdots \\ 1 & s_{200} \end{bmatrix},$$

and

$$y = \begin{bmatrix} \ln w_1 \\ \ln w_2 \\ \vdots \\ \ln w_{200} \end{bmatrix}.$$

In the sample you find that

$$X'X = \begin{bmatrix} 200 & 106 \\ 106 & 106 \end{bmatrix}$$

and

$$X'y = \begin{bmatrix} 475.25933 \\ 263.60782 \end{bmatrix}$$

1. With the information given to you, can you compute unbiased estimates of β_0 and β_1 ? If so, what are the estimates?

2. Do you have sufficient information to compute an unbiased estimate of σ_ε^2 ?
If so, what is it?
3. Say that you reformulate the regression model so that

$$\ln w = \delta_1(1 - s) + \delta_2s + u.$$

Can you find unbiased estimates of δ_1 and δ_2 that are simple linear functions of your estimates of β_0 and β_1 ? If so, what are they?

4. We can define estimated residuals based on estimates of β and δ in both specifications of the model, which we will denote by r_1 and r_2 . Is $\sum_{i=1}^{200} r_{1,i} = 0$?

$$\text{Is } \sum_{i=1}^{200} r_{2,i} = 0?$$

2. W 3.1
3. W 3.6
4. W 3.7
5. W 3.10